

Working Capital Management
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Lecture-50
Uncertainty Model by Miller and Orr

Welcome students, we are in the process of discussing and learning about the cash management models. In the previous we discuss the model which was called as the or which is called as the certainty model given to us by W.J. Baumol and there we applied the concept of economic order quantity considering the cash also like inventory as same as inventory and then we applied the concept of the economic order quantity to find out the optimum amount of the cash, which was C indicated it is EOQ.

But it will it was indicated as C there and we try to find out that C, so that we can find out that if we keep the optimum level of cash balance with us then certainly the cost is also under control and the firm is not going to technically insolvent. But the limitation of that model was that it assumes that everything is certain cash inflows or certain outflows or certain requirements or certain an annual requirement is also means we are able to pre-decide and that is also certain.

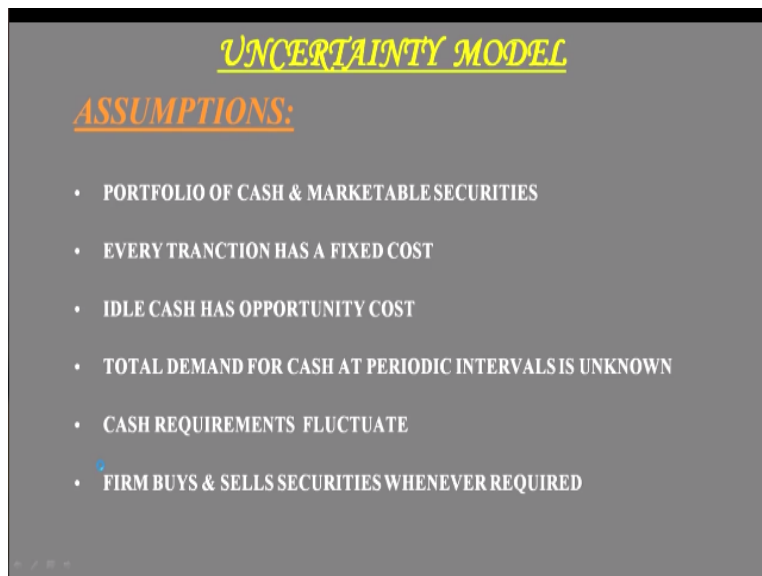
So, in case of the cash these things may sometime not be possible, sometime that element of certainty may not be there as per as the management of the cash is concerned, sometime we end up with having more cash, sometime we end up having less cash. So sometime we need to borrow or arrange funds and sometime we have surplus funds. So all those uncertainties are there, so I told you in the previous class as such that.

Now in this class we will discuss about another model given to us by M.H. Miller and Daniel Orr that talks about or that models name is that is the uncertainty model and that takes into account the limitations of the first model that we called as the certainty model given to us by the W.J. Baumol right. So we will discuss in this class the model given by Miller and Orr and we will address the element of uncertainty that we understand that yes cash flows are sometime uncertain.

And if there is element of uncertainty and everything is not certain then how to take care of that situation. So here is the model which we call as the uncertainty model given to us by M.H. Miller and Daniel Orr and this model is based upon certain assumptions and if you compare these assumptions with the previous model we discussed. So, you will find that the limitations of that model or the assumptions which were unrealistic in that model are not existing here.

So, let us see the assumptions taken here and then we try to understand the model given by these 2 academicians. So, first assumption is portfolio of cash in the marketable securities.

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So, the same assumption was there in the certainty model also that normally a firm should not keep all the amount of cash as cash that should be divided into 2 parts, cash and the marketable securities. So, this assumption is here also, this assumption was there also at both the places assumption is same. So, it means no doubt about that this this assumption is acceptable.

Second assumption is every transaction has a fixed cost, it was there also in the other model also, in the certainty model also that assumption was there also that every transaction has a fixed cost. That when you convert the cash into marketable securities and marketable securities into cash then it has a fixed cost in terms of broker commission, in terms of the storage of securities, in terms of say secretarial say cost.

So all this cost can be easily identified and can be worked out per transaction, so every transaction when you are converting cash into securities or vice versa it has a fixed cost. So, these first 2 assumptions are same in both the models. Now let us talk about the third one ideal cash is the opportunity cost, this assumption was there in the other model also. In the certainty model also Baumol also has assumed almost a same thing.

That if you keep the cash as ideal which is more than your requirements which is not required in the firm in the normal course, in that case that has the cost, that is going to increase the financial cost of the firm. And if we are not able to draw or earn sufficient returns on that in that case the cost is going to out way the benefits and the firm is going to incur the losses sometimes or maybe there is a profit then profit is going to go down.

So, first 3 assumptions are same in both the models, now what is different, total demand for cash at periodic in turbo is unknown. There this assumption was that total demand for the cash at the periodical in turbo is steady and it is known. The demand at the periodical in turbo Baumol says that it is steady but here these 2 people say Miller and Orr say that this is unknown, this is not means the steady it is not same, it is not stable, it is changing and it is not known so easily.

Because of the element of uncertainty in the cash flows or influencing the cash flows right. Then cash requirements fluctuate, there we assumed that cash requirements remain the same, over the different periods or different time in turbo cash requirements remain the same that does not change, but here we are assuming that the cash requirements change. So, it means we have introduce the element of uncertainty here.

And because of this element of uncertainty we are addressing these 2 means we have assumed these 2 assumptions that total demand for cash at the periodical in turbo is unknown and the cash requirements fluctuate, firm buys and sells securities whenever required. So, it is naturally it was same assumption is there also in the other model that we convert the cash into securities and vice versa means securities into cash as and when it is required.

So, it means we have the surplus cash we buy the securities and when there is a shortage of the cash we sell the securities in the market and convert them into cash. So, it means there is the same assumption in these 2 cases. So, other assumptions are different but the assumption number 4 and 5 are different as compare to the previous model and here in this model we are assuming that cash flows are affected by the uncertainty and we must be prepare to address that uncertainty.

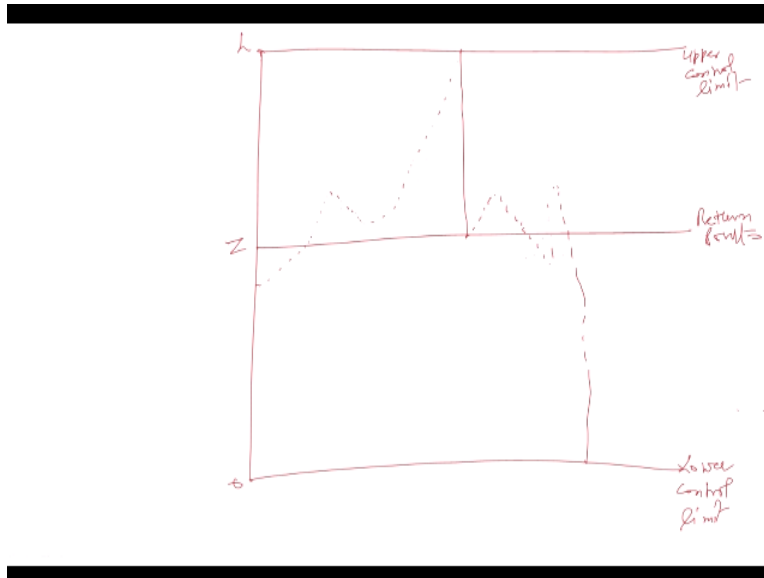
And still we should be able to find out how much cash is there, how much cash is required, how much cash is surplus, how much cash sort. So, if there is a surplus think about investing that in the market. And if it is sort then think of arranging it from some sources, so because of this assumption number 4 and 5 this model becomes different as compare to the previous model and we will now try to understand what the model is right.

So, as I told you that Baumol and this Miller and Orr have given this model of say addressing the element of uncertainty here. So, they have used the concept of control theory and on the basis of the control theory or by using the concept of control theory they have developed it is stockiest cash management model. Because in control theory we set up the 2 limits upper limit and lower limit and in between there is the something which keeps on fluctuating right.

So, we do not do anything but when it reaches up to the upper limit and when it reaches the lower limit then some extends are taken place. So, stockiest model when we are talking about, so we are addressing by say using the stockiest model we are addressing the element of uncertainty in cash balances or the cash requirements or cash balances and cash requirements.

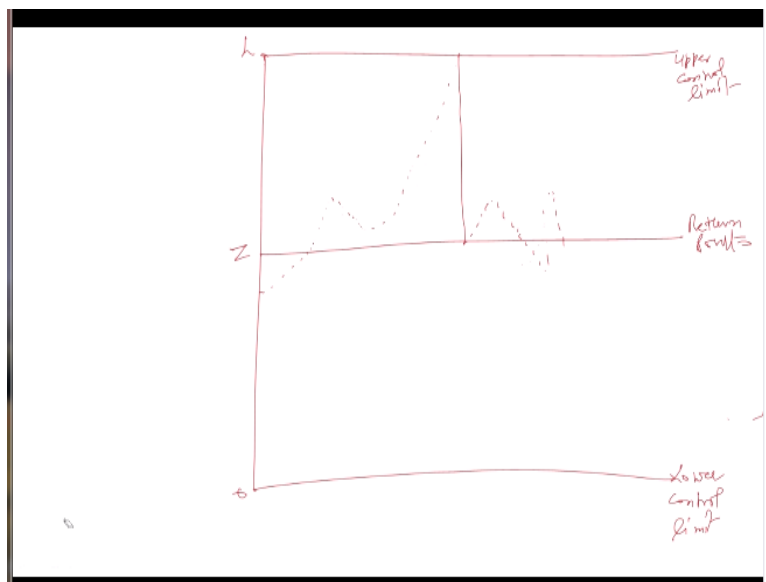
So, how this model is working and how this model has been developed let us see that how this model works first of all say I will draw the structure of the model and then I will explain it to you that what this model means and how they have applied the concept of the control theory and worked out a stockiest model of the inventory management.

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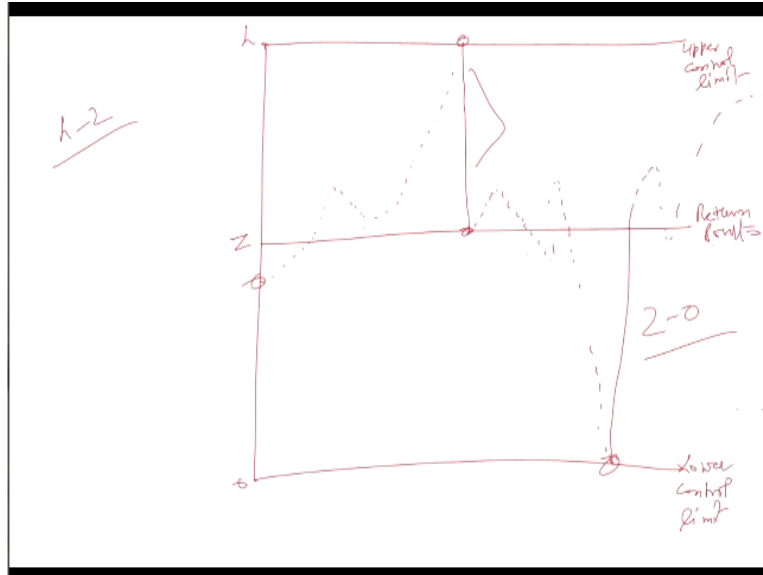
So, if you talk about this part right again this is the up lower control limit and this is the upper this is upper control limit, this is the return point. So, we call this as the 0 this is z and this is h right. Now how the cash hooks here, cash behaves something like this, then when it touches this point it will be brought down then it will be going up like this, so sorry it will come down like this, it will be behaving like this.

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And when it reaches this point we have to make this line as the dotted line right, this line should be dotted line.

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So, this will come up to this point and when it comes this point then it will be taken back state way to this and then it will go up to this level then it will come this level and it will move like this. So, this is how the final structure of the cash movement, we had this much balance of the cash here right, then we started receiving the cash and then we started going up we reached up to here then we use the cash.

But it is not at the z point still it is above the z point it went up like this and when it touch the upper limit for example this amount then $(h-z)$ (10:29) was taken is that this z means $h-z$ amount of the securities were purchased. So, immediately this amount of the cash must again brought down to this level and for this amount z is the $h-z$ amount of the cash the securities are purchased.

And this much of the surplus cash is invested in the market, then again we started receiving the cash and cash started going up then we started using the cash should went down again it is going up which is going down and when it came down to this level that is the control limit that is the lower control limit it means it became a 0 level, when it be reach at this 0 level immediately we took action and sold the securities in the market.

And then we replenish the cash level back to the z which is called as the z point and after that it starts fluctuating. So, sometime it is going up coming down and now it is going up right, so

means maximum level of the cash will go up to h and the lowest level of the cash will be allowed to go down to o or 0 you can say and then if there are any things like that when the cash balance reaches our edge immediately the action will be taken.

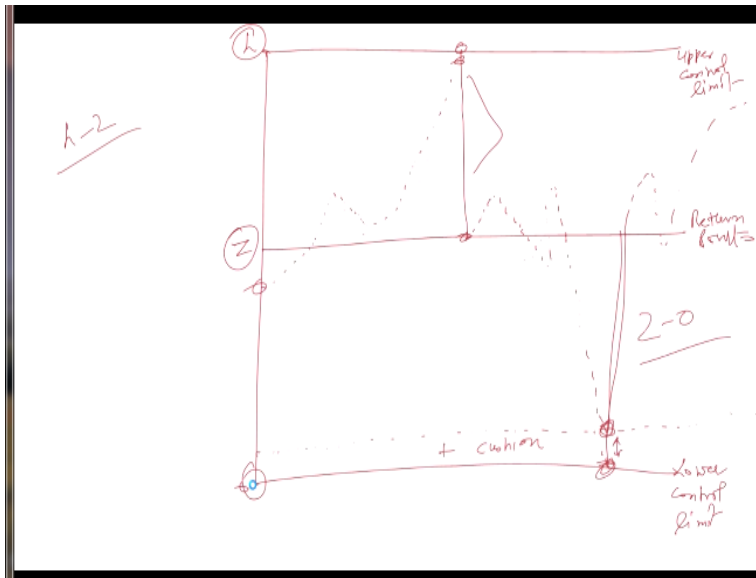
And this much amount that is $h-z$ amount of securities will be purchased and again the cash level will be brought down to the z level. And then we will start again using the cash we will receive the cash also we will pay the cash also and finally when in any case if the cash reaches at this level that is a last level lower control limit then it means now the cash level as totally come down means cash balance is have fully depleted.

So, it means will take action or again we will bring them that level, so it means z amount of the securities will be sold in the market and then we will again bring the cash amount here back to the level z and finally means again it will start fluctuating. So, these are the 2 control limits, upper control limit and the lower control limit and this is the return point. So, when there is a surplus of the cash it is $z-h-z$, so it means we have to buy the securities and invest the surplus cash in the market.

And when there is shortage of the cash that when the cash level is reaches at the o level then we will have to immediately sell the securities and bring the level of the cash back to the level of return point that is z . And when the cash is between these 2 limits that is the h and o then no action will be taken let the cash fluctuate because we know it that this much of the cash can stay with us and we have to keep this much of the cash with us and we have to say we require this much cash for our day today transactions.

So, there is nothing to worry about what when it touches z or it is sorry it touches h or it touches o , then certainly we will have to take action in case of h you buy the securities and in case of the o you sell the securities. So, sometimes what happens expert say that some improvements are there that this level lower control limit should not be at o level.

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Because if it is at the 0 level then the balance of the cash will come down to 0 and once the cash balance become 0 and if we have to make certain payment to somebody to save the situation of the technical solvency then we will be in a trouble. So, what we see that there should be some minimum cash balance maintained here and we should be up to this point. So, it means if we are up to this point it means this is the 0 but we have created a point above 0.

And here if you talk about that when the cash level will come down to this point, this particular point again the action will be taken and then the securities of this amount will be sold in the market. So, that by the time this cash reaches here then we have sold the securities in the market and we have converted those securities into cash and brought back the cash level at the z point or the return point and we are safe.

So, some people say that a margin should be added here, so it means it will not be allowed to the lower control limit but sometime before that lower control limit we should take the action and we should sell the securities and replenish the cash say level of the cash back to the z level. So, that we are safe we have sufficient cash and we can easily make the payments in the market whom or whenever it is becoming due to be made.

So, you call it as this margin is called as the cushion, this you called as the cushion that if there is some requirement then you can say that some cushion should be kept. So, that there is we are

safe and without this cushion if the cash is reaching into 0 and if it is taking time to convert the securities into cash. Then certainly there might be a problem for the firm maybe temporary for a few hours or a day.

But that problem will also come and there should not be allowed to come. So, cushion is a required some margin is required and then we should take action at this level and not at this level. But at the upper limit no cushion is required and when it touches at this level up to this level when the cash reaches goes no action will be taken up to say little before that. But when it touches the control limit then certainly we should take action and then we should buy the securities and bring the level of the cash back to the z point.

So, this is basically the correction of the model and how we apply this model or what we use this model about. So, now the question arises the million dollar question arises here is how to find z, how to find h and z-0 is that is the minimum balance which is already there us. So, after say thorough and the careful analysis and research by these Miller and Orr.

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Handwritten notes on a whiteboard:

$$z = \sqrt[3]{\frac{3bs^2}{4i}} \quad \text{Return point.}$$

$$h = 3z$$

$z = \frac{\text{daily loss of } C}{\text{Fluctuations of daily CF}}$
 Variance of daily CF

$z =$

They have come out with a model to find out the value of the z and the value of the z can be found out as cube root of 3bs square divided by 4i this is the cube root. So, this is the cube root of 3 bs square divided by 4i and this is the value of the z and what is the value of the h, h will be

equal to $3z$, 3 times of the z will be the value of the h . So, this way you can find out the amount of the z which is the return point or normally we have to allow the cash to revolve around z .

And if it is reaching at h then only the action will be taken or if it reaches at the over the minimum level then the action will also be taken. Now in this model what is b , b is same that is the when you talk about the b , what was the b in the previous model the b in the previous model was is the fixed cost of security transaction or you can call it as the fixed cost of transaction for converting the cash into securities and securities into cash, that is called as the b .

And what is i , i is the opportunity cost of the daily cash balances. Now this is the difference you have to mind, that there was the opportunity cost of the ideal cash we will keeping. Here now we are taking it on the daily basis very minutely we have to work it out and we have to calculate this i that this is the opportunity cost of i is opportunity cost of holding, cash balance on the daily basis that i is that balances is on daily basis not on the monthly or fortnightly or weekly basis, it is on the daily basis.

So, what is the opportunity cost for that and now we talk about the s square, s square is basically what you must be understanding what is s square. So, s square is basically the degree of fluctuations in the daily cash balances, degree of fluctuations in the daily cash balances and how we have to calculate that it will be specified by the s square means variance of daily cash flows which is the variance of s square.

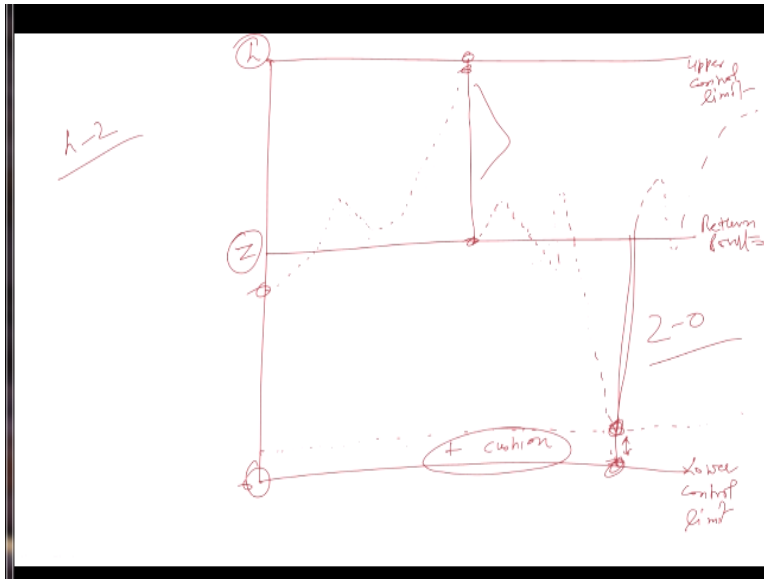
If it is only s or σ then it will be standard deviation but it is the s square that is the variance of the daily which is specified means that is the daily fluctuations specified by the variance of the daily cash flows and that is signified by s square. So, in this model you have got b that is a fixed transaction cost, i is the opportunity cost of the ideal cash on the daily basis that is i that is the opportunity cost of the holding cash balance you can say how much cash balance we are holding that is the opportunity cost of that on daily basis.

But and then s square is the fluctuations in the cash flows and that has to be measured in terms of the variance of the daily flows. So with the help of these 3 important inputs that is the transaction

cost and the opportunity cost or holding cash on daily basis and the variance of the daily cash balances if you take the cube root after solving this. If you take the cube root of that you will get the value of z right that is the return point.

And multiplying the z with the say with 3 then it becomes the 3z but here we talk about is when you calculate the z sometimes what happens is I told you that we need to have some cushion.

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Here we are talking about that here this cushion is required when I was talking to you with a cushion.

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$$z = \frac{3}{\sqrt{4}} \sqrt{\frac{365 \cdot 2}{41}} \text{ Return level}$$

$$h = 32$$

cf = daily loss of c

S^2 fluctuations

Variance of daily cf

$$z = (10000) + \frac{3}{\sqrt{4}} \sqrt{\frac{365 \cdot 2}{41}} = (20) \times 3$$

$$h = 10000 + 32 \frac{10000}{30000} \times$$

If that is the case then how the model will be changed say for example we are keeping a cushion of 10,000 rupees that minimum 10,000 rupees of the cash will be kept. So, the model will become that is the $10,000 + \text{cube root of } 3bs \text{ square}/4i$, now it will be like this right. So, this will be the total, so what with the amount comes up here in that amount, in that balance you will add this cushion amount 10,000 and that will become the total z point right.

Now 1 point of caution here is that when you calculate h, you will do again the same thing, same thing means that when you are saying that z, z is now this z is $10,000 + \text{this the output of this model}$. But when you are going to multiply here you have to do the same thing here that is the actually because the value of z is this much not this one right. We are adding it up to find up only z.

But when you are trying to find out h in this case again you have to do is that is $10,000 + \text{value means the 3 times of this z that is this}$. So, this 3 time of this we have to do not this entire of this total amount we are not do the 3 times, so this will be again remain again will be segregated if you are here only up to z then you calculate the value of this model output of this model and then you add into that 10,000 this becomes the z point, return point.

But you want to calculate the say h, so h will also while calculating h only you will calculate first to know the z value. So, multiplied by 3 and then add 10,000 into this, otherwise what will happen you are making your cushion level also 3 times. So, it maybe in this case if you do it like this that is the total of this if you multiplied by 3. So, what you are doing, you are not taking the cushion as 10,000 but you are taking the cushion as 30,000.

And that is not correct, that is wrong, so we have to again be cautious here the $10,000 + z$ value of the z and then when you are calculating h again $10000 + 3z$. So, with the help of this model or these values or these different items required in the model you can easily find out the value of z and you can find out the value of h and if you have both the values with u then you can easily fix up the control limits minimum cash we know and the maximum we are able to find out, return point we are also able to find out.

So, we are means in a position to find out that yes we can calculate these 2 limits or the balance or the 2 limits and then we know that what action has to be taken. Now let us move further that we try to understand this model implementation of this model with the help of an example or a small situation, so what is the situation say for example a company there is a company for example talk about it is xyz limited right.

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Day	Cash forecast (₹. lakh)
1	+24
2	+13
3	-16
4	-12
5	+36
6	+4
7	-28

Minimum C-B = 0,00
 Current
 Fixed cost = ₹ 1600/-
 ROI m/s = 10% p.a.
 i.e. 10%
 2500
 b = 1600

This is the company xyz limited and its daily net cash flows are something like this we put its days here and then we write here the cash forecast, this is the cash forecast right, this is the days and this is the cash forecast and this amount is rupees in lakhs right. So, if we have the days like 1, 2 here it is 3, it is 4, it is 5, it is 6 and it is 7 and cash flow here is that is the cash flow forecast we are talking about is cash flow forecast is basically make it cash flow forecast.

This is the cash flow forecast is we are writing here that is +24 lakhs we will have that means cash balance +13 lakh on the 7 second day we will have +13 lakhs then it will be -16 cash flow we are talking about that is the daily cash flow. So, it will be -16 then it is -12 and then it is +36 and then it is +4 and then it is -28. If this is a situation 1, we have got the daily cash balances we are going to have +24 means the positive balance + means positive balance we are going to a surplus, we are going to have.

So, in the first day we are going to have a surplus of 24 then second day we are going to have the surplus of 13, third day we are going to have the deficit of 16 lakhs, fourth day we are again going to have the deficit of 12 lakhs. Then fifth day we are going to have the surplus of 36 lakhs, sixth day we are going to have the surplus of 4 lakhs and then seventh day we are going to have the deficit of 28 lakhs right.

This is the basic information given to us and the minimum cash balance for example this is the minimum cash balance, minimum cash balance forms want to maintain is that is the 10000 that is the cushion . I was talking to you about is that is the cushion of 10000 the form want to have that is all the times right. And fixed cost is that is b we were talking about fixed cost of security transaction is say rupees 1600 right this is the fixed cost of security transactions.

And ROI return on investment in the marketable securities is that is somewhere you can call it as 10% per annum. If this is a situation available or this is the information available with us then what should we do, here we have the daily cash balances 1, 2, 3, 4, 5, 6, 7 we have now the cash flow forecast and forecast is we have 24 surplus, 13 surplus, 16 deficit, 12 deficit, 36 surplus, 4 surplus and 28 is the deficit on the seventh day.

Minimum cash balance we want to maintain all the times is 10000 cushion that is a cushion we want to keep, fixed cost of security transaction is 1600 rupees and ROI is the that is ROI of investment that is the investing the surplus cash in the from ROI from the marketable securities it means investing the surplus cash in the marketable securities is 10% per annum right. So, now let us now try to find out the z point out of this information, h point out of this information and let us see how we can solve this particular situation or in this particular situation what we can do.

So, what you have to do here is that as we are saying that these are the daily cash balances and if this case you have to find out is first of all what is their requirement of the model, requirement of the model is that is $3bs^2 + 4i$ right you are given i also you are given the b also not need to find out your i is how much, your i is 10% and then your when you talk about the i is 10% this is per annum.

So, we have to convert it into daily opportunity cost by dividing it 365 and then we are given b also which is 1600.

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Return point.

$$z = 3 \sqrt{\frac{365i}{4}}$$

$$h = 32$$

cf = daily Com of c

S^2 fluctuations

Variance of daily cf

$$z = (10000) + 3 \sqrt{\frac{365i}{4}} = 20000$$

$$h = 10000 + 32 = 10032$$

30000 X

So, in this model what you want to find out, you want to find you want to have the 3 thing, you want to have b, you want to have s square, you want to have i, you are given b, you are given i, you can calculate i because it is per annum. So, divided by 365 it will be daily opportunity cost but you have to find out now here is s square, that is a variance of the daily cash balances or the daily cash flows.

So, we will calculate the variance of these daily cash balances or the cash flows and then we will be trying to find out that what is the z point or how to calculate it right. So, in this case say in this model or this as means looking at this model.

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$$z = \sqrt[3]{\frac{3 \times 365 \times 10000}{4 \times 0.1}}$$
 Return point

$$h = 32$$

$$cf = \text{daily loan of } c$$

$$S^2 = \text{fluctuations}$$

$$\text{Variance of daily } cf$$

$$z = \sqrt[3]{(10000) + \frac{3 \times 365 \times 2}{4}} = 20 \times 3$$

$$h = 10000 + 32 = 30000 \times$$

And the information given here from this what you have to do is you have to calculate the square that is the variance of the cash flows and once you calculate the variance of the cash flows put the all these fixed things in this model that you have 3b into the variance of the cash flows, daily cash flows divided by the 4 into the daily opportunity cost until the cube root of that, that will be the value of z.

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Day	Cash forecast (Rs. daily)
1	+24
2	+13
3	-16
4	-12
5	+26
6	+4
7	-28

XY 2 d flow
 minimum C-R = 10,000
 Current
 fixed cost = Rs. 1600/-
 ROI m/s = 10%
 p/a

$$is \frac{10\%}{2600} \quad b = \frac{1600}{2600}$$

So, with this information we have kept this information here, I have stored this information here and in the next class we will learn how to calculate the z point and then to the h point and how to interfere that and how to understand the different cash limits and the implementation of the uncertainty model given to us by Miller and Orr. So, I stop here at the moment and let us discuss

this remaining or the solution of this particular problem to find out z and h point in the next class,
thank you very much.