

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE  
NPTEL  
NPTEL ONLINE CERTIFICATION COURSE  
Business Analytics & Data Mining Modeling  
Using R – Part II  
Lecture-20  
Time Series Forecasting – Smoothing Methods  
Part II  
With  
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**Business Analytics & Data Mining Modeling  
Using R - Part II**

Lecture-20  
Time Series Forecasting-Smoothing Methods  
Part II



Welcome to the course Business Analytics and Data Mining Modeling Using R – Part 2, so in previous lecture we started our discussion on smoothing methods, so we'll continue from there and this is the last lecture of this particular course, the part 2 Business Analytics and Data Mining Modeling Using R Part 2, so we would like to complete this particular topic in this lecture itself, so let's start.

## Smoothing Methods

- Based on
  - Averaging over multiple observations
    - Idea is to smooth out the noise to uncover the patterns
  - Data driven
    - No pre-determined structure is imposed on data
    - Time series components are estimated directly from the data
  - Suitable when time series components change over time

So let's do a quick recap of what we have discussed in previous lecture, so we talked about smoothing methods and we said that these methods are typically based on averaging over multiple observation, so main idea is to smooth out the noise to uncover the patterns, so the patterns could be you know the time series components like trend, seasonality, level, so these

are the three components we would like to smooth out the noise part, and uncover this patterns, it's a data driven so we don't impose any pre-determined structure, so these things we were able to discuss in the previous lecture.

## Smoothing Methods

- Typically smoothing methods differ by
  - No. of observations used for averaging
    - Length of the considered time series history
  - Formula used to perform averaging
  - Frequency of averaging
  - And so on
- Popular Smoothing Methods
  - Moving average (MA) methods
  - Exponential smoothing (ES) methods



We talked about how different smoothing methods differ from one and another, so few points also we discussed, then we talked about two important, two popular smoothing methods moving average methods and exponential smoothing methods.

## Smoothing Methods

- Moving Average
  - Simple smoother
  - Averaging across a user-specified window of consecutive observations
    - Generates a series of averages
  - Two types
    - Centered moving average
      - Used for time series analysis, since consecutive past and future values of a time point are used for averaging
    - Trailing moving average
      - Used for time series forecasting, since most recent values of a time point are used for averaging

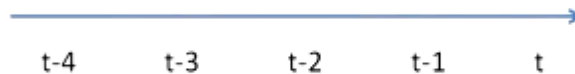


So moving average we talked about that averaging across a user specified window of consecutive observations, so you know typically like we discussed in the previous lecture, typically moving average methods they don't you know make a distinction between you know

different time series components like level, you know trend and seasonality, so therefore the performance of you know moving average methods in specially if all three components are present, so you know that might not be always competitive in comparison to other you know methods, so how do we overcome this? Are there any other methods available to overcome you know this limitation of moving average methods, so we'll discuss you know that part also in this lecture.

## Smoothing Methods

- Selecting window width ( $w$ )
  - For a time series having seasonality component
    - Length of the seasonal cycle could be a choice for  $w$ 
      - Idea is to suppress seasonality and noise to uncover trend
- Trailing Moving Average
  - For example, a trailing window with width,  $w=5$  can be depicted as



## Smoothing Methods

- Trailing Moving Average
  - Value of one-step-ahead forecast ( $F_{t+1}$ ) at time  $t$  is computed as below

$$F_{t+1} = (Y_t + Y_{t-1} + \dots + Y_{t-(w-1)})/w$$

Where  $w$  is window width

- Also,  $k$ -step-ahead forecast:

$$F_{t+k} = F_{t+1}$$

- Open RStudio



So centered moving average, so we have discussed this in previous lecture, trailing moving average also we covered, so then we talked about in the previous lecture this is where we ended

## Smoothing Methods

- MA methods
  - Suitable for forecasting series with no trend or seasonality
  - If seasonality is present
    - Under-forecasting the peak seasons and over-forecasting the non-peak seasons
  - If trend is present
    - Forecasts lag behind
      - Under-forecasting if increasing trend and over-forecasting if decreasing trend
- Solution
  - De-trending and de-seasonalizing



that moving average methods they are suitable for forecasting series with no trend or seasonality, this is because you know typically these methods because this is based on averaging of multiple you know consecutive observations, so typically you know these methods they don't make any distinction between you know the time series components, level, trend or seasonality, and if trend or seasonality are present then you know because of the averaging that is happening you know because of the seasonality as we talked about there are going to be under forecasting for peak season, there is going to be over forecasting for non-peak season, so therefore time series where seasonality is present you know this particular methods might not be, might not work as you know well, in comparison to other techniques.

Similarly if trend is present then also, so the solution is use you know de-trended and de-seasonalized time series, therefore just use these methods to forecast the level component of the time series where only level and noise is present, so noise is going to be averaged out because of the characteristic of these methods, because we take averaging of multiple observations, so noise is going to be smoothed out, and therefore we would you know essentially we forecasting the level component given that you know time series has been de-trended and de-seasonalized or when the time series doesn't have de-trend or seasonal component, so we'll continue our discussion from this point.

## Smoothing Methods

- Regression models can be used to produce
  - De-trended and de-seasonalized series
- Open RStudio
- Selecting window width ( $w$ )
  - User-specified
  - Balance between under-smoothing vs. over-smoothing
  - In case of centered MA (for time series analysis)
    - To uncover global trend, use high value of  $w$
    - To uncover local trend, use low value of  $w$



So how do we you know do this, so regression based model just like we have discussed in the previous topic in this module where we talked about regression based forecasting, so you remember that the, you would remember that in the bicycle riders dataset that we had used, finally we were able to you know model the seasonality and trend part, so we were able to you know find that it was the quadratic trend that was there and the month, you know annual seasonality was you know present there, so we are able to model that in the regression based forecasting seasonality and trend both the components, so therefore the residual series that we had was actually de-trended and de-seasonalized series, so you know why not use that series and you know apply smoothing methods to you know improve the forecast on the level component, specifically short term forecast on the you know level component, so that is something that can be done, so let's open R studio and we'll do an exercise and do this.

```

65
66 resultstest=data.frame(Month=format(fulldf$Month.Year, "%b-%y")[148:159],
67 Actual=tsvtest, Forecast=MAforecasterstest,
68 Error=tsvtest-MAforecasterstest); resultstest
69
70 # Test set performance
71 mmetric(tsvtest, MAforecasterstest, c("MAPE", "MAE", "MSE"))
72
73 #####
74 # Produce de-trended and de-seasonalized series using regression model
75 Season=format(fulldf$Month.Year, "%b")
76 nc=length(fulldf$Month.Year)
77 t=seq(1, nc, by=1)
78 tsq=t^2
79 df=cbind(fulldf, data.frame(t,tsq,Season))
80 head(df)
7520 [Tap level]

```

Environment History Connections

Global Environment

Data

- fulldf 159 obs. of 2 variables
- values Time-Series [1:159] from 200..

Files Plots Packages Help Viewer

```

C:/Users/T Cell/Desktop/MOOC July 2018/Dr. Gaurav Dixit/Session 5/
> # bicycleRidership.xlsx
> fulldf=read.xlsx(file.choose(), 1, header = T)
> fulldf=fulldf[, !apply(is.na(fulldf), 2, all)]
> str(fulldf)
'data.frame': 159 obs. of 2 variables:
 $ Month.year: Date, format: "2004-01-01" "2004-02-01" ...
 $ Riders : num 3710 3626 3975 3815 3976 ...
> tsv=ts(fulldf$Riders, start=c(2004, 1), frequency=12)
>

```

So load this library XLSX, let's import the dataset, you can see data has been imported as you can see in the environment section this is the structure of the dataset we are already familiar with this, so I have just created the you know time series vector for this, so we'll just move on, centered moving average and you know and the trailing moving average this part we have already done in the previous lecture, so we'll just skip this, and we'll come to this part where we will talk about the you know produce de-trended and de-seasonalized series using regression model, so we'll first as we did in the you know in this regression based forecasting methods

```

71 mmetric(tsvtest, MAforecasterstest, c("MAPE", "MAE", "MSE"))
72
73 #####
74 # Produce de-trended and de-seasonalized series using regression model
75 Season=format(fulldf$Month.Year, "%b")
76 nc=length(fulldf$Month.Year)
77 t=seq(1, nc, by=1)
78 tsq=t^2
79 df=cbind(fulldf, data.frame(t,tsq,Season))
80 head(df)
81
82 # Trimming
83 dftrain=df[1:147,]
84 dftest=df[148:159,]
85
821 [Tap level]

```

Environment History Connections

Global Environment

Data

- df 159 obs. of 5 variables
- fulldf 159 obs. of 2 variables
- values
- nc 159L
- Season the (1-159) "Jan" "Feb" "Mar"

Files Plots Packages Help Viewer

```

> df=cbind(fulldf, data.frame(t,tsq,Season))
> head(df)
  Month.year Riders t tsq Season
1 2004-01-01  3710 1  1  Jan
2 2004-02-01  3626 2  4  Feb
3 2004-03-01  3975 3  9  Mar
4 2004-04-01  3815 4 16  Apr
5 2004-05-01  3976 5 25  May
6 2004-06-01  3868 6 36  Jun
>

```

you know topic, those particular lectures so we need to first get the season variable, right as you can see here, then length of this you know, this particular you know dataset, then T, T square will get a data frame out of all this variables, so this is the data set that we would be taking forward for regression based forecasting, so this is something that we have done, so we are

doing this regression based forecasting to produce a de-trended and de-seasonalized series, so residual series of this regression based model is going to be the de-trended and de-seasonalized series and then we would be using this particular residual series to apply moving average you know model.

```

74 # produce de-trended and de-seasonalized series using regression model
75 Season=Format(fulldf$Month.Year, "%b")
76 nc=length(fulldf$Month.Year)
77 t=seq(1, nc, by=1)
78 tsq=t*t
79 df=cbind(fulldf, data.frame(t,tsq,Season))
80 head(df)
81
82 # Trimming
83 dftrain=df[1:147,]
84 dftest=df[148:159,]
85
86 mod=lm(Riders~t+tsq+Season, dftrain[, -c(1)])
87
88 restsv=ts(mod$residuals, start=c(2004, 1), frequency=12)
89
90 |
91 (Top level)
R Script

```

Environment History Connections

Global Environment

Data

- df 159 obs. of 5 variables
- dftest 12 obs. of 5 variables
- dftrain 147 obs. of 5 variables
- fulldf 159 obs. of 2 variables

Files Plots Packages Help Viewer

```

C:/Users/T Cell/Desktop/MOOC July 2018/Dr. Gaurav Dixit/Session 5/
1 2004-01-01 3710 1 1 Jan
2 2004-02-01 3626 2 4 Feb
3 2004-03-01 3975 3 9 Mar
4 2004-04-01 3815 4 16 Apr
5 2004-05-01 3976 5 25 May
6 2004-06-01 3868 6 36 Jun
> # Trimming
> dftrain=df[1:147,]
> dftest=df[148:159,]
>

```

So let's do the trimming, so as you can see like the you know previous lectures we can create this model riders versus TT square, so we are fitting quadratic model plus seasonality, so let's run this, so we have got the model so from this you can see we are now going to create a time series vector you know from the residuals, so you can see mod\$residuals so this is we are going to use this particular vector and create a you know time series vector for residual series, so let's have a look at the plot, so this is the plot for residuals, so as you can see like we did in

```

90 readLDT
91
92 # Trimming
93 dftrain=df[1:147,]
94 dftest=df[148:159,]
95
96 mod=lm(Riders~t+tsq+Season, dftrain[, -c(1)])
97
98 restsv=ts(mod$residuals, start=c(2004, 1), frequency=12)
99
100 plot(restsv, xlab="", ylab="Residuals", las=2)
101
102 # Trailing moving average model on residual series
103 restrailingMA=filter(restsv, rep(1/12, 12), sides = 1)
104
105 resMAforecaster=ts(data=NA, start=c(2004, 1), end=c(2016, 3), frequency=12)
106
107 |
108 (Top level)
R Script

```

Environment History Connections

Global Environment

- dftrain 147 obs. of 5 variables
- fulldf 159 obs. of 2 variables
- mod List of 13

Values

- nc 159L
- restsv Time-Series [1:147] from 200..

Files Plots Packages Help Viewer

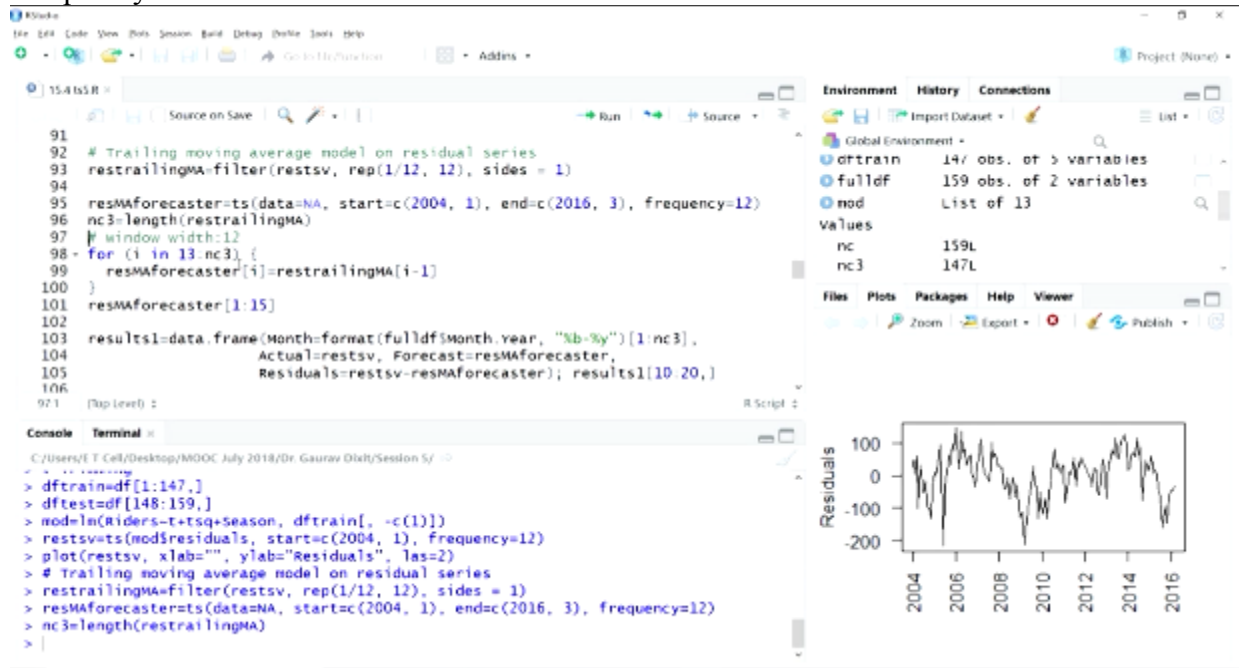
```

C:/Users/T Cell/Desktop/MOOC July 2018/Dr. Gaurav Dixit/Session 5/
4 2004-04-01 3815 4 16 Apr
5 2004-05-01 3976 5 25 May
6 2004-06-01 3868 6 36 Jun
> # Trimming
> dftrain=df[1:147,]
> dftest=df[148:159,]
> mod=lm(Riders~t+tsq+Season, dftrain[, -c(1)])
> restsv=ts(mod$residuals, start=c(2004, 1), frequency=12)
> plot(restsv, xlab="", ylab="Residuals", las=2)
>

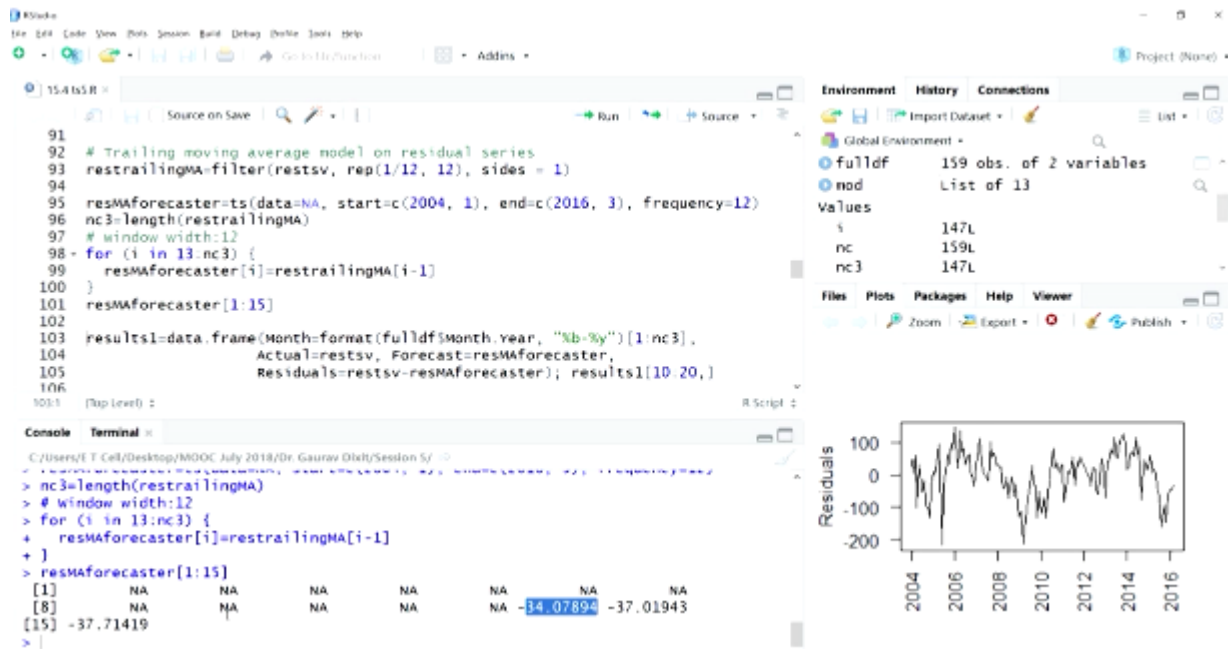
```



the previous lectures on regression based forecasting, the trend component and the seasonality component has been adequately modeled which is also clearly visible in the you know in this plot also, and we had also seen in the ACF plot also in the previous lectures that you know there is, there was nothing you know outside the bounds and therefore those components were adequately modeled.



Now can we use moving average methods to improve the forecast even further, so this is something that we are going to attempt now, so we are going to apply as you can see here trailing moving average model on this residual series, so filter is the function like we use in the previous lecture, and we are going to apply this on this residual series as you can see here, so let's run this code, so we get this values, this factor, and then let's apply this model, first we will initialize another time series, take the length of the you know the computation that we have just done and we are taking this window width as 12 we are taking that you know the series has annual seasonality, and with this we'll use this window and do our forecast, so this forecast is like we did in the previous lecture using the trailing moving average model and so we are going to take average of 12 values, 12 consecutive values and going to assign that as forecast for in different points in the training set.



So if you are interested in looking at the forecast you can see that forecast is available from this point onwards that is 8, 9, 10, 11, 12, 13 points onwards, another forecast is 12 points are required for averaging, so this is the forecast that we have, then we can also have a look at this particular data frame to get the idea about how this trailing model is been used, you can see this is the thirteenth observation from where we can, starting where you know our moving average model has been applied, so you can see this is the actual and then the you know the forecast and the residual, so this is what we have done, so we have applied trailing moving average model on this residual series and we have got our forecast, so hopefully this forecast is going to further improve the performance of model, so essentially what we have done we have applied regression based forecasting, got the residual series adequately model the trend and you know seasonal components and the residual series, then we are applying moving average you know method, so that any short term forecast you know that could be improved further, so level component essentially we are working on the level component and trying to improve the short term forecast.

The screenshot displays the RStudio interface. The script editor contains the following R code:

```

99 resMAforecaster[i]=restrailingMA[i-1]
100 }
101 resMAforecaster[1:15]
102
103 results1=data.frame(Month=format(fulldf$Month.Year, "%b-%y")[1:nc3],
104                    Actual=restsv, Forecast=resMAforecaster,
105                    Residuals=restsv-resMAforecaster); results1[10:20,]
106
107 # Training set performance
108 mmetric(restsv[13:147], resMAforecaster[13:147], c("MAPE", "Mdae", "MSE"))
109
110 # Forecast for April 2016
111 resMAforecstertest=restrailingMA[147]; resMAforecstertest
112
113 ### Simple exponential smoothing
114
115 (Tap level) :

```

The console shows the following output and error:

```

C:/Users/T T Cell/Desktop/MOOC July 2018/Dr. Gaurav Dixit/Session 5/
> # Training set performance
> mmetric(restsv[13:147], resMAforecaster[13:147], c("MAPE", "Mdae", "MSE"))
Error in mmetric(restsv[13:147], resMAforecaster[13:147], c("MAPE", "Mdae", :
could not find function "mmetric"
> library(rminer)
> # Training set performance
> mmetric(restsv[13:147], resMAforecaster[13:147], c("MAPE", "Mdae", "MSE"))
      MAPE      Mdae      MSE
287.40634  44.96552 4114.21214
>

```

The Environment pane shows the following objects:

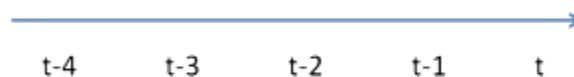
- Global Environment
  - fulldf: 159 obs. of 2 variables
  - nod: List of 13
  - results1: 147 obs. of 4 variables
- Values
  - Gctorture: FALSE
  - i: 147L

The Plots pane shows a line plot of Residuals over time (Year). The y-axis ranges from -200 to 100, and the x-axis shows years from 2004 to 2016. The plot shows a fluctuating line representing the residuals of the model.

So we can have a look at the performance of this model on training set so you can see here, so for these observation for which we have the forecast available, we can compute the performance, so first we need to you know load this library K miner, R miner, so we are going to do this, so once this, so once this library is installed so this is loaded, so let's compute this, so these are the values as you can see here, so we can use these values to compare the performance how, with the performance that we had using the regression based forecasting and how it has been improved, so you would see that if we you know compare these numbers with the, numbers that we have just, when we just use the regression based forecasting then whether moving average method has been able to improve this performance, right, so then you know for the you know next point if we want to forecast the next point that is April 2016, so if we want to create a forecast for that so this is how we can do it, so this value, this vector, this value is going to be the forecast for this point, first point is, this point is the first point in the test dataset, so this is the forecast.

## Smoothing Methods

- Selecting window width ( $w$ )
  - For a time series having seasonality component
    - Length of the seasonal cycle could be a choice for  $w$ 
      - Idea is to suppress seasonality and noise to uncover trend
- Trailing Moving Average
  - For example, a trailing window with width,  $w=5$  can be depicted as



## Smoothing Methods

- Regression models can be used to produce
  - De-trended and de-seasonalized series
- Open RStudio
- Selecting window width ( $w$ )
  - User-specified
  - Balance between under-smoothing vs. over-smoothing
  - In case of centered MA (for time series analysis)
    - To uncover global trend, use high value of  $w$
    - To uncover local trend, use low value of  $w$



Now let's go back to our discussion, now how do we select window width, so few things that we have discussed in the previous lecture also, so let's revisit, so this was the point that we discussed so far a time series having seasonality component, length of you know seasonal cycle could be a choice for  $W$  that is something that we discussed, the main idea being that we can suppress the seasonality and noise, and we would be able to uncover the trend, so this is mainly you know kind of rule of thumb for a time series having the seasonality component. Few other aspect about how do we go about selecting the window width  $W$ , so few points have been discussed so as we have you know discussed before also  $W$  is something that is user specified, then while you know selecting a particular value for  $W$  we have to look for the balance between

under smoothing versus over smoothing, so we want you know, we don't want to average out you know everything otherwise the average series won't have you know enough information for us to make forecast, however you know we don't want to under smooth you know the series, so you know we don't want to under smooth the series so that the noises are still present and the forecast performance is quite bad, so the balance between under smoothing versus over smoothing that has to be done.

So specifically we talked about the centered MA that is typically used for time series analysis to uncover global trend we can use high value of  $W$ , because global trend that is typically going to be applicable in the longer term, so therefore if we keep the higher value of  $W$ 's and the averaging out will happen in more number of observations, and therefore the global trend that is visible, more chances that we would be able to uncover that, if we want to uncover the local trend then probably we would be you know better with having using the low value of  $W$ , because the you know local aspect of local trend that is present in that particular part of the series, that would be you know easier to uncover.

## Smoothing Methods

- **Selecting window width ( $w$ )**
  - In case of trailing MA (for time series forecasting)
    - Depends on the relevance of recent values
    - Rate of change of series
    - Experiment with different values of  $w$  and compare performance of candidate models on validation set
      - Avoid overfitting
- **Simple Exponential Smoothing**
  - Idea is to give higher weightage to recent values, while retaining older information with lower weightage



Similarly if we talk about the trailing moving average and which is typically used for time series forecasting, so we have to look at the you know the relevance of recent values, so how many you know, what is the relevancy of the historical values that are there, whether they are going to help us in terms of improving the forecast, and how the series itself has been changing, so the rate of change of series and the relevance of the recent values, the past values of the series, so that we have to look at in terms of you know for deciding the window width  $W$ , so we can do experiment with different values of  $W$  and compare performance of candidate models on validation sets, so something like we did in the previous course, the first part where for example in  $K$  you know we followed a mechanism where we you know compared the performance of different candidate models on validation set to find out the best  $K$  for the model, similarly here also we can do the same thing and you know try out different values of  $W$  and compare the performance of different candidate models on validation set and get the best  $W$  for us, however in such a situation if we follow that mechanism we should keep in mind that it might lead to

over fitting, so we have to guard for this over fitting scenario as well, if we follow this approach.

Now let's move to our next you know, next technique that is simple exponential smoothing, so let's discuss how this particular technique, simple exponential smoothing is different from the moving average methods, so the idea is in this particular technique the idea is to give higher weight is to recent values while retaining older information with lower weightage, so if we look at the moving average what we have been doing we have been selecting the number of observations and taking average, so we did not make a distinction between you know which values are going to be more you know useful in forecasting and which are going to be less useful, however in this case we take a weighted average in the sense and we give more weightage through the you know more, you know recent values and less weightage to the older values, so the exponential smoothing you know methods, they typically implement the weighted average formulas.

## Smoothing Methods

- Simple Exponential Smoothing
  - Weightage average of all past values is taken and weights decrease exponentially into the past
  - Suitable for forecasting series with no trend or seasonality
  - Value of one-step-ahead forecast ( $F_{t+1}$ ) at time t is computed as below

$$F_{t+1} = \alpha Y_t + \alpha(1 - \alpha)Y_{t-1} + \alpha(1 - \alpha)^2 Y_{t-2} + \dots$$

Where  $\alpha$  is the smoothing parameter and takes values between 0 and 1

So you can see here weightage average of all past values is taken and weights decrease exponentially in to the past. Suitable for forecasting series with no trend or seasonality, it's just like for the moving average method we talked about smoothing methods in general you know specifically you know simple moving average methods or simple exponential smoothing method, you know they don't make distinction as we talked about distinction in terms of different time series components, level, trend and seasonality, so therefore you know if the trend and seasonality are not present or the series has been de-trended or de-seasonalized, so these you know methods, so these smoothing methods can be helpful in terms of improving the short term forecast.

So let's have a look at the expression for simple exponential smoothing, so value of one step ahead forecast  $F_{T+1}$  at time T can be computed using this expression can be written in this form, so  $F_{T+1}$  is  $\alpha Y_T + \alpha(1 - \alpha)Y_{T-1} + \alpha(1 - \alpha)^2 Y_{T-2}$ , so you can see here that starting from the first term, then the second terms the weights are continuously decreasing giving that  $\alpha$  is a smoothing parameter and takes the value between 0 and 1, so because of the value that it takes between 0 and 1, so the second term

onwards the weight if you just look at the weight from alpha to alpha times you know 1-alpha and so on, the weight will continuously decrease so therefore we are giving more weightage to the you know higher weightage to the, you know recent values that is Y<sub>T</sub>, Y<sub>T-1</sub>, and lower weightage to values which will come later that is Y<sub>T-3</sub>, Y<sub>T-4</sub>, and so on.

## Smoothing Methods

- Simple Exponential Smoothing

- F<sub>t+1</sub> can be rewritten as

$$F_{t+1} = \alpha Y_t + (1 - \alpha)[\alpha Y_{t-1} + \alpha(1 - \alpha)Y_{t-2} + \dots]$$

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

$$F_{t+1} = F_t + \alpha E_t$$

Where E<sub>t</sub> is the forecast error at time t

- Also, k-step-ahead forecast:

$$F_{t+k} = F_{t+1}$$

So this same expression can be rewritten in this form, so F<sub>T+1</sub> can be rewritten as this as you can see alpha Y<sub>T+1</sub> – alpha times and we have taken this factor as common 1-alpha and in the brackets we get this expression and this expression can now be rewritten as F<sub>T</sub> as you can see here, so this we can see here, this can be rewritten as F<sub>T</sub> here, and this is how now we can again rewrite this expression you can see here F<sub>T+1</sub> F<sub>T</sub>, now we are taking alpha as common, now if we look at, looking the parenthesis Y<sub>T</sub>-F<sub>T</sub> so this is actually nothing but actual value minus the forecasted value, so this is the error part, so the same thing we have written, this is the forecast error at time T, so in a sense F<sub>T+1</sub> can be rewritten as combination of F<sub>T</sub> + alpha times E<sub>T</sub>, so where E<sub>T</sub> is the error, so if we look at it the F<sub>T+1</sub> the forecasted value is actually improvement on the previous value, so value at T+1, the forecasted value for T+1 is actually improvement on the forecasted value at T, you know plus this error, so alpha being the you know smoothing parameter and that, and this alpha decides the rate of learning, so this learning is coming from this E<sub>T</sub> that ways you know that is component and how much learning is to be done, to be able to make the forecast, so we can see from here.

Also if we can see that, if we can see here K step ahead forecast F<sub>T+K</sub> is nothing but you know F<sub>T+1</sub>, reason being we don't have any further information to improve this forecast, right, so because of this all you know two step ahead, three step ahead, four step ahead, all those forecast are essentially going to be the same as one step ahead forecast, if we use this expression, simple exponential smoothing equation.

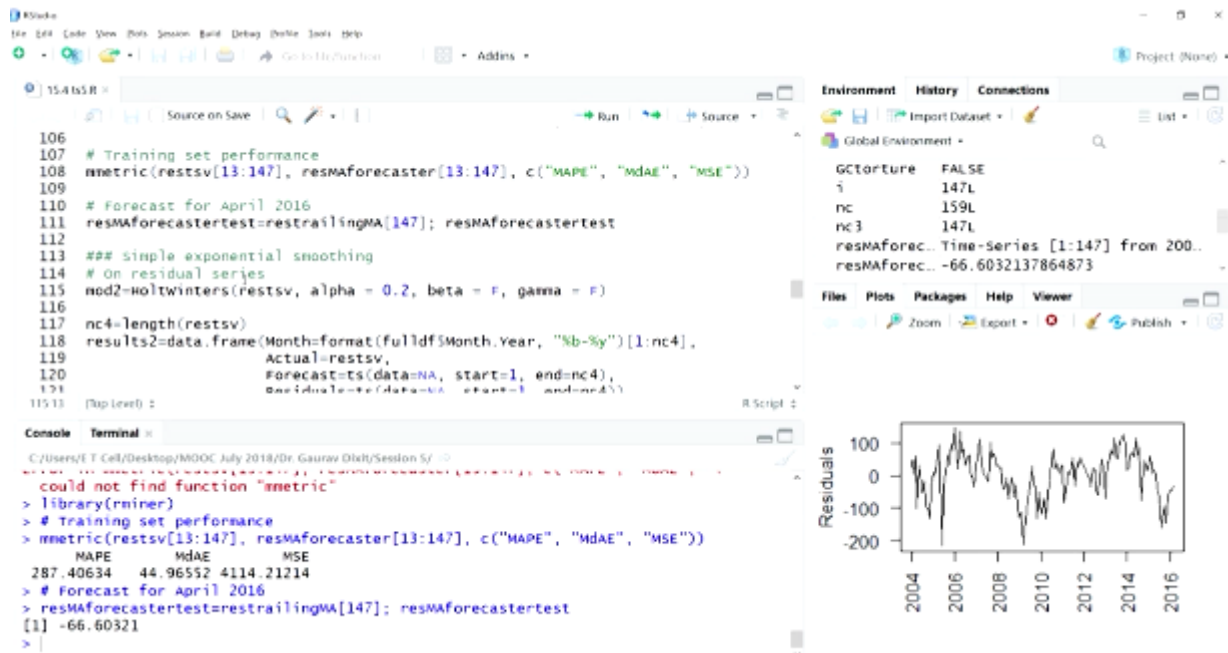
## Smoothing Methods

- Selecting smoothing parameter ( $\alpha$ )
  - Determines the rate of learning
    - Value near 1: fast learning
    - Value near 0: slow learning
  - Depends on relevance of past values and required amount of smoothing
  - Experiment with different values of  $\alpha$  and compare performance of candidate models on validation set
    - Avoid overfitting
  - Default values: 0.1-0.2



Now let's discuss this parameter you know alpha, smoothing parameter alpha and how do we go about selecting the value for alpha, so as we have talked about determines the rate of learning, so is being used for the you know to compute the weightage you know weighted average also, and in a sense that also decides, so if the value is near one then of course the fast learning is going to take place, because we would be learning you know more from the you know this value, this fast values,  $Y_T, Y_{T-1}$ , if the alpha value is close to 1. If the alpha value is close to 0 then the learning is going to be a slow and we are going to learn a slowly from the past values, so depending on the you know scenario, depending on the relevance of past values and the amount of smoothing that is required we can decide on a value of alpha, and of course just like the, just like for the moving average method and the  $W$  that we talked about, we can also do experiment with different values of alpha and compare the performance of candidate models on validation set, and find out the best alpha for our problem, however as we said for you know window width here also in this approach also we have to avoid the over fitting.





If we look at the default values typically you know point 1 value between point 1 and point 2 is considered good for learning in most of the cases, so what we are going to do we'll open R studio and do an exercise for simplest exponential smoothing, so you can see here so we are going to use the same residual series that we have computed in this lecture, so now we are going to use this Holt Winters formula, so this Holt Winters function is actually gives us the flexibility of applying the advanced exponential smoothing, however right now we are going to just use it for simple exponential smoothing and not for the advanced exponential smoothing, so to be able to use this function for simple exponential smoothing we need to make these two arguments beta and gamma as false, what these arguments are about that something we are going to discuss in this lecture itself, right now let's focus on this alpha which is the learning parameter, so this is the residual series restsv and the first argument, and the second argument we are giving the rate of this learning as 0.2 the smoothing parameter is this one, this much, so let's run this code and we'll get the model for our simple exponential smoothing.

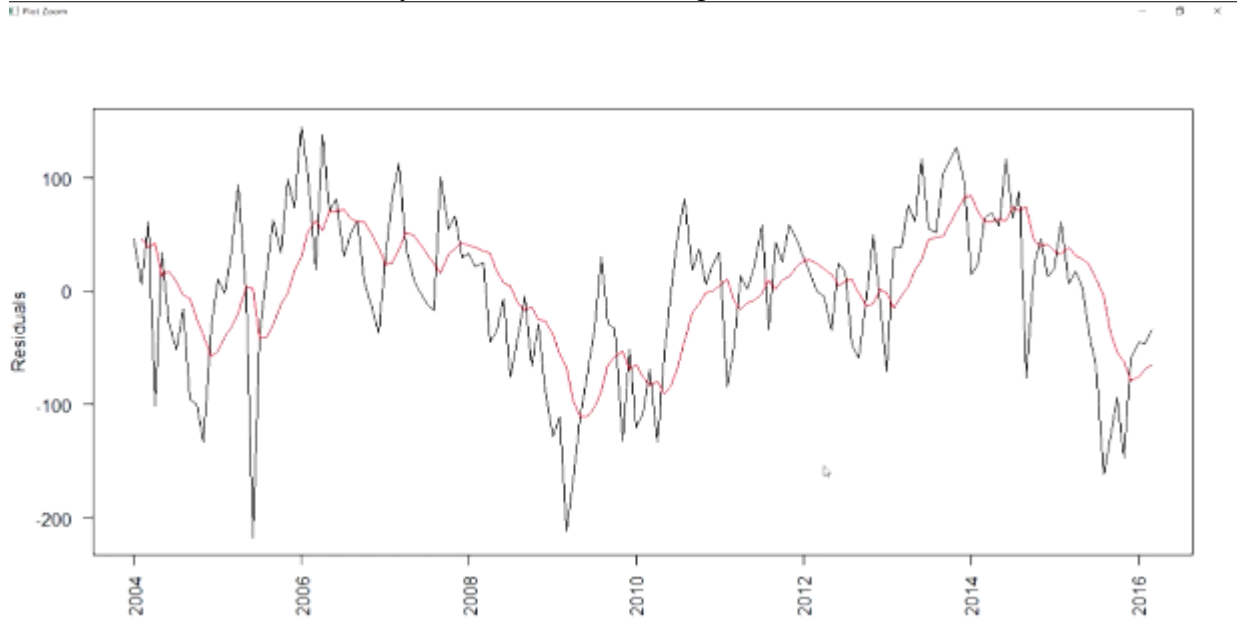
```

114 # UN RESIDUAL SERIES
115 mod2=holtwinters(restsv, alpha = 0.2, beta = F, gamma = F)
116
117 nc4=length(restsv)
118 results2=data.frame(Month=format(fulldf$Month.Year, "%b-%y")[1:nc4],
119                    Actual=restsv,
120                    Forecast=ts(data=NA, start=1, end=nc4),
121                    Residuals=ts(data=NA, start=1, end=nc4))
122 results2[2:147,3]=mod2$fitted[, "xhat"]
123 results2[2:147,4]=restsv[2:nc4]-mod2$fitted[, "xhat"]
124 results2[1:10,]
125
126 # Plot residual series and forecasted residual series
127 plot(restsv, xlab="", ylab="Residuals", las=2)
128 tindex=time(restsv)
129 points(tindex, results2$Forecast, type="l", col="red")
130
131 [Top level]

```

Month	Actual	Forecast	Residuals
1 Jan-04	46.172451	NA	NA
2 Feb-04	5.018508	46.172451	-40.15394
3 Mar-04	60.941487	38.141663	22.79982
4 Apr-04	-101.303369	42.701628	-144.00500
5 May-04	34.061926	13.900628	20.16130
6 Jun-04	-27.239445	17.932888	-45.17233
7 Jul-04	-52.374149	8.898421	-61.27257
8 Aug-04	-15.592187	-3.356093	-12.23609
9 Sep-04	-95.310775	-5.803317	-89.50641

So let's have a look at the forecast and residuals from this model, so certain computation to compute this table where we'll have, these are the actual values from the residual series, and we can see from second row onwards we have the forecast also and the residual part also, so this is based on applying simple you know exponential smoothing. Now if we want to plot this, how this you know model is performing so we'll let's plot the residual series, let's add the forecast, so this is the plot that we can see here, we can see red line is the forecasted points, and the actual residual series plot in black colour so we can see how this single exponential smoothing model how it has been able to you know forecast this particular series.



We can have a look at the performance, training set performance also here, so you can see that performances if we look at this parameter and second and third parameter with the provinces has slightly improved from the moving average method.

The screenshot shows the RStudio environment with the following components:

- Source Editor:** Contains R code for plotting residuals and forecasting. The code includes:
 

```

126 # Plot residual series and forecasted residual series
127 plot(restsv, xlab="", ylab="Residuals", las=2)
128 tindex=time(restsv)
129 points(tindex, results2$Forecast, type="l", col="red")
130
131 # Training set performance
132 mmetric(restsv[2:147], mod2$fitted[, "xhat"], c("MAPE", "Mdae", "MSE"))
133
134 # Forecast for April 2016
135 library(forecast)
136
137 resESforecasttest=forecast.holtwinters(mod2, h=1); resESforecasttest
138
139 ### Advanced exponential smoothing
140 # on raw (original) series
      
```
- Console:** Shows the execution of the code, including the output of the `mmetric` function:
 

```

> tindex=time(restsv)
> points(tindex, results2$Forecast, type="l", col="red")
> # Training set performance
> mmetric(restsv[2:147], mod2$fitted[, "xhat"], c("MAPE", "Mdae", "MSE"))
      MAPE      Mdae      MSE
1 280.36306  41.71738 3567.92019
      
```

 It also shows an error message: `Error in library(forecast) : there is no package called 'forecast'`.
- Environment:** Lists the loaded packages and objects, including `resMAforec`, `restrailin`, `restsv`, `Season`, `t`, and `tindex`.
- Plots:** A residual plot titled "Residuals" showing the residuals of the time series from 2004 to 2016. The y-axis ranges from -200 to 100. The plot shows a black line for the original data and a red line for the forecasted values, which are plotted as points connected by lines.

Similarly if we want to, if we want to forecast for this April 2016 point we can use this `forecast.holtwinters` function but for this we need to have this forecast package, so before you know we move ahead let's install this, let's install this package, so what we are going to do is we'll install this package so that we are able to use this library, so let's type `library(forecast)`, so this package actually helps us in making certain forecast using function `forecast.holtwinters` so we'll just install this and then load this library and use it for our forecasting. This is something that this particular is something that is we are going to use later on for the advanced exponential smoothing as well, so let's do this first, so let's again run it, it is seems to be some problem let's look at the key for this, so this as you can see the help section `forecast.holtwinters` and the

The screenshot shows the RStudio interface. The main editor contains an R script with the following code:

```

127 plot(resfv, xlab= , ylab= res1000is , las=2)
128 tindex=time(resfv)
129 points(tindex, results2$forecast, type="l", col="red")
130
131 # Training set performance
132 mmetric(resfv[2:147], mod2$fitted[, "xhat"], c("MAPE", "MAE", "MSE"))
133
134 # Forecast for April 2016
135 library(forecast)
136
137 resESforecasttest=forecast.HoltWinters(mod2, h=1); resESforecasttest
138
139 ### Advanced exponential smoothing
140 # on raw (original) series
141 tsvtrain=ts(dftrain$Riders, start=c(2004, 1), frequency=12)
142
143
144
145
146
147
148
149
150
151 [Tap level]

```

The console shows the following error messages:

```

C:/Users/T T Cell/Desktop/MOOC July 2018/Dr. Gaurav Dixit/Session 5/
> could not find function "forecast.HoltWinters"
> resESforecasttest=forecast.HoltWinters(mod2, h=1); resESforecasttest
Error in forecast.HoltWinters(mod2, h = 1) :
could not find function "forecast.HoltWinters"
> # Forecast for April 2016
> library(forecast)
> resESforecasttest=forecast.HoltWinters(mod2, h=1); resESforecasttest
Error in forecast.HoltWinters(mod2, h = 1) :
could not find function "forecast.HoltWinters"
>

```

The Environment pane on the right shows the following objects:

- resMAforec: -66.6032137864873
- restrailin...: Time-Series [1:147] from 200..
- resfv: Time-Series [1:147] from 200..
- Season: chr [1:159] "Jan" "Feb" "Mar" ..
- t: num [1:159] 1 2 3 4 5 6 7 8 ..
- tindex: Time-Series [1:147] from 200..

The Files pane shows the following files:

- forecast.HoltWinters (forecast)
- R Documentation

The Help pane shows the following content:

### Forecasting using Holt-Winters objects

Description

Returns forecasts and other information for univariate Holt-Winters time series models.

Usage

```
## S3 method for class "HoltWinters"
```

forecast package this is there, but we are getting some error here for this one, let's see whether the model 2 was created, so model 2 is there we are getting this error for some reason. Okay, so let's move forward, we'll get back to this problem and sorted it out and get back, let's move to our discussion on slides, so we'll discuss some more important aspects of smoothing methods, so let's first discuss moving average versus simple exponential smoothing, so if we look at you know the parameters that have being used in these two types of methods, so we use window width  $W$  for moving average and simple exponential smoothing you know, for simple exponential smoothing we use the smoothing parameter  $\alpha$ , so these are the two parameters which are specified by the user and in a sense they set the importance of recent values over old values.

## Smoothing Methods

- Open RStudio
- Moving average vs Simple Exponential Smoothing
  - Window width ( $w$ ) vs. smoothing parameter ( $\alpha$ )
    - Used to set the importance of recent values over old values
- Limitation of MA and ES methods
  - Suitable for forecasting series with no trend or seasonality
    - Series with only a level and noise
  - One solution is to either de-trend and de-seasonalize the series using regression models

Let's look at the limitation of these two methods MA and ES methods, so as we have talked about that these methods are suitable for forecasting series with no trend or seasonality, so that means series would be left with only a level and noise component, because these two methods as we have discussed typically take the average or weighted average of the consecutive values, past consecutive values so therefore they don't make a distinction, they don't try to identify the level, trend or seasonality component separately, so typically so you know they tend to just you know forecast the level, so they are useful for short term forecasting, however we have to make sure that trend or seasonality is not present, otherwise for you know the performance will take a hit there.

## Smoothing Methods

- Limitation of MA and ES methods
  - Second solution is to use advanced versions of exponential smoothing methods to capture trend and seasonality components
- Advanced exponential smoothing methods
  - For series with a trend (no seasonality), double exponential smoothing method can be used
  - Double exponential smoothing
    - In this method, we assume that trend can change over time (no global trend; focus is on local trend)
    - Local trend and level are estimated from data and updated dynamically

So as we have discussed one solution is to either de-trend and de-seasonalize the series using regression models something that we have done already, right, so second solution is to use advanced versions of exponential smoothing methods to capture trend and seasonality components, so there are some advanced versions of exponential smoothing method that are available which could be used to overcome this problem, so this is the second you know solution that we have, so that is something that we are going to discuss now, so this comes under the advanced exponential smoothing methods category, so we are going to discuss three scenarios here, first one where the series is having the trend component, but no seasonality component then a series is having both the component trend and seasonality, then a series is having you know just the seasonality component and no trend component, so we'll talk about the advanced exponential methods, so that could be used for this, so for a series with a trend component and no seasonality component we can use double exponential smoothing as we can see here, so in double exponential smoothing what we assume is that trend can change overtime, so therefore the idea is that there is no global trend so therefore this structural model, the regression based model probably they are not going to be suitable in this scenario, the focus is on local trend so which is where this exponential smoothing and in general smoothing methods are going to be useful, so local trend and level are estimated from data and updated dynamically, so if you see this statement in the earlier or you know simple moving average and simple exponential smoothing we were typically able to you know forecast the level component but here in this scenario trend and level both components are estimated from data and updated.

## Smoothing Methods

- **Advanced exponential smoothing methods**

- **Double exponential smoothing**

- k-step-ahead forecast ( $F_{t+k}$ ):

$$F_{t+k} = L_t + kT_t$$

- Level at time t ( $L_t$ ):

$$L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$

- Trend at time t ( $T_t$ ):

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

- Also,

$$F_{t+1} \neq F_{t+2} \neq F_{t+3} \dots$$

Where  $\alpha$  and  $\beta$  are smoothing parameters for level and trend respectively

So let's have a look at the equations here, so this is called double exponential smoothing because we can look at two components, simple exponential smoothing it looks at just one component that is level, double exponential smoothing it can look at two components that is level and trend, so now K step ahead forecast  $F_{t+K}$  is going, can be rewritten as this  $F_{t+K} = L_t + K T_t$  that is level component at time  $t + K$  times trend component at time  $t$ , so this is trend at time  $t$ .

Now how this you know level and trend component are going to be computed, so we can see the equations for these two components  $L_t$  can be written as you know combination of alpha

$Y_{T+1} - \alpha(L_{T-1} + T_{T-1})$ , so you can see in the first part  $\alpha Y_T$  we are you know using the actual value to you know define the level + the previous value is also there  $L_{T-1}$  so that is also here and adjustment using the trend, so trend component is being used to adjust also, and the actual value is also being used to determine the dynamically determine the level at time T.

Similarly for trend component you can see here the second part  $(1-\beta)T_{T-1}$ , so the previous value of the trend is also you know in a way determining the trend at time T, and also from the level also the difference in the levels that is also being used to determine the trend at time, so dynamically it is going to be determined, so in this fashion we can compute the K step ahead forecast. Here from this you can clearly see that now one step ahead forecast that is  $F_{T+1}$  is not going to be equal to you know forecast at  $T+2$ ,  $F_{T+2}$ ,  $F_{T+3}$  and so on, reason is obvious because now trend component is there which is going to vary, right, which is going to change.

Now we are going to use two smoothing parameters alpha and beta, so these parameters are respectively to be used for level and trend.

## Smoothing Methods

- **Advanced exponential smoothing methods**
  - For series with a trend and seasonality, Holt-Winter's exponential smoothing method can be used
    - An extension of double exponential smoothing
  - Holt-Winter's exponential smoothing
    - k-step-ahead forecast ( $F_{t+k}$ ) for multiplicative seasonality:

$$F_{t+k} = (L_t + kT_t)S_{t+k-m}$$

Where m is indicating the presence of seasonality with m seasons, also  $t > m$

- Level at time t ( $L_t$ ):

$$L_t = \frac{\alpha Y_t}{S_{t-m}} + (1 - \alpha)(L_{t-1} + T_{t-1})$$

Now second scenario where if both trend and seasonality both the components are present then the Holt Winters exponential method that could be used, so this method we saw in the R studio also that we had the arguments for beta and gamma, so now you can see that beta is for the trend component and gamma is later on we'll see is going to be as used for the, in this scenario itself we'll see this is use for the seasonality, so three smoothing parameters are to be used now, one for level alpha, one for trend that is beta, one for seasonality that is gamma, so this Holt Winters exponential smoothing is also an extension of double exponential smoothing so let's have a look at the some of the expressions for this, so now we can see K step ahead forecast  $F_{T+K}$  and these equation we are writing for multiplicative seasonality, so we are assuming here that multiplicative seasonality is present so therefore values from one season to other seasons are changing in percentage amounts, in additive seasonality they change you know by a constant amount, but in multiplicative seasonality you know it is you know they change by percentage amount, so therefore we multiplied by that factor, so you can see  $F_{T+K}$  and we are

multiplying by this seasonality factor to you know compute this values FT+K can be written as level component LT + KT you know trend component and this whole expression is multiplied by the seasonality you know value index, seasonality factor FT+K –m this is small m is actually indicating the presence of seasonality with M seasons, right, so from there here you'll also see that to be able to use this expression the T value should at least be greater than M, so that this you know this S could be meaningful there and could be used to compute this FT+K.

Similarly if you see the level at time T so there is some you know change in this expression, you can see now we are adjusting for this multiplicative seasonality, so first term alpha YT is now divided by this capital ST-M value + the second term is edges LT-1 + T-1, so level is being adjusted by the trend itself, and also the actual value is being adjusted by the seasonality.

## Smoothing Methods

- **Advanced exponential smoothing methods**

- **Holt-Winter's exponential smoothing**

- Trend at time t ( $T_t$ ):

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

- Seasonality at time t ( $S_t$ ):

$$S_t = \frac{\gamma Y_t}{L_t} + (1 - \gamma)S_{t-m}$$

- Also,

$$F_{t+1} \neq F_{t+2} \neq F_{t+3} \dots$$

Where  $\alpha$ ,  $\beta$ , and  $\gamma$  are smoothing parameters for level, trend and seasonality respectively

If you look at the trend component it is same like it was in the previous scenario, if you look at the seasonality component now how we can see, the gamma is the smoothing parameter here and we can see in the first expression seasonality is you know being determined by the actual value, but after adjusting for the level, the second you know component in this is this, you know previous you know seasonality value, right, so we can see as T-M, so this is how, these are the equations that can be used if the multiplicative seasonality is present, here also this one step ahead forecast, two step ahead forecast or these are going to be different, and as we can see alpha, beta, and gamma are the smoothing parameters for level, trend and seasonality respectively.



## Smoothing Methods

- Advanced exponential smoothing methods
  - For series with seasonality (no trend), Holt-Winter's exponential smoothing method can be used
    - Delete the trend terms in the equations
  - Holt-Winter's exponential smoothing
    - k-step-ahead forecast ( $F_{t+k}$ ) for multiplicative seasonality:

$$F_{t+k} = L_t S_{t+k-m}$$

Where m is indicating the presence of seasonality with m seasons, also  $t > m$

- Level at time t ( $L_t$ ):

$$L_t = \frac{\alpha Y_t}{S_{t-m}} + (1 - \alpha)(L_{t-1})$$

## Smoothing Methods

- Advanced exponential smoothing methods
  - Holt-Winter's exponential smoothing

- Seasonality at time t ( $S_t$ ):

$$S_t = \frac{\gamma Y_t}{L_t} + (1 - \gamma)S_{t-m}$$

- Also,

$$F_{t+1} \neq F_{t+2} \neq F_{t+3} \dots$$

Where  $\alpha$  and  $\gamma$  are smoothing parameters for level and seasonality respectively

- Open RStudio

Now third scenario where the series is having just the seasonality and no trend, so the same expression that we have discussed the Holt Winters exponential smoothing method the same method can be used, we just need to delete the trend you know trend terms in the equations, so if we do this we'll have these equations as you can see here K step ahead forecast, level, and the seasonality and these are the equation that can be used, so with this we have covered the advanced exponential smoothing methods also.

```

132 metric(restsv[2:147], mod2$fitted[, "xhat"], c("MAPE", "MAE", "MSE"))
133
134 # Forecast for April 2016
135 library(forecast)
136
137 resESforecasttest=forecast.HoltWinters(mod2, h=1); resESforecasttest
138
139 ### Advanced exponential smoothing
140 # on raw (original) series
141 tsvtrain=ts(dftrain$Riders, start=c(2004, 1), frequency=12)
142
143 mod3=HoltWinters(tsvtrain, alpha = 0.2, beta = 0.15, gamma = 0.05,
144                 seasonal = "multiplicative", start.periods = 12)
145
146 nc5=length(tsvtrain)
147 results3=data.frame(Month=format(dftrain$Month, year = "%b-%y"),
148                    Actual=tsvtrain,
149                    Forecast=ts(data=NA, start=1, end=nc5),
150                    Residuals=ts(data=NA, start=1, end=nc5))
151 results3[13:nc5,3]=mod3$fitted[, "xhat"]
152 results3[13:nc5,4]=tsvtrain[13:nc5]-mod3$fitted[, "xhat"]
153
154 }

```

Environment History Connections

- Global Environment
- tsvtrain 147 obs. of 2 variables
- fulldf 159 obs. of 2 variables
- mod List of 13
- mod2 List of 9
- results1 147 obs. of 4 variables
- results2 147 obs. of 4 variables

Files Plots Packages Help Viewer

R: Forecasting using Holt-Winters objects

forecast.HoltWinters (forecast) R Documentation

### Forecasting using Holt-Winters objects

Description

Returns forecasts and other information for univariate Holt-Winters time series models.

Usage

```
## S3 method for class 'HoltWinters'
```

So let's go back to R studio and do an exercise for advanced exponential smoothing also, so let's also hope that maybe this forecast.holtwintersthis will work this time, so first we'll create a TSV time series vector, time series observed for the training set so we can see here, we are using this DF train you know riders which was already trimmed, so we'll create a time series observed from here.

Now the Holt Winters function we are using, now you can see the argument alpha is specified as 0.2 for level, beta is 0.15 for the you know trend, and gamma 0.05 for the seasonality, we can see the seasonal we have specified as the multiplicative start.periods has been specified as 12, so we are specifying annual seasonality here, so we can run this function you'll get the model, now we can summarize the results for this, so you can see from 13 you know row onwards we have the you know forecast and residual numbers here, so let's plot these numbers and see how

```

137 resESforecasttest=forecast.HoltWinters(mod2, h=1); resESforecasttest
138
139 ### Advanced exponential smoothing
140 # on raw (original) series
141 tsvtrain=ts(dftrain$Riders, start=c(2004, 1), frequency=12)
142
143 mod3=HoltWinters(tsvtrain, alpha = 0.2, beta = 0.15, gamma = 0.05,
144                 seasonal = "multiplicative", start.periods = 12)
145
146 nc5=length(tsvtrain)
147 results3=data.frame(Month=format(dftrain$Month, year = "%b-%y"),
148                    Actual=tsvtrain,
149                    Forecast=ts(data=NA, start=1, end=nc5),
150                    Residuals=ts(data=NA, start=1, end=nc5))
151 results3[13:nc5,3]=mod3$fitted[, "xhat"]
152 results3[13:nc5,4]=tsvtrain[13:nc5]-mod3$fitted[, "xhat"]
153
154 }

```

Environment History Connections

- Global Environment
- results3 147 obs. of 4 variables

Files Plots Packages Help Viewer

R: Plot residual series and forecasted residual series

plot(tsvtrain, xlab="Year", ylab="riders", las=2)

Console Terminal

```

C:/Users/E T Cell/Desktop/MDOC July 2018/Dr. Gaurav Dixit/Session 5/
> # Plot residual series and forecasted residual series
> plot(tsvtrain, xlab="Year", ylab="riders", las=2)
>

```

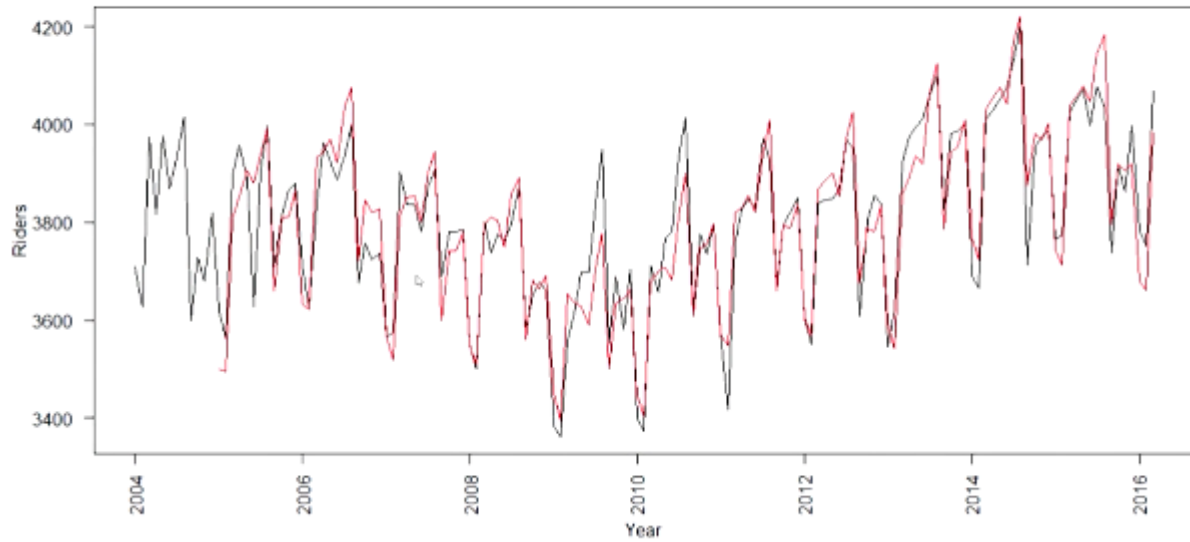
9 Sep-04 3598 NA NA  
10 Oct-04 3728 NA NA  
11 Nov-04 3679 NA NA  
12 Dec-04 3819 NA NA  
13 Jan-05 3619 3500.434 118.56594  
14 Feb-05 3563 3494.774 68.22608  
15 Mar-05 3896 3806.384 89.61645

Riders

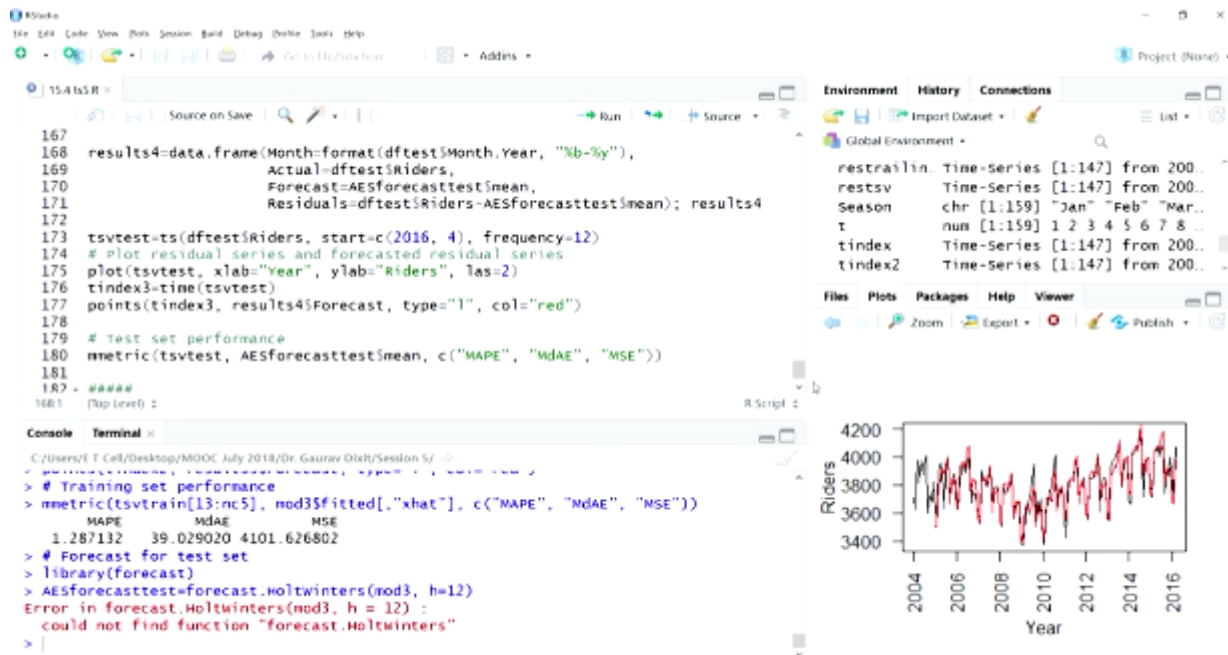
Year

we are able to forecast, so this is the you know TSV train, this is the, so this is the, so right now let me tell you that this advance exponential is loading, this is being done on raw or original series, right, so all this is on original series, so advanced this particular you know function Holt Winters function is going to deduct all components from the data itself, so this series we already know this bicycle ridership dataset we already know it, has a trend and seasonality component, so that is why you saw that we had specified alpha, beta and gamma while modeling.

Plot Zoom



So let's plot our forecasted values, so we can see in this plot the actual values and the forecasted values also, so we can see the forecasted value are quite you know closely following the actual values, so you can see the Holt Winters function can be used to forecast the series with trend and seasonality components as well, you can have a look at the performance, you can see performance is less than, so you can see second value MDAE this value is even less, so the comparable performance can be seen something that you know when we use regression based forecasting and then applied you know trailing moving average and the performance was you know similar to this, and in this case also you know similar kind of performance we are able to see.



So this was the performance on the training set, so for the test set also we can do this, so again let's hope that forecast this you know works this time, so it's not working for some reason, however if we use this function we'll get the forecast for next well, and then we would have been able to see the results for the you know test set as well, so that is left for you to complete this exercise, and we can also have looked at the plot for how the model is performing you know on the test partition.

## Smoothing Methods

- Advanced exponential smoothing methods
  - Holt-Winter's exponential smoothing

- Seasonality at time  $t$  ( $S_t$ ):

$$S_t = \frac{\gamma Y_t}{L_t} + (1 - \gamma) S_{t-m}$$

- Also,

$$F_{t+1} \neq F_{t+2} \neq F_{t+3} \dots$$

Where  $\alpha$  and  $\gamma$  are smoothing parameters for level and seasonality respectively

- Open RStudio

So let's go back, so we have, we able to cover, we have been able to cover this smoothing methods, so that completes our time series modeling as well, so in this particular course Business Analytics and Data Mining Modeling Using R – Part 2 we have been able to cover

both the modules unsupervised learning methods as well as time series forecasting. So with this we complete our course, hope you enjoyed the course, good luck for future. Thank you.



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