

Project Management for Managers
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Lecture - 11
Methods of Project Selection (MCDM-II)

Hello friends, I welcome you all in this session. In previous session we were discussing about multi criteria decision making techniques and we have seen the list of several MCDM techniques and we did this discuss goal programming also, and we also discussed analytical hierarchy process that is called AHP. So, as I told you that any in AHP you have got different levels at the top level you have got the ultimate goal and then there are several criteria each criterion can have multiple sub criteria and so on So, in AHP we try to rank let us say different suppliers if there are let us say 5 site, sites to be selected then we will rank those 5 sites and so on So, it is basically prioritization tool and we will look at couple of examples related to AHP now.

So, let us say there are 4 projects P 1, P 2, P 3 and P 4 and we have to select one of these projects. In each project has got different quality levels different price levels, different service levels and different delivery levels. So, we will solve this problem let us get started. So, first of all we will ask experts about their opinion on different pairs and there are different pairs in this matrix.

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Pair wise comparison matrix and computations: evaluation criteria				
Original matrix	Quality	Price	Service	Delivery
Quality	1	2	4	3
Price		1	3	3
Service			1	2
Delivery				1

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So, let us say the first pair is quality versus price, quality versus service, quality versus delivery. So, you can have different pairs and we will ask experts to give how these criteria are important with respect to some other criteria. So, let us say if you are asking expert how quality is important compared to price. So, expert said price is twice important than quality, service is 4 times important than service sorry this service is 4 times important than quality, and delivery is 3 times important than quality similarly let us say this cell. So, here we will say that delivery is two times important than service.

Now, when I say price is two times important than quality, in other words quality is 1 by 2 time important than price. When I say service is 4 times important than quality I say quality is 1 by 4 time important than service. So, the remaining sales of this matrix can be filled very easily. So, you need not get data from these empty cells, all these cells would be filled by just taking inverse of these values. So, here it would be 1 by 2, here it would be 1 by 4, here it would be 1 by 3 right here it is 1 by 3, again 1 by 3 and 1 by 2 right so this is how you can fill the remaining cells are the lower triangle of the matrix.

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Pair wise comparison matrix and computations: evaluation criteria				
Original matrix	Q	P	S	D
Q	1	2	4	3
P	$\frac{1}{2}$	1	3	3
S	$\frac{1}{4}$	$\frac{1}{3}$	1	2
D	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	1
Column total	25/12	11/3	17/2	9

Once you are done with filling up of these remaining sales what you should do you just take the total of each of these columns. So, let us say this is 25 by 2, so 1, then 1 by 2 plus 1 by 4 plus 1 by 3. So, you will get this value is 12 by it is 25 by 12. Similarly for P and S and then finally, D right, so if you look at total of this columns, so 3 plus 3 6 plus 2, 8 plus 1 so this is 9.

So, just take the total of all these columns the next step is divide all those cells by their respective total values for example, divide 12 by 25 by 12.

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Adjusted matrix	Q	P	S	D	Weights (row avg.)
Q	12/25	6/11	8/17	3/9	0.457
P	6/25	3/11	6/17	3/9	0.300
S	3/25	1/11	2/17	2/9	0.138
D	4/25	1/11	1/17	1/9	0.105
				Total	1.0

So, it would be 12 by 25, when you divide 12 by 25 by 12 it would be 6 by 25 and so on So, similarly you can divide all other cell values by their respective column totals.

Let us look at how to find out weight rate now. So, after finding all these values you just take average of all these points in this particular row. So, it is 12.25, 12 by 25 6 by 11, 8 by 17, 3 by 9 divided by 4. So, you will get averages 0.457 similarly for delivery it is 0.105. Keep in mind that the weight age sum should always be one it cannot exceed more than one. So, we will say that out of these 4 criteria the most important criterion is quality, the second important criterion is price third is service and forth is delivery right. So, what we have done we have done pair wise comparison amongst different criteria right.

Now, we will go for finding weights of different projects just look at this. So, now, we will go for pair wise comparison of different projects with respect to quality. So, far what we did? We did compare pair wise comparison of all the criteria with each other. Now with respect to quality criterion we will compare different projects. So, if you are asking to an expert how the quality of project P 2 is important then P 1.

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Supplier pair wise comparison matrices and priorities


A: wrt quality

	P1	P2	P3	P4	Wts
P1	1	5	6	1/3	?
P2		1	2	1/6	?
P3			1	1/8	?
P4				1	?

A: wrt price

	P1	P2	P3	P4	Wts
P1	1	1/3	5	8	?
P2		1	7	9	?
P3			1	2	?
P4				1	?

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The expert says quality of project P 2 is 5 times important than quality of P 1 project. The quality of P 3 project is 6 times important than quality of P 1 project and so on. The remaining cells can be filled similarly for example, when I say P 2 is 5 times important than P 1 it means P 1 is 1 by 5 time important then P 1. If you look at this if you look at this this cell P 4 is 1 by 3 time important than P 1 it means P 1 is 3 times important than P 4 and so on. Similarly you just go for the totaling of all these columns divide each cell value by the total of each of those columns and then take the average right. Similarly you can do this comparison amongst projects with respect to price. So, whatever you did for quality you can do it for price also right.

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Supplier pair wise comparison matrices and priorities


A: wrt service

	P1	P2	P3	P4	Wts
P1	1	5	4	8	?
P2		1	1/2	4	?
P3			1	5	?
P4				1	?

A: wrt delivery

	P1	P2	P3	P4	Wts
P1	1	3	1/5	1	?
P2		1	1/8	1/3	?
P3			1	5	?
P4				1	?

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Let us similarly for with respect to service and with respect to delivery. So, you will have to get data from experts how many times total. So, first of all you would be collecting data on 4 different criteria and then second time with respect to quality, with respect to price, with respect to service, and with respect to delivery. So, you will be taking data 5 times from experts and if there is one expert. So, he would be filling you 5 different questionnaires right.

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Calculation of the weights A: wrt quality										
	P1	P2	P3	P4			P1	P2	P3	P4
P1	1.00	5.00	6.00	0.33		P1	1/4.37	5.00/12.50	6.00/17	0.33/1.63
P2	0.20	1.00	2.00	0.17		P2	0.20/4.37	1.00/12.50	2.00/17	0.17/1.63
P3	0.17	0.50	1.00	0.13		P3	0.17/4.37	0.50/12.50	1.00/17	0.13/1.63
P4	3.00	6.00	8.00	1.00		P4	3.00/4.37	6.00/12.50	8.00/17	1.00/1.63
Column total	4.37	12.50	17.00	1.63						
	P1	P2	P3	P4	Row Sum	Normalized				
P1	0.229008	0.4	0.352941	0.205128	1.187077	0.296769				
P2	0.045802	0.08	0.117647	0.102564	0.346013	0.086503				
P3	0.038168	0.04	0.058824	0.076923	0.213915	0.053479				
P4	0.687023	0.48	0.470588	0.615385	2.252996	0.563249				

So, let us find out the weight age for quality criterion and this comparison is amongst projects right. So, this is what was your initial data 15 6 3 1 by 3. So, here I have just divide 1 by 3, and I have taken this value as 0.3 right. So, you can also do such type of calculation, after doing this just calculate the total of each of these columns 4 columns divide this one value by 4 by 4.37, 0.2 by 0.37. So, you will get all these values after dividing this for P 1, P 2, P 3, P 4 and this is the sum of all these P 1 to P 4 just divided by 4. So, this is known as normalization right. So, the highest if you look at the quality criterion the most important project is point ya; this is P 4, 0.56 because the this value is highest this the next important one is P 1 then P 2 and finally, this. So, we will find out the weight ages of all these projects with respect to other criteria as well right. So, this was for quality criteria. So, you can do it for price right.

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A wrt Price				
	P1	P2	P3	P4
P1	1.00	0.33	5.00	8.00
P2	3	1	7	9
P3	0.166667	0.142857	1	2
P4	0.125	0.111111	0.5	1
Column Sum	4.29	1.59	13.50	20.00

	P1	P2	P3	P4	Row Sum	Normalized
P1	0.23301	0.21	0.37037	0.4	1.213380079	0.30334502
P2	0.699029	0.63	0.518519	0.45	2.297547645	0.574386911
P3	0.038835	0.09	0.074074	0.1	0.302909026	0.075727256
P4	0.029126	0.07	0.037037	0.05	0.186163251	0.046540813

So, for price again these are different totals first second third and fourth after finding out these values you just divide the cell values by this total value. So, 1 by 4.29, 3 by 4.29 you will get these values right 0.23, 0.69 and similarly you can calculate other values also and then finally, take the average of first row second third and fourth row right. So, with respect to price which is most important project now can you tell me? Yes the most important project is P 2 because the weight is 0.574, after this the most important project is 0.30 right then 0.075 then 0.046 right similarly for other two criteria you can calculate these normalized scores right.

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Supplier pair wise comparison matrices and priorities						
A: wrt quality						
	P1	P2	P3	P4	Wts	
P1	1	5	6	1/3	0.297	
P2		1	2	1/6	0.08	
P3			1	1/8	0.53	
P4				1	0.563	
A: wrt price						
	P1	P2	P3	P4	Wts	
P1	1	1/3	5	8	0.303	
P2		1	7	9	0.573	
P3			1	2	0.078	
P4				1	0.046	
A: wrt service						
	P1	P2	P3	P4	Wts	
P1	1	5	4	8	0.597	
P2		1	1/2	4	0.140	
P3			1	5	0.214	
P4				1	0.050	
A: wrt delivery						
	P1	P2	P3	P4	Wts	
P1	1	3	1/5	1	0.151	
P2		1	1/8	1/3	0.060	
P3			1	5	0.638	
P4				1	0.151	

So, this is the complete weight ages for all these projects with respect to all those criteria. So, for quality you have got this for price you have got this these weights for service you have got these weights and for delivery these weights and this is very simple I think you can easily calculate.

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Computation of weights: supplier alternatives					
Adjusted matrix			Weights (row avg.)		
Quality			0.457		
Price			0.300		
Service			0.138		
Delivery			0.105		
	Quality	Price	Service	Delivery	Wts (Q*P*S*D)
P1	$457 * .297$	$.300 * .303$	$.138 * .597$	$.105 * .151$.325
P2	$457 * .087$	$.300 * .573$	$.138 * .140$	$.105 * .060$.237
P3	$457 * .053$	$.300 * .078$	$.138 * .214$	$.105 * .638$.144
P4	$457 * .563$	$.300 * .046$	$.138 * .050$	$.105 * .151$.294

Now, once you are done with this you are also having the weights of criteria initially what you calculated. So, 0.457 was there for quality and 0.1205 was for delivery. Now which project is to be selected at the end of the day now for this what you should do you

have got these weights for each of those criteria. So, 0.457 multiply it by 0.297 by this value; similarly 0.457 this weight of quality is to be multiplied with this weight of project P 2.

Similarly, 0.53 is to be multiplied by 0.457 and 0.457 to be multiplied by 0.563 right. So, this way you would be getting the total weight again is 0.325. So, weights here is this is the multiplication of all these plus and you can divided by 4 right. So, it is 0.325 for first project, for forth project 0.294. So, finally, which project you will be selecting at the end of the day you would be selecting project P 1 right because it has got the highest weight age. The second number project would be P 4 project then P 2 and then finally, P 3. So, this how you can solve any question related to AHP, and as I said AHP has got multiple applications whether you are selecting different sites for your manufacturing plant, you are selecting a supplier from several suppliers, if you want to let say after engineering or after doing any degree if you are if you want to go for let us say a job or if you want to go for higher studies are, if you want to start your own business is not it. So, there are different options which you can select for your future work right. So, you can apply AHP in that case also.

So, let us look at another important MCDM technique is called topsis; it is the full form of topsis is technique of order preference by similar to ideal solution. This is also a technique which gives you ranking at the end of the day similarly what we have seen in AHP; AHP rank different projects right.

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TOPSIS METHOD

Technique of Order Preference by Similarity to Ideal Solution

This method considers three types of attributes or criteria

- Qualitative **benefit** attributes/criteria
- Quantitative **benefit** attributes/criteria
- **Cost** attributes or criteria

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And we selected finally, in AHP this was our selection right P 1 project was our final selection. So, in topsis method also we can select one of the projects out of several choices, and it has got several other applications. So, this method considered 3 types of attributes. So, you will have some qualitative benefits you will have some quantitative benefit, attributes are criteria and you have got cost attributes are criteria. Let us see what these criteria are; for example, if I say I am making a let us say a product. So, I would always like to minimize cost of the product right. So, that can be one of the criteria. Suppose if I want to if I want to maximize profit then that would be again one of the criteria.

So, what we do generally in this method? We come up with two artificial alternative solutions, the first solution is known as ideal solution and the second one is negative ideal solution. When I say ideal solution it means all attributes are at their best levels. So, let us say if I want to maximize my profit, I want to increase reliability of the system, I want more and more quality right. So, all of those would be at their best levels when I say negative ideal solution it means the one which has the worst attribute values.

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- In this method **two artificial alternatives** are hypothesized:
- **Ideal** alternative: the one which has the **best level** for all attributes considered.
- **Negative** ideal alternative: the one which has the worst attribute values.
- TOPSIS selects the alternative that is **closest** to the **ideal** solution and **farthest** from **negative** ideal alternative.

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So, it is a solution which is completely opposite to your ideal solution right. So, let us say if peace let us a P plus the negative solution would be P minus right. So, that kind of situation you can think of.

So, what we generally do in topsis it selects the alternative that is closest to the ideal solution and farthest from the negative ideal solutions. So, that is the principle of topsis method. So, what are different input to topsis. So, you have got different alternatives and different criteria. So, you can have m by n matrix.

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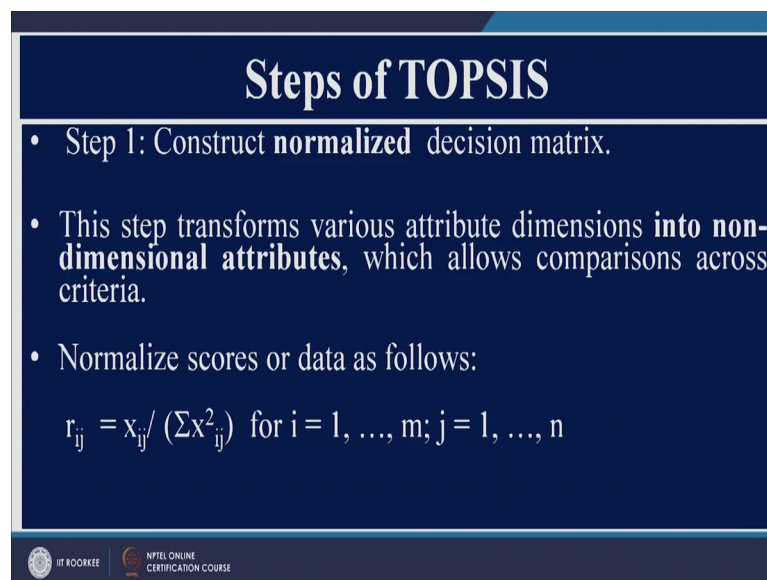
Input to TOPSIS

- TOPSIS assumes that we have m **alternatives** (options) and n **criteria** and we have the **score** of each option with respect to each criterion.
- Let x_{ij} score of option i with respect to criterion j
We have a matrix $X = (x_{ij})$ $m \times n$ matrix.
- Let J be the set of **benefit attributes** or criteria (more is better???)
- Let J' be the set of **negative attributes** or criteria (less is better???)

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So, you can have let us say 4 alternatives 3 criteria, 5 alternatives 4 criteria and so on right. So, you will have excisable would represent score of opinion i with respect to criterion j. So, you will have an m by n matrix. So, let j b the set of benefit attributes. Benefit attributes means those attributes which you would like to more and more which you would like to maximize for example, let us say again profit you want to maximize the efficiency, you want to maximize let us say space utilization is not it and z h be negative attributes it means we would like to minimize them for example, you want to minimize cost right you want to minimize you know conflicts in the organization, you want to minimize waste is not it. So, you can have several negative attributes also.



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Steps of TOPSIS

- Step 1: Construct **normalized** decision matrix.
- This step transforms various attribute dimensions **into non-dimensional attributes**, which allows comparisons across criteria.
- Normalize scores or data as follows:

$$r_{ij} = x_{ij} / (\sum x_{ij}^2)^{1/2} \text{ for } i = 1, \dots, m; j = 1, \dots, n$$



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So, there a couple of steps in topsis the first step is we normalize our decision matrix. So, whatever data you are having initially you are supposed to normalize only. The reason for normalization is because you are input data would be having different units. So, at the end of the day you would be coming up with non dimensional attribute for each of those criteria are those attribute.

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- Step 2: Construct the **weighted normalized** decision matrix.
- Assume we have a set of weights for each criteria w_j for $j = 1, \dots, n$.
- Multiply each column of the **normalized** decision **matrix** by its associated **weight**.
- An element of the new matrix is:



$$v_{ij} = w_j r_{ij}$$

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Then after normalizing matrix you are supposed to multiply those that particular matrix by it is better j s and the weight ages would be given by experts. So, multiply a column of normalized matrix by it is associated.

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- Step 3: Determine the **ideal and negative** ideal solutions.
- **Ideal** solution.
 $A^* = \{v_1^*, \dots, v_n^*\}$, where
 $v_j^* = \{ \max_i (v_{ij}) \text{ if } j \in J; \min_i (v_{ij}) \text{ if } j \in J' \}$
- **Negative** ideal solution.
 $A' = \{v_1', \dots, v_n'\}$, where
 $v' = \{ \min_i (v_{ij}) \text{ if } j \in J; \max_i (v_{ij}) \text{ if } j \in J' \}$

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With the third point is determine the ideal and negative ideal solution, as I said what is the ideal solution ideal solution is a solution where all the attributes are at their best levels and negative ideal is just opposite to ideal solution. So, they these two solutions more in exactly opposite direction; calculate the separation measures for each alternative.

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- Step 4: Calculate the **separation** measures for each alternative.
- The separation from the **ideal** alternative is:
$$S_i^* = \left[\sum_j (v_j^* - v_{ij})^2 \right]^{1/2} \quad i = 1, \dots, m$$
- Similarly, the separation from the **negative ideal** alternative is:
$$S_i' = \left[\sum_j (v_j' - v_{ij})^2 \right]^{1/2} \quad i = 1, \dots, m$$

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So, we will we will calculate separation measures for ideal solution as well as for negative ideal solution. In fact, this thing should be clearer when I will teach you an example then finally; we calculate relative closeness to the ideal solution.

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- Step 5: Calculate the **relative closeness** to the ideal solution C_i^* .

$$C_i^* = S_i' / (S_i^* + S_i') , \quad 0 < C_i^* < 1$$

Select the option with C_i^* closest to 1.

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So, this is the relative closeness would always be between 0 to 1. So, this ratio of negative solution negative ideal solution divided by some of negative solution and positive ideal solution. So, select the final answer which is closest to one or just got highest value.

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Applying TOPSIS Method to Example				
Weight	0.1	0.4	0.3	0.2
	Style	Reliability	Fuel Eco.	Cost
Civic	7	9	9	8
Saturn	8	7	8	7
Ford	9	6	8	9
Mazda	6	7	8	6

So, let us look at this question would be applying Topsis method to this particular example, let us say there are 4 different cars Civic, Saturn, ford and Mazda and you have to select one of these cars and there are 4 criteria on which you would be selecting these cars. So, you have got style, you have got reliability, you have got fuel economy and you have got cost and these are the weights given by experts for each of these criteria. So, will say that the most important weight age is given by expert is 0.4 right. So, this is your input matrix right.

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Applying TOPSIS to Example	
<ul style="list-style-type: none">• $m = 4$ alternatives (car models)• $n = 4$ attributes/criteria• x_{ij} = score of option i with respect to criterion j $X = \{x_{ij}\}$ 4×4 score matrix.• J = set of benefit attributes: style, reliability, fuel economy (more is better)• J' = set of negative attributes: cost (less is better)	

So, now let us move towards applying topsis to this particular example. So, this is a case of 4 by 4 matrix, because there are 4 car models and you have got 4 criteria right.

So, what would be your benefit attributes? The benefit attributes would be what you would like to maximize you want more and more stylish car, you want more and more reliable car, you want more and more fuel economy right. So, these are known as benefit attributes right and let us say j dash is set of negative attributes right. So, you want something to be less and less right. So, you would definitely like cost to be less and less right. So, this is how you can have 4 by 4 matrix.

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Steps of TOPSIS				
• Step 1(a): calculate $(\sum x_{ij})^{1/2}$ for each column				
	Style	Rel.	Fuel	Cost
Civic	49	81	81	64
Saturn	64	49	64	49
Ford	81	36	64	81
Mazda	36	49	64	36
$\sum x_{ij}$	230	215	273	230
$(\sum x)^{1/2}$	15.17	14.66	16.52	15.17

So, let us first of all normalize the matrix right for normalization what you need to do you just take the square of those values which are there in your initial matrix take the sum of all these columns all these 4 columns, after this just take the under root of these values right, so this 15.17, 14.66, 16.5 and 15.17 right.

So, the next step is divide each of those values divide each column by this value.

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Steps of TOPSIS				
<ul style="list-style-type: none"> Step 1 (b): divide each column by $(\sum x_{ij}^2)^{1/2}$ to get r_{ij} 				
	Style	Rel.	Fuel	Cost
Civic	0.46	0.61	0.54	0.53
Saturn	0.53	0.48	0.48	0.46
Ford	0.59	0.41	0.48	0.59
Mazda	0.40	0.48	0.48	0.40

So, let us say how you are getting this 0.46. So, initial value in this cell was 7 right. So, divide 7 by 15 by 15.17 right. So, how did you get this value 0.61, what was the original value here in the very first matrix? It was 9 right. So, divide 9 by what 14.66 right you will get this right 0.61 right. So, this how you can get your normalized matrix right very simple just a simple arithmetic.

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<ul style="list-style-type: none"> Step 2 : multiply each column by w_j to get v_{ij}. 				
Weight	0.1	0.4	0.3	0.2
	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

Now once you are done with this process the next step is you need to get weighted normalized matrix right. So, just multiply the previous matrix by it is weights right. So,

0.1 is to be multiplied by 0.6, 0.4 by 0.4 by 0.2 4 6 and so on right. So, this is how you will be getting your weighted normalized matrix right.



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- Step 3 (a): determine ideal solution A^* .

$$A^* = \{0.059, 0.244, 0.162, 0.080\}$$

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

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Once you are done with this determine ideal solution and what is ideal solution? Ideal solution you would like to choose where all these style reliability and fuel economy are having their best values right. So, for the style the highest value is 0.5 0.059. So, write this value over here, for reliability the highest value is 0.244 for fuel the highest value is 0.16 and since you want to minimize cost.


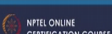
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- Step 3 (a): find negative ideal solution A' .

$$A' = \{0.040, 0.164, 0.144, 0.118\}$$

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

↓
↑






So, you need to select this value the least one right. So, this is your ideal solution similarly you can have a negative ideal solution right. So, select the least of style least of reliability, least of fuel economy and maximum of cost right. So, write all those 4 points these are your negative solution, these two are your positive ideal solution and negative ideal solutions right.

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- Step 4 (a): determine separation from ideal solution
 $A^* = \{0.059, 0.244, 0.162, 0.080\}$
 $S_i^* = [\sum (v_j^* - v_{ij})^2]^{1/2}$ for each row

	Style	Rel.	Fuel	Cost
Civic	$(.046-.059)^2$	$(.244-.244)^2$	$(.162-.162)^2$	$(.106-.080)^2$
Saturn	$(.053-.059)^2$	$(.192-.244)^2$	$(.144-.162)^2$	$(.092-.080)^2$
Ford	$(.059-.059)^2$	$(.164-.244)^2$	$(.144-.162)^2$	$(.118-.080)^2$
Mazda	$(.040-.059)^2$	$(.192-.244)^2$	$(.144-.162)^2$	$(.080-.080)^2$

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Then find out separation matrix from ideal solution and separation matrix for negative ideal solution right. So, this is again a simple one. So, this value is there this for negative solution.

Let us say this is for positive solution. So, 0.4 0.046 minus this whole square, 0.053 minus this whole square similarly for this how would you write 0.144 minus 0.162 whole square right and in this way you can calculate separation from ideal solution a very simple arithmetic right. Similarly separation from negative ideal solution, but before that let us write what are what is the total of all these rows write. So, this is how you can calculate the total and then take the under root of that value you right.

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- Step 4 (a): determine separation from ideal solution S_i^*

	$\Sigma(v_j^* - v_{ij})^2$	$S_i^* = [\Sigma(v_j^* - v_{ij})^2]^{1/2}$
Civic	0.000845	0.029
Saturn	0.003208	0.057
Ford	0.008186	0.090
Mazda	0.003389	0.058

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So, this is total of all these values right. So, this is your separation from ideal solution right similarly you can have separation from negative ideal solution the same approach you have to apply right.

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- Step 4 (b): find separation from negative ideal solution $A' = \{0.040, 0.164, 0.144, 0.118\}$

$S_i' = [\Sigma(v_j' - v_{ij})^2]^{1/2}$ for each row

	Style	Rel.	Fuel	Cost
Civic	$(.046-.040)^2$	$(.244-.164)^2$	$(.162-.144)^2$	$(.016-.0118)^2$
Saturn	$(.053-.040)^2$	$(.192-.164)^2$	$(0)^2$	$(.092-.118)^2$
Ford	$(.059-.040)^2$	$(.164-.164)^2$	$(0)^2$	$(0)^2$
Mazda	$(.040-.040)^2$	$(.192-.164)^2$	$(0)^2$	$(.080-.118)^2$

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And then you can have separation from negative ideal solution right, now what we need to calculate? We need to calculate ratio right.

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- Step 4 (b): determine separation from negative ideal solution S_i'

	$\Sigma(v_j' - v_{ij})^2$	$S_i' = [\Sigma (v_j' - v_{ij})^2]^{1/2}$
Civic	0.006904	0.083
Saturn	0.001629	0.040
Ford	0.000361	0.019
Mazda	0.002228	0.047

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Ratio of what negative ideal solution to the sum of negative plus positive ideal solution, right. So, this is the ratio for civic it is 0.714 this is the ratio for Saturn this is the ratio for ford and is the ratio for Mazda right.

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- Step 5: Calculate the relative closeness to the ideal solution $C_i^* = S_i' / (S_i^* + S_i')$

	$S_i' / (S_i^* + S_i')$	C_i^*	
Civic	0.083/0.112	0.74	← BEST
Saturn	0.040/0.097	0.41	
Ford	0.019/0.109	0.17	
Mazda	0.047/0.105	0.45	

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So, which car would you select at the end of the day? Civic car right. Why because this is this solution is closest to the ideal solution and farthest from the negative ideal solution and this value is highest in this particular column right. So, you should be selecting civic

car. So, I hope that you would have understood topsis method and if you can apply the same method for any other example also right.

So, similarly you can have let us say for 4 vendors you can have different criteria for example, cost of the product delivery time credit period quality and so on right and this is how you can get solution to the other problem as well right. So, let me summarize what we did in this session we did an example on AHP and we have also solved topsis method, we just took an example and we solved it using topsis method. So, with this let me complete this session.

Thank you very much.