Project Management for Managers

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Lecture - 43
Simulation of Networks- I

Hello friends, I welcome you all in this session. In previous session we discussed probability issues in network models and we have seen couple of examples. In this session, we will be looking at simulation of networks though we have studied simulation when you are studying about management of stand along risk of projects there we have seen how to use simulation analysis to find out NPV of a project and we did use random numbers and on that topic. So, we will be using random numbers to simulate the duration of the activity.

Now, let us get started with this particular topic simulation of networks. We have seen that an activity can be either deterministic it can take a beta distribution shape it means it can have three time estimates. It can be a normally distributed activity with mean and standard deviation. It can have only two time estimates for example, TO and TP optimistic time and pessimistic time. But suppose if there is an activity which is taken time in uncertain manner. When I say uncertain manner it means sometimes it takes let us say two days, sometimes it takes 20 days sometimes it takes 25 days and so on. So, it is quite random it is very difficult to predict the nature of that particular activities duration. So, since the situation is uncertain we will use simulation methodology for solving problem of it is a uncertain time of an event right.

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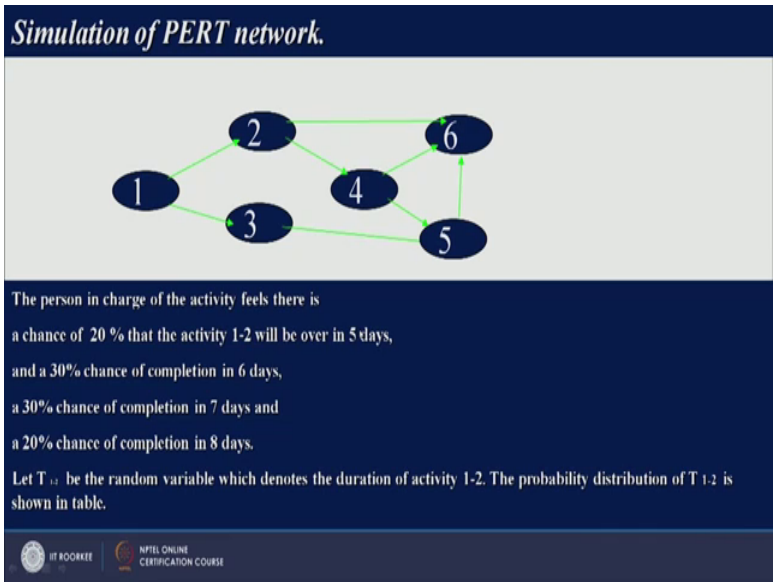

So, let us look at a project where in the chance that an activity 1, 2 will take 5 days is 20 percent, it will be completed in 6 days its probability is 30 percent, 7 days 30 percent, 8 days 20 percent. Now, let activity 1-2 with a random variable which denotes the duration of activity 1-2. Now, the duration of the activity 1-2 would be determined by the random let us take this as a random variable the duration is random variable. The probability distribution of T 1-2 is shown in this. So, this is probability distribution of activity 1-2.

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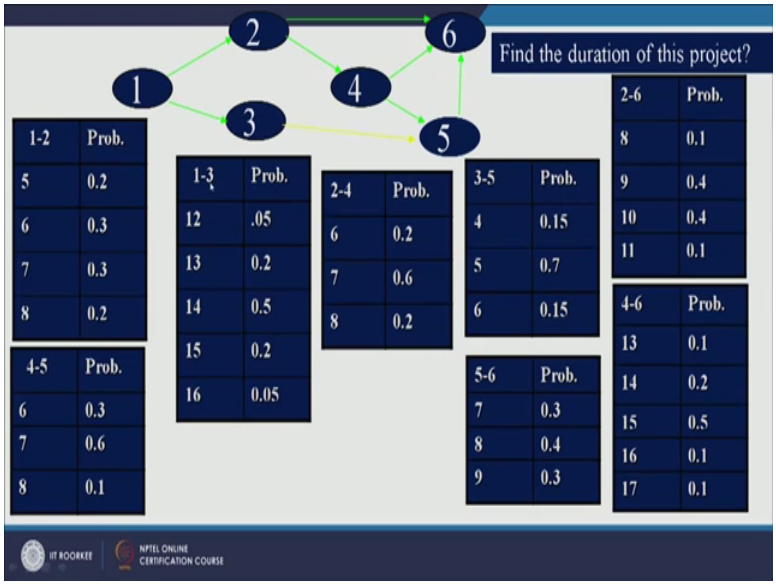

Similarly the probability distribution of activity 4-5, 1-3, 2-4, 3-5, 5-6, 2-6 and 4-6. Now, we have to use this information plus use of random numbers, we will find out duration of this project.

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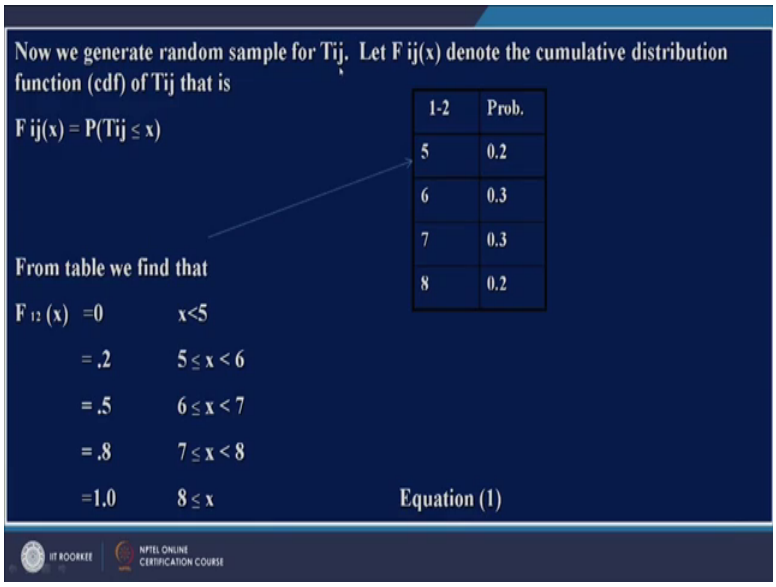

Now, let us generate random sample for T ij activity, activity T is duration is ij. Let F ij denotes let F ij denotes the cumulative distribution function of T ij that is F ij x is equal to probability that T ij is less than and equal to x. So, if you look at this activity 1-2 for which probabilities are given we can write this equation like this. So, 1 2 x is equal to 0, x is less than 5. So, x is between 5 and 6 probabilities 0.2 then 0.5, 0.8, and 1. Now this is equation 1.

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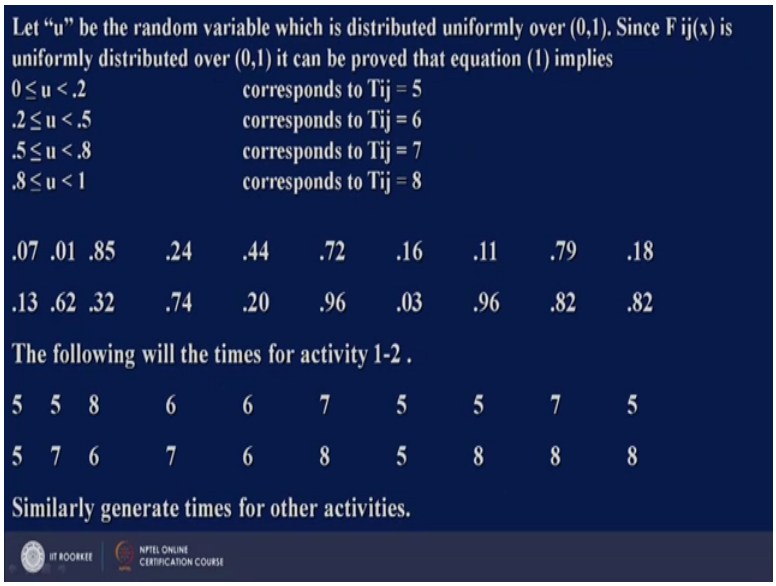

Now, we can rewrite this equation as this. So, let u be the random variable which is distributed uniformly over 0 and 1. Since F ij x is uniformly distributed over 0, 1, it can be proved that equation one implies this. So, random variable u is in between it is if it is the probability that the activity T ij whatever is ij whether it is 1- 2 or 4-5 whatever is that activity that activity will take 5 days if u value is equal to or greater than 0 and less than 0.2 then less than or equal to 0.2, but less than 0.5 that the duration would be 6, similarly, 7 and 8. How did we get this 0.2 0.58? So, for activity 1-2 so 0.2, 0.5, 0.8 and 1 that is how we get right. So, let us look at how to find out duration of activity 1-2 given some random numbers.

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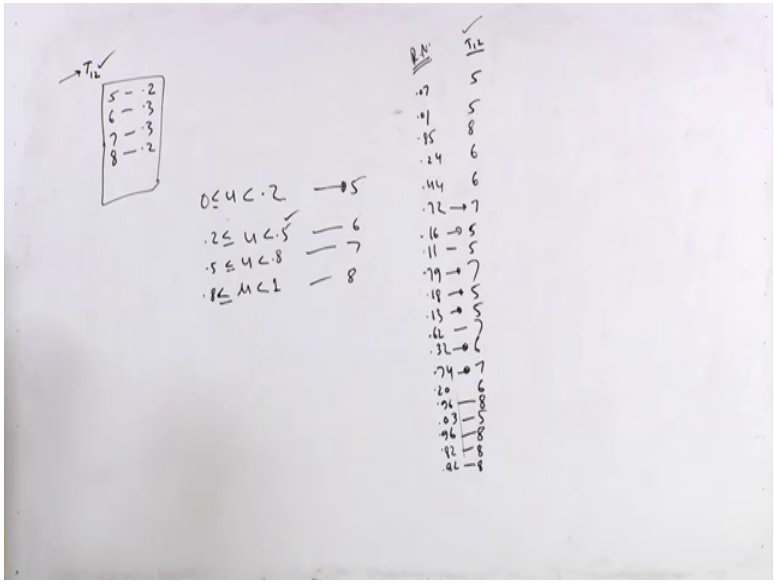

So, we are talking about activity 1-2. So, activity 1-2, so we will say that we have the probabilities of this particular activity and the probabilities are given in tabular form to be like this. So, the probability is 0.2 and it is duration is 5 days right, duration is 6 days, 7 days, 8 days, this is 0.3, this 0.3 and this is 0.2. So, we can write on the duration of this activity using random numbers right. So, we will say that this activity will take; if your random number is in between this probability then its duration is 5, so this is u. So, 0.2 greater than less than or equal to u less than 0.5, so this 0.2 plus 0.3, so this is 0.5. 0.5 less than or equal to u less than 0.8 is not it 0.5 plus 0.3, 8 right; and finally, 0.8, 0.8 u less than equal to 1. So, this is 6, this is 7, and this is 8. So, for this particular activity, we have given certain random numbers. And using random numbers we will find out duration of this activity 1-2. So, random numbers are what is there.

So, 0.07, so let us say 0.07 then 0.01, then 0.85, 0.24 0.44, 0.72, 0.16, 0.11, 0.79, 0.18, 0.13, 0.62, 0.32, 0.74 then you have a 0.20, 0.96, 0.03, again 0.96, 0.82 and 0.82 once again right. So, these are random numbers, random numbers. Now, we want to find out duration of this particular activity. Now, this random number is 0.07, 0.07 falls in which range here; it falls here right, it is between 0 and less than 0.2. So, the duration of the activity 1-2 would be 5; 0.01 which range in which range this 0.01 is falling gain here, so duration is 5. 0.85, 0.85 is here in this range right, so duration is eight. 0.24, 0.24 is here right 6; 0.44 is here right, so less than 0.5 is 0.44, so this is 6; 0.72, 7; 0.16, it is 5. 0.11, what would be the duration 0.11, 5. 0.79, 7; 0.18, 5; 0.18 5, 0.13 5; 0.62, 0.62 is here

right 7; 0.32 is here, 6; 0.74, 0.74, 7. 0.20, so this is less than 0.20 right; for equal to 0.20, this is 6. 0.96 of course 8; 0.03, 5; 0.96, 8, 0.82, 8; 0.82, 8.

So, these are different durations of activity 1-2 from these random numbers. Now, you have to calculate these time estimates for all other activities which have been given in this question right. So, all these for all other activities you need to find out what these different time estimates. And we have done this for 20 times, 1 to 20. So, we are simulating the durations of all these activities 20 times. So, this is for activity 1-2, you can see all these values here.

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SN	RN	T12
1	.07	5
2	.01	5
3	.85	8
4	.24	6
5	.44	6
6	.72	7
7	.16	5
8	.11	5
9	.79	7
10	.18	5
11	.13	5
12	.62	7
13	.32	6
14	.74	7
15	.20	6
16	.96	8
17	.03	5
18	.96	8
19	.82	8
20	.82	8

Now, you have to do it for all other activities this is for activity 1-2.

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SN	RN	T12	RN	T13	RN	T24	RN	T35	RN	T26	RN	T45	RN	T46	RN	T56
1	.07	5	.54	14	.41	7	.19	5	.34	9	.09	6	.17	14	.21	7
2	.01	5	.26	14	.78	7	.19	5	.96	11	.23	6	.29	14	.33	8
3	.85	8	.26	14	.69	7	.34	5	.89	10	.71	7	.55	15	.89	9
4	.24	6	.62	14	.56	7	.90	6	.96	11	.10	6	.93	17	.88	9
5	.44	6	.90	15	.27	7	.17	5	.96	11	.07	6	.38	15	.88	9
6	.72	7	.53	14	.98	8	.76	5	.55	10	.60	7	.31	15	.21	7
7	.16	5	.34	14	.73	7	.94	6	.28	9	.62	7	.17	14	.47	8
8	.11	5	.83	15	.87	8	.15	5	.23	9	.27	6	.26	14	.72	9
9	.79	7	.44	14	.52	7	.54	5	.13	9	.99	8	.56	15	.75	9
10	.18	5	.82	15	.14	6	.30	5	.37	9	.73	7	.25	14	.44	8
11	.13	5	.99	16	.73	7	.33	5	.94	11	.71	7	.57	15	.39	8
12	.62	7	.26	14	.90	8	.02	4	.12	9	.08	6	.29	14	.04	7
13	.32	6	.89	15	.43	7	.38	5	.80	10	.00	6	.97	17	.44	8
14	.74	7	.53	14	.33	7	.73	5	.65	10	.99	8	.50	15	.27	7
15	.20	6	.42	14	.29	7	.37	5	.11	9	.23	6	.71	15	.58	8
16	.96	8	.38	14	.66	7	.81	5	.69	10	.63	7	.76	15	.98	9
17	.03	5	.55	14	.36	7	.77	5	.98	11	.09	6	.16	14	.71	9
18	.96	8	.63	14	.46	7	.37	5	.12	9	.41	7	.59	15	.59	8
19	.82	8	.91	15	.83	8	.42	5	.37	9	.98	8	.75	15	.71	9
20	.82	8	.59	14	.49	7	.79	5	.01	8	.06	6	.34	15	.78	9

Now just see this table quite a completed table. So, serial number 1 to 20, random number for activity 1-2 is 0.07 and these duration, so all these durations of activity 1-2. Now, you have got different random numbers for activity 1-3. Now, this 0.54, 0.4, in fact, you will have to again change this rangers according to the probabilities activity 1-3. These are different random numbers these are different durations for activity 1-3, random numbers and durations for activity 2-4, random numbers activity 3-5, activity 2-6, 4-5, 4-6 and 5-6. So, what we have done we have generated time durations for these activities using random numbers. And we are already having their probabilities with us.

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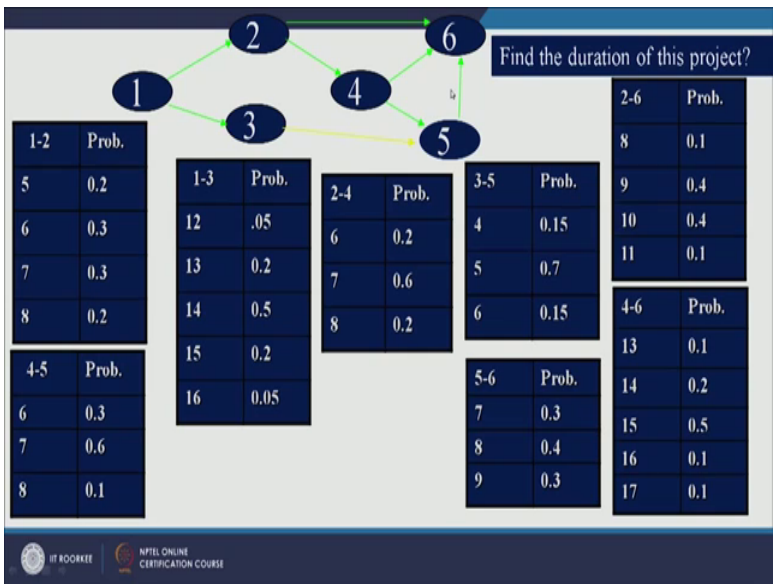

Now, you need to find out what is the duration of this project 1, 2, 3, 4, 5 and 6. So, this is a project. We want to find out duration of this project using these random numbers. So, the first is this four serial number is this right. And the duration for activity 1-2 is 5 for activity 5-6 it is 7. So, let us find out what is the duration of this network. So, you have got network and we will find out its duration.

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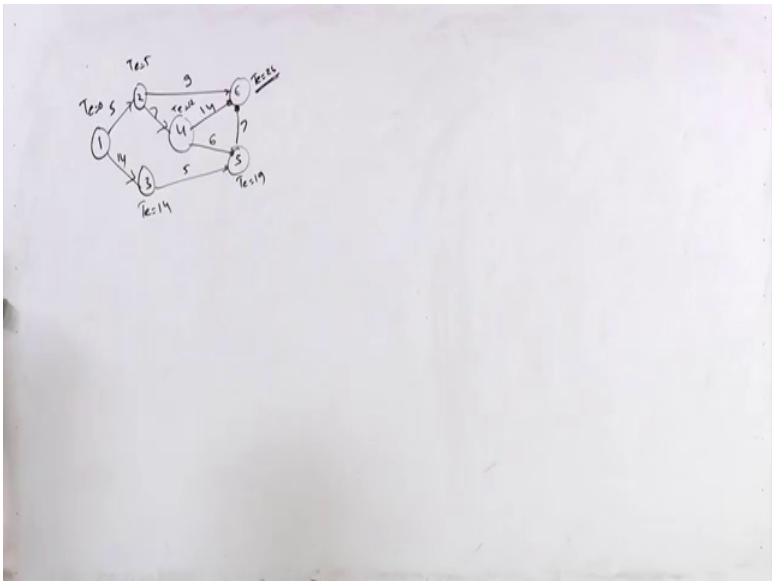

So, network is this 1, 2, 3, 4, this is 6 and this is 5, this is 5. Now, if you look at this question then for few activities saying in fact there is no deterministic time for even any of the activities for activity 1-2 what is the time for activity 1-2 time is 5. So, just write down 5 here. For activity 1-3 we have got 14, the duration is 14. For activity 2-4, duration is 27.

So, let me write all these durations for activity 1-2; 1-3, it is 14; for activity 2-4, it is 7. It is there in table also. For activity 3-5, it is 5; for activity 2-6, it is 9; for activity 4-5 it is 6; and for activity 4-6, it is 14, 4-6 this is also there, 4-6 – 14. And finally, 5 to 6, it is 7. So, these are the durations of these activities for first run first simulation run. Let us find out what is the total duration of this project. So, let us look at this T e is equal to 0, T e is equal to 5, T e is equal to 14, T e at this point is 5 plus 7 – 12; 14 plus 5 is 19. And from here it would be 18 right, so this 19 is correct. Now, this is 20, 12 plus 14 - 26 and from here also 26. So, the duration of this project is 26. Now you have to find out duration for

all those remaining 19 then. So, you will have 20 times 20 different time estimates for this particular network.

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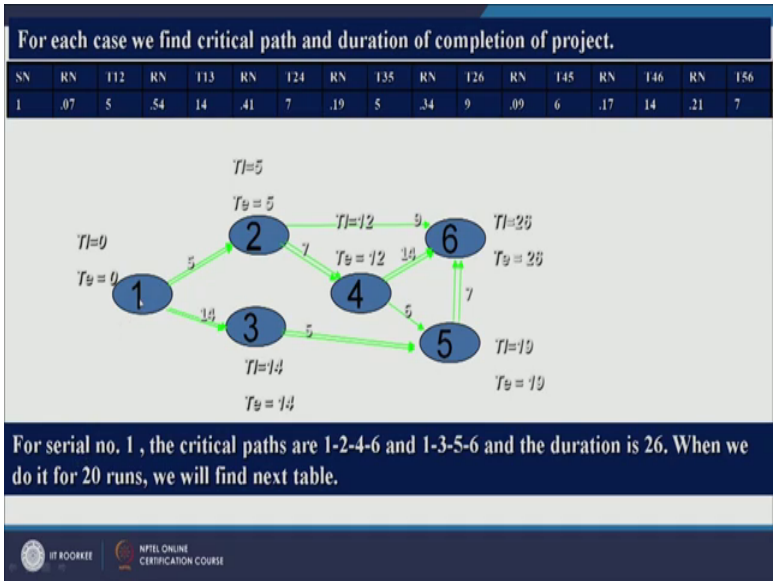

So, let us look at what are those values. So, in this particular network which I have drawn here, there are two critical paths. So, first is 1-2-4-6, 1-2-4-6, 1-2-4 and 6; and this second one is this 1-3-5-6 right and the duration of this network is 26. So, now you have to find out duration of this network for all other runs right. For first run duration is 26, for second, third and for other runs, you need to calculate total duration of project. So, let us look at what are those values. So, for first run, we have seen that the time duration is 26.

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Critical paths for sr. no. 1 are :1-2-4-6 and 1-3-5-6

Sr.No.	1-2	1-3	2-4	3-5	2-6	4-5	4-6	5-6	T
1	1	1	1	1			1	1	26
2		1		1				1	27
3	1		1			1		1	31
4	1		1				1		30
5		1		1				1	29
6	1		1				1		30
7		1		1				1	28
8		1		1				1	26
9	1		1			1		1	31
10		1		1				1	28
11		1		1				1	29
12	1		1				1		29
13	1		1				1		30
14	1		1			1	1	1	29
15	1		1				1		28
16	1		1			1		1	31
17		1		1				1	28
18	1		1			1	1	1	30
19	1		1			1		1	33
20	1		1			1	1	1	30
Crit Ind.	.65	.40	.65	.40	.00	.35	.45	.75	Avg:29.3

And what are critical activities in first run. So, if an activity critical just mark it as 1 right. So, for 1-2, it is critical; 1-3 is critical, 2-4 is critical, 3-5 is critical, 2-6 is not critical, just see 2 to 6 is not a critical, activity 4-5 is also not a critical, activity 4-6 is critical, 5-6 is critical and the total duration is 26. When you do it second time, its duration would be 27; when you do it nineteenth time the duration would be 33; twentieth time duration would be 30. And these are different critical activities in twentieth run. So, 1-2, 2-4, 4-5, 4-6 and 5-6. Now, you just take the average of all these time estimates and average is 29.3. So, what is your conclusion, you will say that the duration of this project is 29.3.

And if you look at there is something called critical index. So, if you repeat this project let say for 100 times or if you take up this project for 100 times than 75 percent of the time activity 5-6 will be critical, so that is the meaning of critical index. So, in this particular project most of the times the critical activity would be 5-6 after that activity 1-2 would be critical 1-2 and 2-4, same value 65 percent and 65 percent and then this one. So, in this way you should calculate the duration of the project.

Now, what is the probability that the project we will take 26 days. What is the probability? If you look at this table then out of twenty times 26 is and total times how many times 1 then, 2, 2 times right. So, this is 2 by 20. How many times total duration was 7, so was 27 just once? So, this is 1 by 20. How many times it is 28, just check here

28, this second time, third time and fourth time, so 4 by 20. And similarly there is you did not get 32 as the total duration you mean a single time right and 33 days just one time. So, probability that project will take more than 29 days. If I ask you a question what is the probability that this project will take more than 29 days for more than 29 days means 30, 31, 32, 33. So, just add this 5, 5 by 20 plus 3 by 20 plus 1 by 20, so you will get 9 by 20.

If I ask you question; what is the probability that the project will take less than 29 days less than 29 days. So, what it would be it would be 2 plus 1 - 3 plus 4 - 7 by 20 right. So, in this way you can simulate duration of an activity. So, since we have got computers these days, you can simulate it for let us say 20,000 times. So, you will get a precise time for an activity, so the more the number of runs better the duration of activity.

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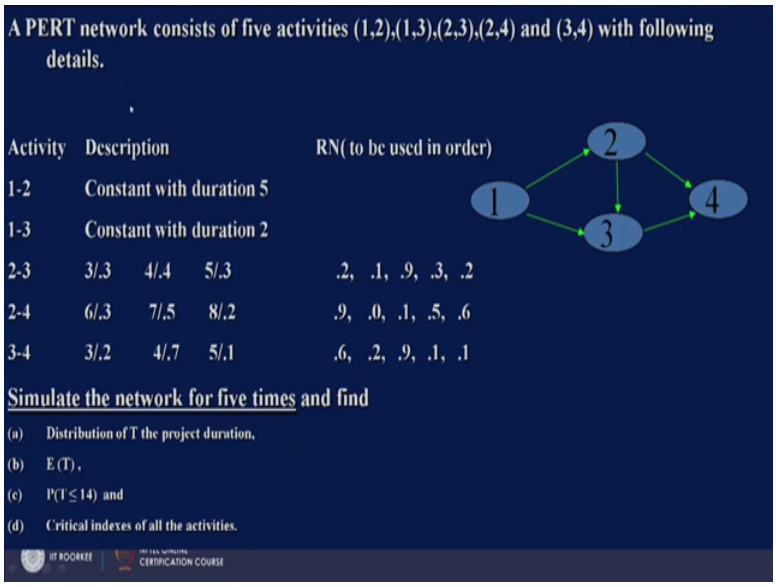

Let us take this example very interesting example. We have been given this five activities 1-2, 1-3, 2-3, 2-4 and 3-4 right the duration of a of the activity 1-2 is constant is 5 right, so there is no variances right. Activity 1-3, its duration is 2 right, but activity 2-3, 2-4 and 3-4. So, the probability that activity 2-3 would be completed in 3 days is 0.3. It would be completed in 5 days, the probability is 0.3 it would be completed in 4 days probability is 0.4; similarly, for activity 2-4 and 3-4. Now, these are different random numbers given for activity 2-3; similarly for activity 2-4 for activity 3-4.

Now, you have to simulate the network for five times and find out what is the distribution of it is time duration. Find out it is expected time how much time this project will take when you do it for five times. What is the probability that the project will be completed in less than 14 days, and find out critical indexes of all the activities. We will take up this example and we will solve it.

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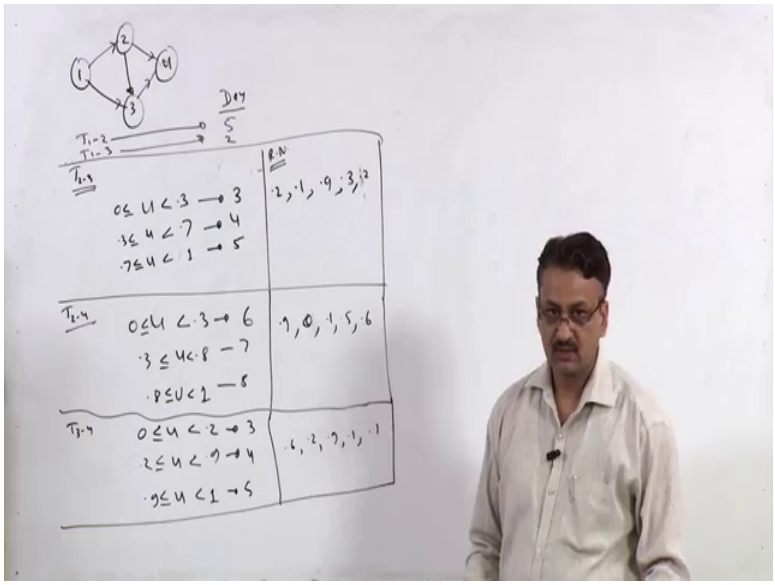

So, let us first draw this network. 1, 2, 3 and 4 right very simple network. So, this is 2 to 3, activity 1-2 has got constant duration right; 1-3 is also got constant duration, but for activity 2-3 if you look at activity 2-3, T 2-3 right the duration for activity 2-3, you have been given probabilities right. So, we will follow the same method which we did in previous example. So, u is between 0 and less than 0.3 right is less than or equal to u in between 0 and equal to 0 and more than 0, but less than 0.3. The duration is 3 days. It is between, but less than 0.7, we have 0.3 and 0.4. So, u is equal to and less than 0.3, but less than 0.7. What would be the duration, duration is 4 right less than or equal to 0.7, but less than point 1, it is 1 right. So, 0.3 0.7 plus 0.3 is equal to 1, and the duration is 5. The random numbers are first time for activity 2-3 right, so 0.2, 0.1, 0.9, 0.3 and 0.2, this is for activity 2-3.

Now, for activity 2-4, for 2-4 also you have been given probabilities. So, 0 to 0.3 and the duration would be 6 days then 0.8, 1, this is 0.3, 0.8, this is u and this is u. So, duration is 6, 7 and 8. And the random numbers are 0.9, 0 usually let it be 0, then 0.1, 0.5, 0.6 this

for activity 2-4. For activity 3-4, 0, it is 0.2 and the duration is 3 days; 0.9 is 0.2, 0.9, 1 and the duration is 4 and 5. And the random numbers are 0.6, 0.2, 0.9, 0.1 and 0.1. And we have also been given time durations of 1-2. So, for activity 1-2 and for activity 1-3 for this the time is 5 and 2. So, let me write these things over here this or in this column these are random numbers.

Now, we have to for the time being, let me stop here and the solution to this question, we will see in next session. So, I hope you would have understood how to how to find out time duration of an activity which is quite uncertain using random numbers.

Thank you very much.