Supply Chain Analytics
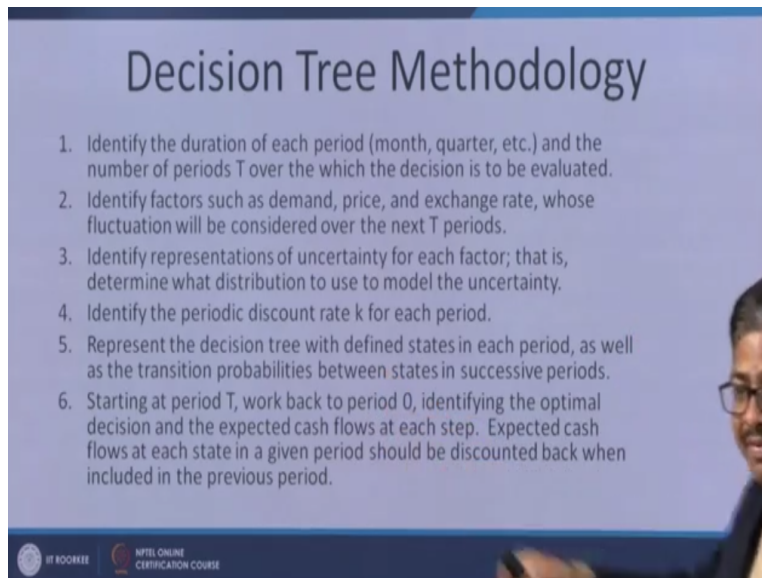**Prof. Dr. Rajat Agrawal**
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Lecture-37
Example of Using Decision Tree Incorporating Uncertainty in Single Factor

Welcome back, in our last sessions we were discussing about use of binomial uncertainty in our network design decisions. And, in the last session we discussed in detail about the use of decision tree for the purpose of evaluation of uncertainties in the future decisions. Now we have already gone through the representation of uncertainty, we have already gone through the concepts of time value of money and now in the last session we were discussing that what are the important things we should think into the mind when we are going for decision tree model.

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And, we discuss that we need to have the period, we need to have the duration of each period, then we need to find that what are the underline factors for which we are evaluating the uncertainty then we need to represent the factor of uncertainty for all these types of underline factor. Then we also need to know the time value of money and for that purpose we require some kind of discounting rate.

Then, we also need to have the what is the probability from one state to another state. And then we start calculating from period t, the last period to period 0. From where, we have started the

building of this decision tree. So, all these things in a very detailed manner we discussed in our last session.

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Then we came to a particular specific example about deciding the warehousing situation that we have alternatives where, we can take the long term lease decision, and we can take a spot market also. There are certain issues related to long term lease and spot market. A spot market is currently costlier, but there may be fluctuations with respect to prices of the a spot market at in future the prices of a spot market get go down.

Then there is a situation of demand also, today there is a demand and according to that demand, I take lease space, but it is quite possible that tomorrow demand may go down and in that case when demand can go down I have some kind of unused a space in my warehouse, so that is also extra cost, so all these things we discussed.

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ABC Logistics: Three Options

- Get all warehousing space from the spot market as needed
- Sign a three-year lease for a fixed amount of warehouse space and get additional requirements from the spot market
- Sign a flexible lease with a minimum change that allows variable usage of warehouse space up to a limit with additional requirement from the spot market

Now coming very specific so, there are 3 possibilities considering all those situations which we discuss in the last session and in the previous slide as well. We have three options available to this company, these options are get all warehouse in this place from the spot market as required. So, today you require 60 square feet, so that take much. Tomorrow you require 100 square feet, you take much, then you require 90 square feet leave some 10 square feet, and take only 90 square feet.
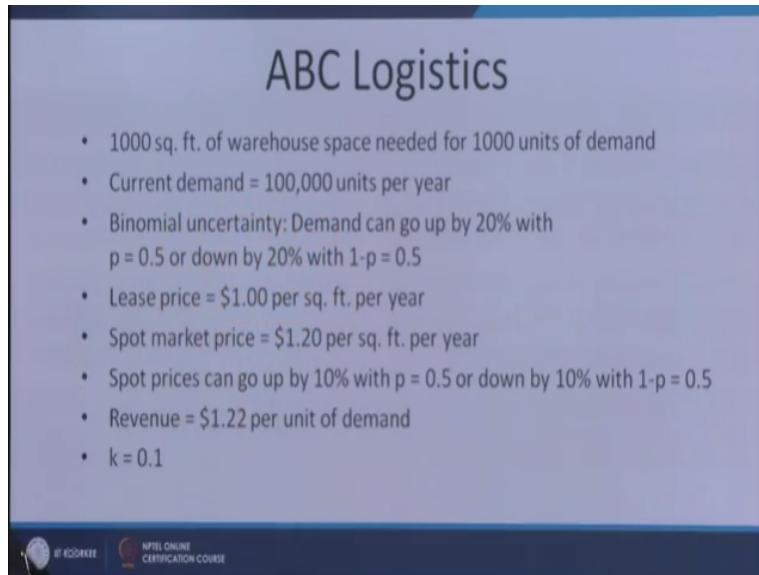
So, that type of flexibility is there in this first option, the second option that you sign a contract for 3 years, and it is a fix amount of warehouse space, and get additional requirements from the spot market. So, you have taken let us say some 100 square feet area, from the lease agreement, and if demand exceeds beyond that so the additional requirement can be fulfilled from the spot market at whatever price it may be available at that period.

And the third is sign a flexible lease that you have taken a minimum amount of fix area. and then you have some kind of variable uses of warehouse space, up to a limit and additional requirement from the spot market. So, this is taking the advantage of the spot market condition, as well as for the leases condition you can have this kind of flexible ware for some space.

May be for, 60 square feet you have this rate of uses, and beyond that, you have some other rate of uses, and then there is the upper limit of the availability of the space in this agreement. And,

then even you go beyond that, if demand increases beyond that the additional requirement can be fulfil from the spot market. So, it takes the case of both spot as well as your lease agreement.

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Now, coming to some specific data. So, the data says you require 1000 square feet of ware house space, for 1000 units. One unit take 1 square feet, so for 1000 units you take 1000 square feet. See current demand is 100,000 units per year. That is the current level of the demand. The binomial uncertainty is demand can go up by 20% with a probability of 0.5, and demand can also decrease by 20% with a probability of 1-p, that is again 50%.

So, chances of increasing and decreasing are same, demand can increase same here, we are taking the same values 20%, for the increase in 20% for the decrease. But, in reality it can be different also, it is possible that the chances that demand can increase by 30% is 50%, and demand can decrease by 10% is 50 %. So, that type of data also there. It can also be modified that you have same value of U and D but different probabilities.

The demand can go up by 20% with the probability of 60%, and demand can decrease by a factor of 20%, with a probability of 40 %. So, all these combinations are possible. Here just for the sake of simplicity you can say that simple data is taken. Where you small u if you remember introduce this term small u, and small d in our last sessions. So, that small u and, small d these are same. Small u becomes in this case.
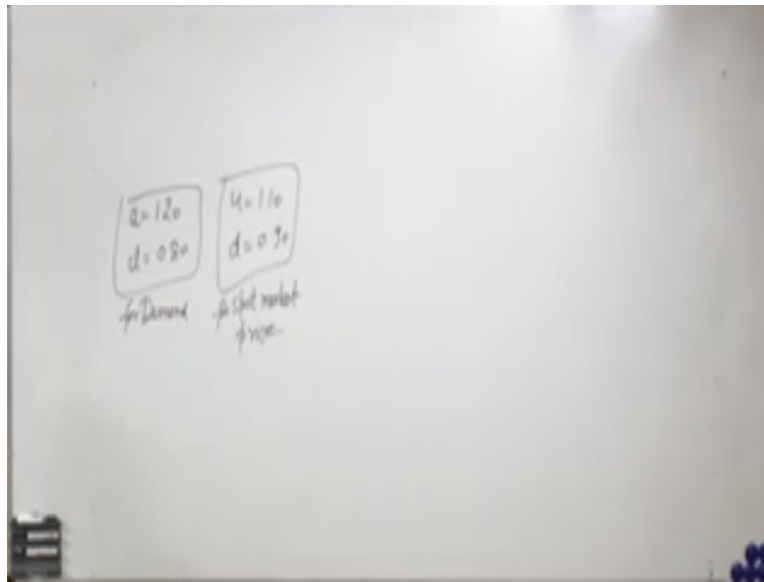
The value of small u becomes 1.20, and value of small d becomes 0.80. And, the probabilities for both these things are 50% and 50%. Though in reality it can be different also. The u can have a probability of 60%, it can have a probability of 40%, and all other alternative combinations are possible, so as long as you have p and 1-p. So, you can have a right view of cases. Then lease price is given to us that is that 1 dollar per square feet per year.

That is the lease price, if I go for lease agreement, so I will be charged at this rate. The spot market price currently is 1.2 dollar per square feet, per year. That is the spot market price. But, there can be an uncertainty in a spot market price also, and a spot market price can also go up or down, by a factor of 10%. So, this u and d is for the demand. And, there will be another u and d.

That is 1.10 and d is 0.90, this if for a spot market prices. So, you have 2 u and d, one is and it is a matter of chance that for both these the probabilities are 50% again. So, spot market prices can go up by 1.10. The probability is 50% can decrease by a factor of 0.90, probability is 60%. And, the revenue is from these units, which you are storing in your warehouse 1.22 dollar per unit.

And, for determining the present value of the future cash flows. The discount rate is 10%. So, that is the discount rate K is discount rate. So, that is 10%. Now, let us see the development of decision tree for this set of data.

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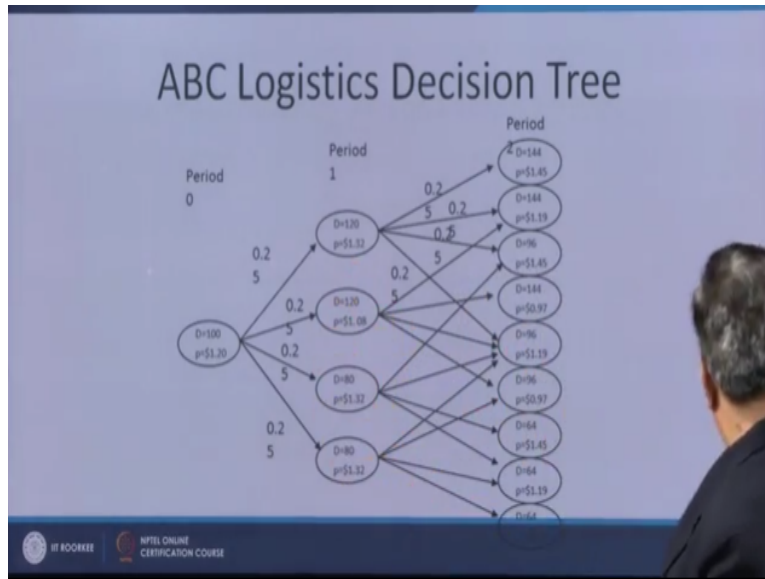
So, it is good if to understand this development of decision tree, we keep this data readily available on one side of our notebooks somewhere you note down this data. And with the help of this data we will see that how we will develop or what we develop this decision tree, on the board, and then finally we will see that whether our looks like that the decision tree which there in the next slide.

So, let us see how to develop the decision tree with this data, and I expect with that you will also be able to develop decision trees on your own.

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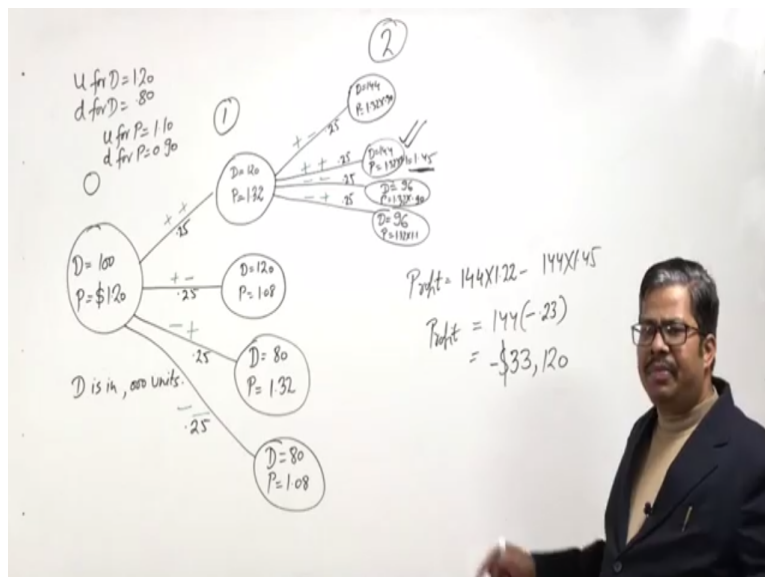
First is the time t0, this is the period 0, where you have some demand, and the price of the spot market, so initially is current demand is let us say 100,000 units. So, for the sake of simplicity I am simply writing the 100. And, it is into d is n units, so you are clear that three zeros are more, after this 100. And, the initial price of the spot market is 1.2 dollar per square feet per year. So, that is the initial price that is 1.20.

Now, we move to period 1, in period 1 what are the different possibilities 1 possibility is that demand increases, and price also increases. So, demand is increasing, let me give you this with the help of + +. Demand is increasing and, price is also increasing. Both these things are having the positive things. So, the factor of u for demand is 1.20 and so, this demand will become 120.

And, price for that factor is 1.10, so price will become 1.20 into 1.1, so 1.32. So, you can see that this is the first case that in period 1. From here you are moving to this place. That your demand is becoming 120, and price is becoming 132. The next is second case where your demand can increase, demand is increasing, but price is decreasing, so demand is increasing, so demand becomes 120.

But, initial price is 1.20, and u is 0.90 for price, so it becomes 1.20 into 9, so it remains 1.08 that is the another state, that d becomes 120, and price becomes 1.08. Then another possibility is, now demand can decrease and price can increase, so demand is decreasing by a factor of d, and d is 0.80, we wrote initially so d becomes 0.80. And, p is increasing p is +, so p becomes 1.32.

And, similarly another possibility is that demand is decreasing, and price is also decreasing, both negative. So, in that case d becomes 80, and price becomes 1.08. So, these are the 4 possible states, 2 states where demand is increasing, 2 states where demand is decreasing. 2 states where price is increasing, and 2 states where price is decreasing. So, in all these 4 states are possible. And since the price and demand, these 2 things are independent of each other not affecting each other.

So therefore the probabilities, now we need to assign if you go back to our earlier slide. That we need to assign the transition probabilities also. This is step number 5, so we will assign the

transition probabilities, that the probability of increasing the price is 0.5 probability of increasing the demand is also 0.5, so the transition probability here is 0.25.

And, so on for all other decisions also all other states which we are getting for all these different states, the transition probabilities are 0.25, 0.25, 0.25 each. So, you get the complete set of 4 states at time period at all. Now, we need to go further, because we are considering for 3 years, so now this is the next year decisions, from here you can have another 4 possibilities. And, these possibilities are same, your demand can increase.

Demand can increase there are 2 possibilities, here price is decreasing here price is increasing, then demand can decrease 2 possibilities, and in these 2 possibilities again price can decrease, and price can increase. So, further you have 4 possibilities. But, these 4 possibilities will be evaluated from this state now. So, if demand is increasing let us say if demand is increasing, so her it is 120, demand increase factor is 1.2. So, it may become 144.

Here also it can become 144, when demand is decreasing by a factor of 80, 0.80, do demand can become 0.80multiplied by 120, that is 96. So, you should remember that I should write here u for d is 1.20, a small for d is 0.80, u for price is 1.10, and d for price is 0.90. So, similarly when my price is changing, when I am having these 2 values where price is increasing. So, 1.32 multiplied by u that is 1.1. So, this value will come here, 1.32 into 1.1, 1.32 into1.1.
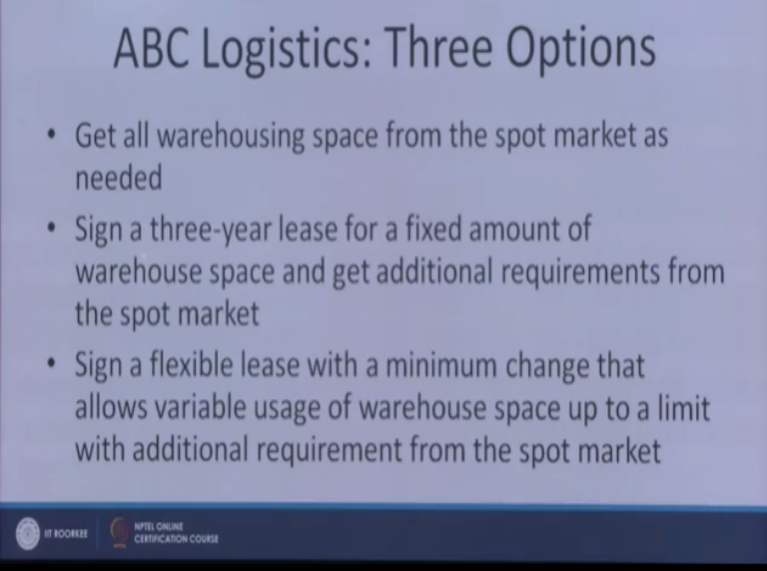
And, when price is decreasing this is here, price equals to 1.32 multiplied by 0.90. And, 1.32 multiplied by 0.90. And, we need to have the transition probabilities, chances of increasing the demand, 50% chances are decreasing the demand. 50% chances of increasing the price, 50% chances of decreasing the price 50%. So, again the transition probabilities are 0.25 for all this 4 stages in period 2.

So, I request my participants that for remaining 3 states of period 1, you can develop similar type of stage for period 2. And, when you develop states for these 3 states. The successive states many of those states will be the common which you get here in the, this discussion. Like when the

demand is 120, if it increases it will become it will become increase by a factor of 20%, so 1.2, and then in that case it may become 144.

Demand may become 144.So, many of those are possible that where you have the common values in these final state. So, I request all my participants that you please develop, and once you develop this entire tree. Decision tree, so this is the representation of our various states, the transition probabilities. And, now the evaluation of various alternatives.

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That these 3 alternatives which are there, will start. So, but before that please complete this decision tree. Because, this decision tree will help us and, I have this please be careful in reading this values. These are 0.25, 0.25, 0.25, and 0.25. So, slightly misadjusted values are there. So, all these I have written very clearly on the board. So, these are 0.25, 0.25, 0.25 as my transition probabilities. Similarly, in this case of period 2.

This has 4 possibilities, which are represented here, and from this also you have 4 different possibilities, from this state also you have 4 different possible states which are emerging. And, from here also we have 4 different states which are emerging. And, now you can see, now when you make your own decision tree diagram, you will see that many of these states are common.

For example, if we talk of this state d equals to 96 and, price is 1.19, this is common in almost, because you have 1 arrow coming from here, 1 arrow coming from here, 1 arrow coming from here, and 1 arrow coming from here, so this state is being represented by all the 4 states of the period 1. You can have possibility. So, it is not exclusive 4, you are going to get from here.

Your states will have various you can say common is gets, in these 3 other calculations. So, otherwise 4 into 4 you will have 16 unique states, but you will not have 16 unique states you can just count that we have in period to 1, 2, 3, 4, 5, 6, 7, 8, 9. 9 unique states, so that is something we should understand that and this again rooted to that type of discussion which we have in last to last session,

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Where, when I am talking of Pu, when it can increase, and it can decrease. So, when Pu decreases in becomes Pud, when Pd increases it becomes Pud, so this state becomes common for both these previous states. So, same logic apply in this particular case also. That there are various common states in developing this decision tree.
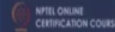
But, developing this decision tree, is very good visual representation of our entire state of affairs which may be there in the future. Now, let us start discussion on the evaluation of the alternatives. That how are we going to evaluate these alternatives?

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ABC Logistics Example

- Analyze the option of not signing a lease and obtaining all warehouse space from the spot market
- Start with Period 2 and calculate the profit at each node
- For D=144, p=$1.45, in Period 2:

$$C(D=144, p=1.45, 2) = 144,000 \times 1.45 = \$208,800$$

$$P(D=144, p=1.45, 2) = 144,000 \times 1.22 -$$
$$C(D=144, p=1.45, 2) = 175,680 - 208,800 = -\$33,120$$

And, for that purpose first we are going to analyse the option of not signing a lease, and obtaining all warehouses from the spot market. So, we are considering this option first, that I am going to take the entire space on the basis of a spot market. So, as we mention that we need to start from the last period. So, we will start with the period 2. And, calculate the profit at each note, so at this case for d equals to 144, and price 1.45.

So, this is probably this situation where, 1.32 into 1.1. This is 1.45. So, I am starting with this particular state, and we will go backward, that you will come to know. For that purpose to determine the cost, here we will write it in such fashion, d 144, p is 1.45, comma 2, this comma 2 signifies that the time period it is 2 if I write comma 1, so this is the state at period 1. So, d 144, p 1.45 this is the state and comma 2 this is the time period.

So, this is the situation, and what will be the cost, because the demand is 144 here. You see how do we calculate this demand is 144, and price is 1.45, a square feet of space. So, and my company is committed to fulfil the entire demand. Whatever demand is there I am going to fulfil the demand completely, so I will multiply this price with this demand. So, and I already mention t is in 1000 units.

So, this 144,000 into 1.45, this is the cost of the space, and if I am having this much demand 144, so my revenue is for this option, my revenue is I will be selling, because my revenue is 1.22

dollar per unit of the demand , so I will multiply because I am fulfilling this demand 144,000. So, 144,000 x 1.22. That is my revenue. And, when I subtract cost from the revenue. So, 144,000 into 1.22 – this cost. That will become my profit.

So, I am subtracting 144 –this cost, and this is 175,680- 2800, this is 33,120 dollars negative. It means that if this is the situation 144 is the demand, and I am fulfilling the complete demand using the spot market I will be charged at the rate of 1.45. My revenue is not going to change, whether I am using spot market or lease. So, my customer is not worried about that.

So, I will get the fix amount of revenue on at the rate of 1.22. So, the simple logic is that my revenue is 1.44 into 1.22. t do that is the revenue which I am going to get. Revenue –cost, cost is 144 into 1.45 that is my cost, and so, this is the profit. So, 144 is common, so you can say that this -0.23. So, this multiplication yields me 33,120 dollars. So it is a loss, so I will evaluate all possibilities of such.

And, will develop the tabular kind of arrangement, and then will see that which option is better. So, we stop here, we understood the method of calculation, but we stop her today, and in our session, we will develop the tables and then we will evaluate, we will compare the 3 alternatives which are available to our warehousing company. Thank you very much.