Project Management for Managers
Dr. M.K. Barua
Department of Management
Indian Institute of Technology, Roorkee

Lecture - 36
Project Time Management- PERT Networks

Hello friends. I welcome you all in this session. As you are aware in previous session we were finding earliest start time of different nodes. In this session also we will continue with the same exercise and we will also try to find out latest completion time of a network right I or different nodes of a network right. So, let me take one more example which would help you in finding a list start time of different nodes right.

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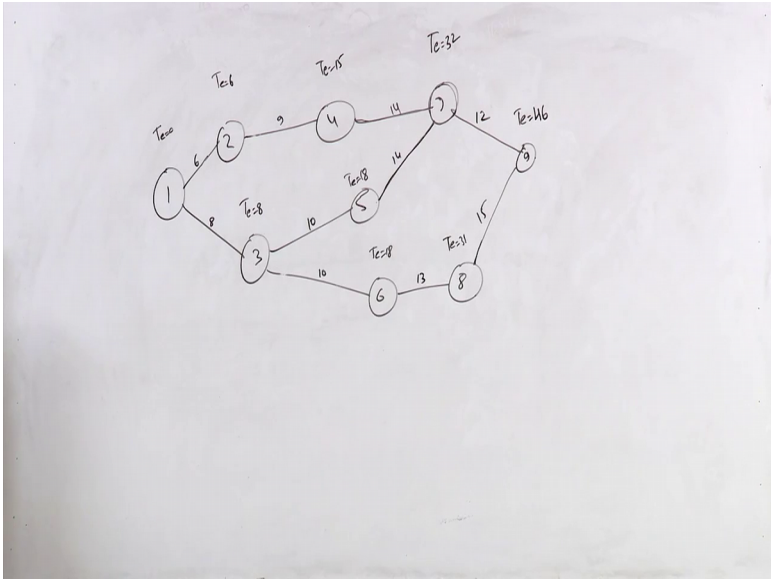

So, let us look at this example. So, you have got first node then this is second third and the activity time is 6 minutes right. The duration of this particular activity is 8 minutes right then you have got one more node here and the duration of this activity is 9 minutes, node 5, node 6 the duration of activity is this 10 and for this also 10 minutes then seventh node the duration of this activity is 14, this activity duration is also 14, then you have got one more node and the duration is 13 minutes and finally, node number 9 which is the last node in this network and the duration of this particular activity is 12 and duration of this particular activity is 15 right now let me find out what is the total completion time right.

So, we will calculate earliest start time of all these nodes right and the least start time which you get over here would be the total time of the project or the project will take that much time to complete right. So, as I said this these 2 activities right 1 2 and 1 3 you can start as early as 0 unit of time right. So, earliest start time is 0 right what is the earliest start time at this node T e here is 0 plus 6. So, this is 6 right earliest start time at this node is 0 plus 8 right earliest start time at this particular node is 6 plus 9 is 15 right. Earliest start time at this node is 8 plus 10 18 right earliest start time at this node is again 8 plus 10 it is 18 right.

Now, what is the earliest start time at this node? T e at this is what that is the question because at this node we are reaching from this path and from this path right. So, when you calculate earliest start time of this node you have got 2 earliest start times right first is 15 plus 14, 29 and 18 plus 14, 32. So, you have to select the maximum of these two. So, 29 and 32 the maximum value is 32. So, earliest start time of 7th number node is 32. Now what is the earliest start time of this node its 18 plus 13. So, earliest start time is 41 or 31 it is 31 what is earliest start time at this node because you are reaching here from this particular node as well as from this node.

Again you will have 2 earliest start time sure you have to select the maximum one right. So, 32 plus 12 is 44, 31 plus 15 is 46 right. So, we will say we will complete this project in 46 minutes right. So, this is how you should calculate earliest start time in forward pass right forward calculation right. So, you just keep on adding the T e value of a particular node and duration of that activity and you will get T e right. If you are reaching at a particular node from let us say 2 or 3 different nodes then take the highest value right. So, this is how you should calculate earliest start time in forward pass right. So, this is the solution to this question and answer is 46.

Let us look at some other times. So, apart from we have got earliest start time you have got latest completion time right. So, how as late as possible you are completing a particular activity right.

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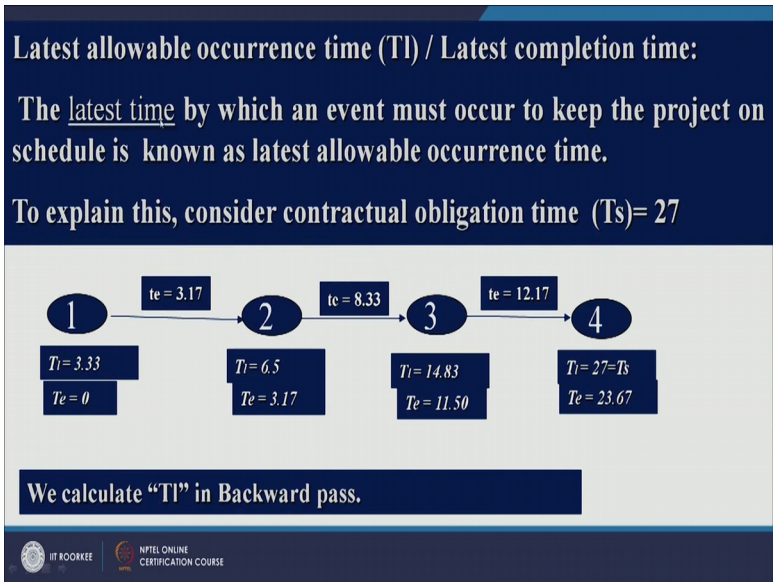

So, that is known as latest completion time right, and the latest time by which an event must occur to keep the project on schedule is known an latest allowable occurrence time right. So, if you do not complete your activity in that time, your project would be delayed right. So, you should complete an activity by that particular time right. So, to explain this let us consider this particular example right I will take this example. So, we will take up this example.

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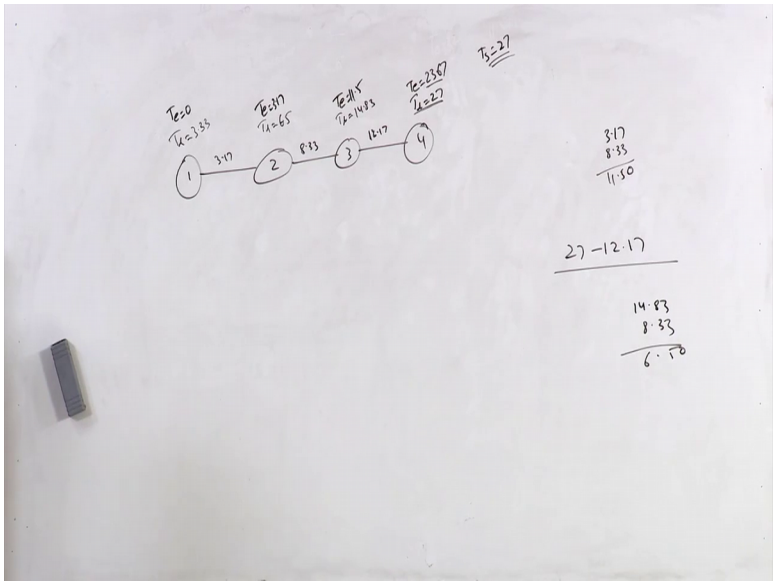

This is your first node, second node, third node and forth not right. The duration of these activities 3.17, duration of this activity is 8.33, duration of this activity is 12.17 right and we have already calculated T e for this particular network let us re calculate right.

So, T e for this is 0, T e for this is 3.17, T e for this is just add these two. So, it would be 12 point just add these two. So, 3.17 plus 8.33 right. So, this is 11.5 its 11.5 and this is T e here is this is 12.17. So, 23.67 right. So, this is what we have already calculated in forward pass. Now let us calculate Tl a latest completion time at all these nodes right. So, for latest completion time in fact, there is some there is one more time latest let us define it as t s this this this is known as scheduled completion time. Scheduled completion time is a time in which you have been given that you should complete a project in this time for example, if I ask you to complete this project in 20 days in 25 days, though I can complete this project in 23.67 days, but I have been given 25 days. So, that 25 is nothing, but scheduled completion time if you not been given scheduled completion time, then take this as scheduled completion time right.

So, let us find out latest completion time for the node number 4 right. So, we will as far as this particular question is concerned you have been given t s as 27 right and we will take Tl as 27, latest completion time is 27 right we have to complete this project within 27 days right. So, this is Tl at node number 4 now how to find out Tl at node number three. So, for finding Tl at node number 3 what we should do? 27 minus this duration right 27 minus 12.17, that would be the Tl and what would be that value? It is 14.83 I think ya it is 14.83 ya 14.83 right is not it how did we get this? We just subtracted twelve point one seven the duration of this activity from Tl of this. Now to get Tl of this node you have got Tl at this node right it is 14.83 minus 8.33 what this value would be? 14.83 14 14.8 minus 8.33 0.5 and this is 6, 6.5 right. So, you will get Tl as 6.5 year.

What would be the Tl at this point? 6.5 minus 3.13. So, you will get 3.33 right. So, this is how you should calculate Tl right and these known as backward pass right. So, we calculate earliest start time in forward pass latest completion time in backward pass right very simple we will take up one or 2 more networks to calculate for calculating this latest completion time, now let us look at some more slide.

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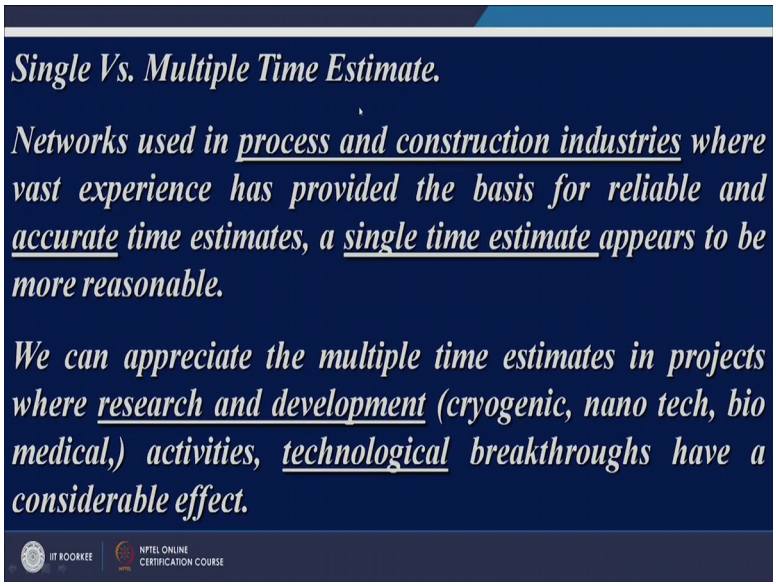

Here you have got single versus multiple time estimates. As I said you need single time you will have single time in c p m network, and in and multiple time estimates in pert network right. So, there are some you know sectors some industries where in you know a priory how much time a particular activity will take right. So, those are nothing, but single time estimates activities right. As for as r and d projects are concerned I have already told you need to have multiple times. So, you can have 3 times actually there is something called optimistic time, there is something and pessimistic time and the most likely time right.

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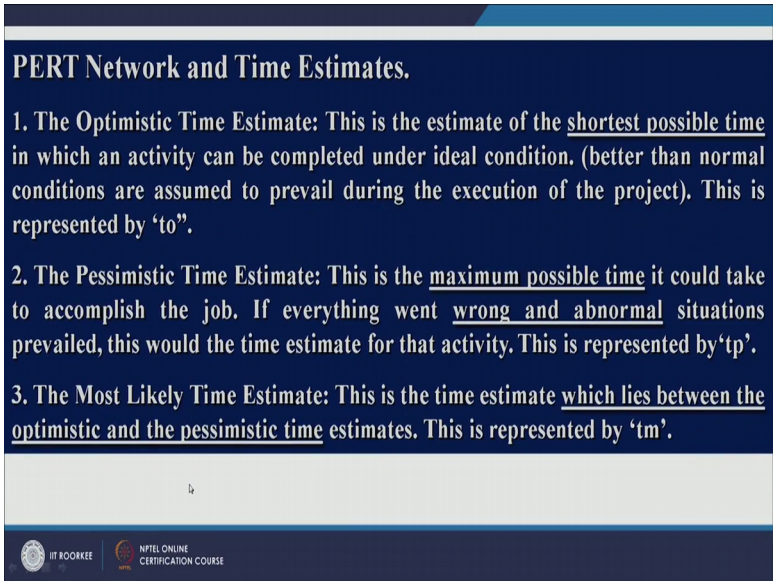

So, there are multiple time estimates in projects where r and d activities take place, and those in pert network we are we will always have 3 time estimates right and these time estimates are optimistic time, pessimistic time and most likely time right. So, optimistic time is a time the shortest time, which an activity will take. So, you are assuming that things are favorable for you right everything is in favor of you.

So, you would be able to complete an activity in shortest possible time right. And this time optimistic time is actually better than normal conditions, are assumed to prevail during the execution of project right. You are assuming that everything would be according to your wish right nothing would be wrong. So, let us define it as t o right t optimistic right or optimistic time.

Then you have got pessimistic time estimate: The maximum possible time an activity takes right and you are assuming that everything will go wrong. So, situations will always be wrong for you right and there is something out most likely estimate and how to designate this pessimistic time it is by t p right. So, the symbol for pessimistic time is t p, for optimistic time t o and the third one is most likely time. This the time estimate which is which is in between these 2 time estimates pessimistic and optimistic and this is represented by t m right t m right, but who will give you these time estimates for an activity.

So in fact, you these time estimate should be given by experts, let us say if you are doing an r and d activity, but you do not know how much time that activity will take. So, you can just talk to experts different exerts and they will give you different time estimates, you just ask them what is the maximum minimum and normal time this particular r and d project will take right. So, you will get these time estimates from experts right.

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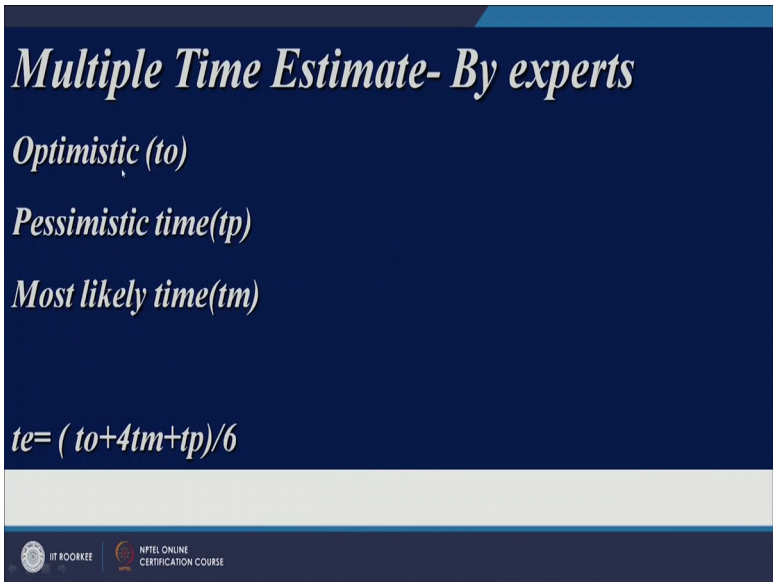

So, you have got optimistic time, pessimistic time and most likely time. And there is something called expected time of the event right it is either you call it mean time or expected time right. So, this is given by formula it is called t o optimistic time plus 4 times most likely time plus pessimistic time divided by 6 right. So, this is expected time are average time or mean time of an activity right especially a pert activity right ok.

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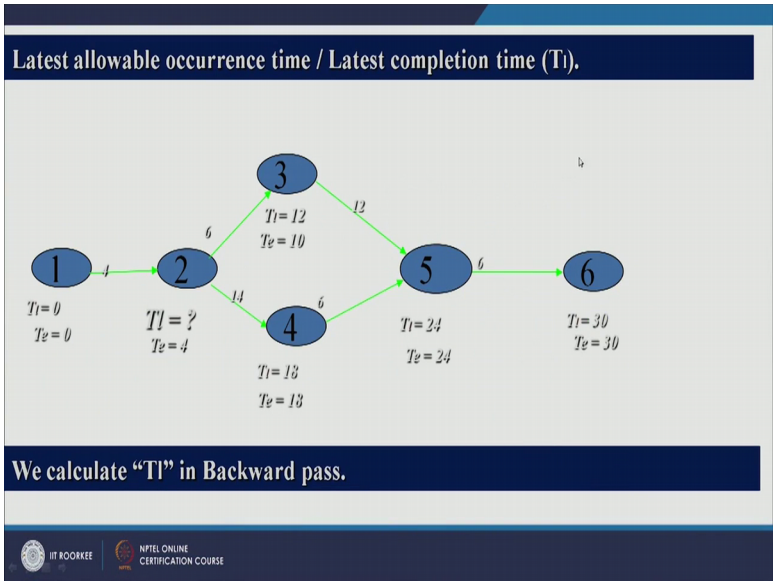

So, this is what we have seen latest completion time, let us find out latest completion time for this particular network, and will do this in backward pass right. So, let us draw

this network. So, you have got 1 2 3 4 5 6 which is the last event, and the duration is 4 6 14, 12, 6 and 6 right.

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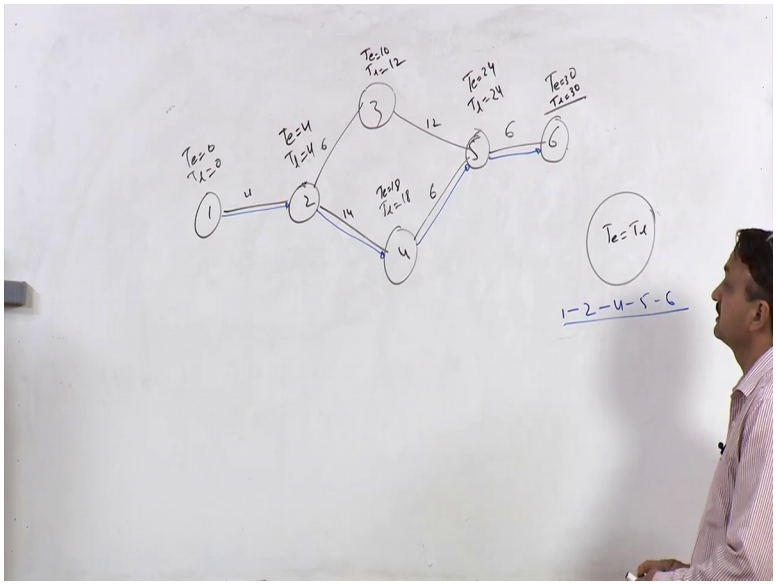

So, you know how to calculate T e just quickly, 0 at this is 4, 4 this is 10, this is T e here is 4 plus 14 18, T e here is 22 from this side and from this side it is 24. So, we will take 24 and this is 30. So, this we will take this T e 30 equal to scheduled completion time and Tl is also 30. If you are not given t s then take Tl equal to t e. So, Tl latest completion time here is 30, now I want to find out what is latest completion time at node number 5. So, Tl here is what 30 minus 6 24 right what is Tl here? Tl T e is 10, Tl is what its 24 minus 12 its 12, and Tl s at this point is 24 minus 6 it is 18.

Now, what would be the Tl value here? That is important because you are getting 2 Tl s at this node, one is from this side and the other one is from this side. So, 12 minus 6 is 6 and 18 minus 14 4. So, 6 and 4 select the lowest one Tl is equal to 4 and here Tl is equal to 0 right and now if you look at except this node except this node all other nodes have got T e and Tl equal to their values are same right. So, this is the solution to this question and now this is the time to define critical path once again. So, we have already defined what is critical path, the longest path which connects first and last nodes, but which path is critical path? a critical path is a path where all T e s and Tl s are equal when they are equal. So, that path will be the critical path. So, how to write that path?

So, you have got T e and Tl equal here right T e and Tl are equal here, T e and Tl are equal here and here. So, what is critical path? Critical path 1 2 4 5 and 6 and project completion time is 30 minutes or 30 days right. So, this how you should calculate critical path we will we will be defining the definition of critical path as we move forward right for the time being critical path is that path where T n Tl s are equal right.

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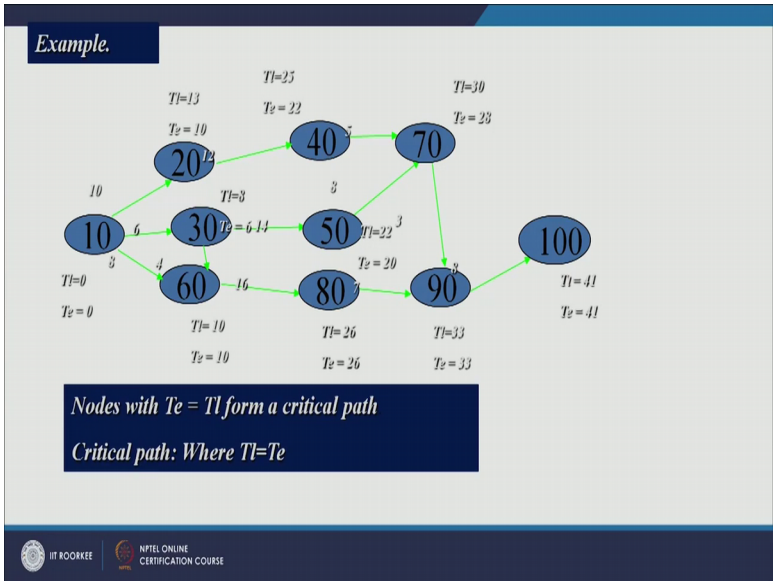

So, this is another example in which you can calculate critical path. So, I am not solving this example, but just for simplicity, let me help you how did I get all these T e s and Tl s at different nodes.

So, in this network the first event has been numbered as 10 not one, and the last one is 100 right why not why not 10 right. I could have no number them right from 1 to 10 also right, but what happens whenever you start a project initially you do not know what are different activities will be there right. So, many times you add some more activities into it. So, it is good to number events as 10 20 and so on. So, suppose if you add one more activity let us say from here to here. So, you can or let us say there is one more node here right. So, you can a number that that node as 91 is not it or 71.

So, that will help you in adding one more event in a network right if you do numbering like this right. So, there are how many nodes total 10 nodes right. So, activity duration is given. So, this is 10, 6 and this is 8 and similarly other durations are also given, you need

to just calculate earliest start time at all the nodes and latest completion time at all the nodes and critical path would be that path where T e and T s are equal right ok.

Now, let me defined one more term it is called slack. So, for slack, slack is something which will help you in delaying some of the activities right. So, let us look at what is slack.

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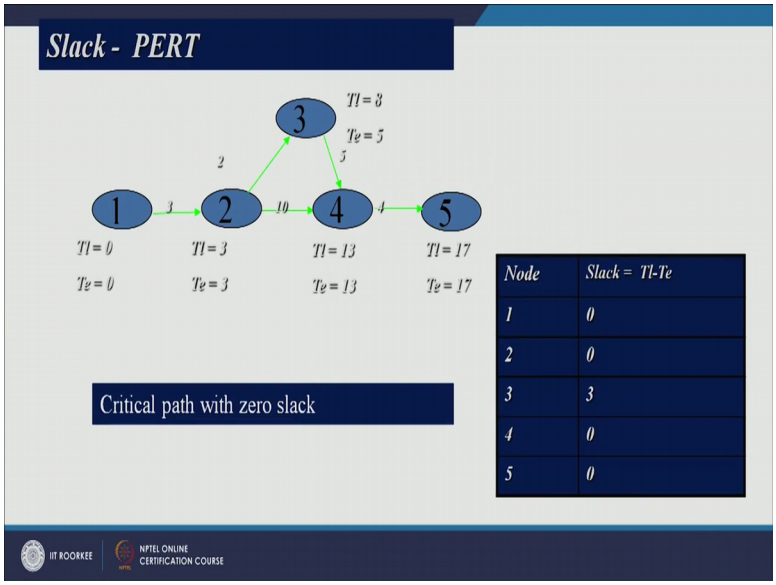

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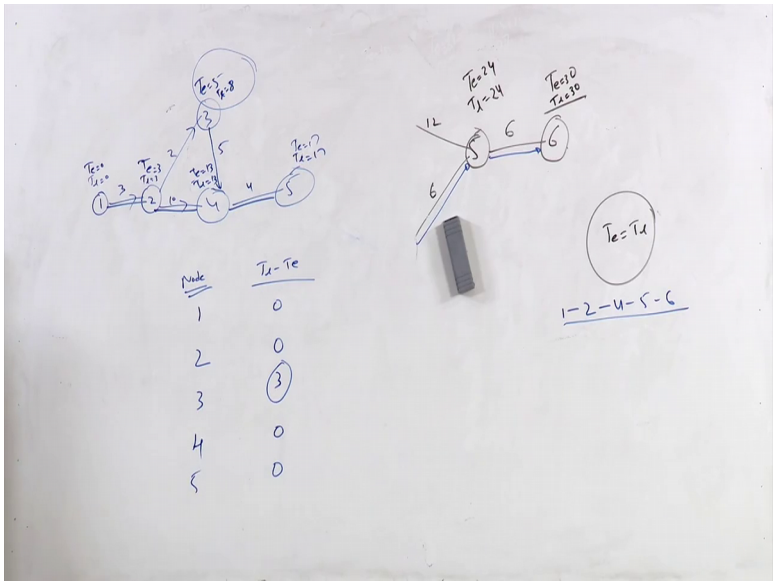

If you look at this particular network you have got activity 1, you have got activity 2 you have got activity 3, you have got activity 4, you have got activity 5 right. So, 4 and 5

and. So, the duration is 3, 2, 10, 5, 2, 10, 5 and finally, this is 4 right now if you look at if you calculate T e and Tl s at all these nodes, let us say T e is equal to 0, T e is equal to 3 here, T e is equal to 5, T e is equal to. So, you will get 2 T e s here right 13 and from here 10 right. So, this is 7 plus ya 5 plus. So, this is 3 plus this 13 right and this is 17 right Tl of course, seventeen tis is Tl here is 13 right. So, T e is 13 Tl is also 13 right Tl here Tl is Tl is 13 minus 5 8 right. Tl from this side is it would be 3 from this side it would be more than 3 right. So, this is 3 and Tl is 0 here.

So, if look at all these nodes except this node you do not have T e and t s same right. So, if you look at node one, what is slack? Slack is the difference between Tl minus T e right. So, node one Tl minus T e this difference is 0 at node 2 this difference is 0 at node 3 this difference is 3 difference between Tl and T e this 3. So, this is some positive value right at node 4 this is 0 node 5 this is 0 right. So, even if we delay activity let us say 2 3 and 3 5 by 3 days nothing will happen to the project completion time, you can complete your project in seventeen units 17 units of time right.

So, this is the meaning of slack. So, there is a slack available for activity 3, 4 and 2 3. So, if you delay either 2 3 or 3 4 by 3 minutes or you can delay both of them, but the total delay if it is up to 3 minutes or 3 days then also you can complete the project in time. So, this is critical path or what which path is critical path here, critical path is a path where all these slacks are 0. So, critical path is this this is your critical path right.

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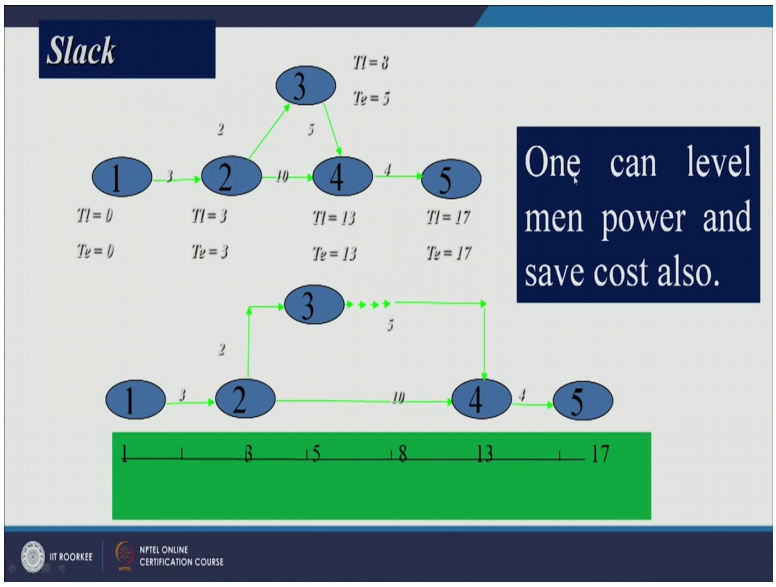

Let us look at the same network. So, this is slack right at node number 3 and this is how I have drawn a timed scaled version of this network. So, on this axis this is nothing, but time, time scale right. So, first day and s17th day right. So, activity one to 2 slack is completing at this much unit of time 3 at T e is equal to 5. So, this is 5 then this is slack right.

So, either even if you delay this activity 3 4 by 3 days, then also you would be able to reach at this node at 13th unit of time and this is the last point in this network and you can complete this particular project in 17 days right. Now this slack will help you in in in doing several things right let us say if you do not have sufficient worker at let us say on ninth day right, if you do not have a particular worker or let us say you do not have particular machinery or equipment or you do not have material.

So, you can delay this particular activity by at least 3 days right. So, slack will help you in you know moving couple of activities and it will also help you in resource profiling right in other words it will also help you in smoothing out the resources right. So, this is slack right. So, let me summarize what we have done in the session, in the session we have seen how to find out latest completion time, how to find out critical path what is slack; and we have defined slack is the difference between Tl and T e right and critical path has been defined as those nodes where T e and Tl s are same.

With this let me complete this session here itself and we will start couple of new topics in next session.

Thank you very much.