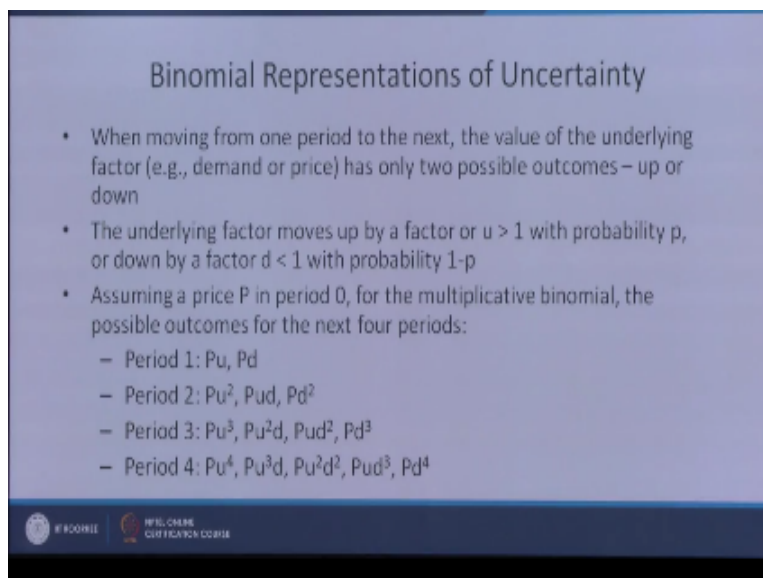


Supply Chain Analytics
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Lecture-36
Using Decision Tree for Handling Uncertainty

Welcome back and we were discussing in our last session about use of uncertainty in the predictive modelling and there are different types of uncertainties, we can have uncertainty related to demand, related to price, related to information, related to supply. So all those uncertainty is are there. There are uncertainties which we need to handle immediately and then there are uncertainties which are we handle in the long-term.

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Binomial Representations of Uncertainty

- When moving from one period to the next, the value of the underlying factor (e.g., demand or price) has only two possible outcomes – up or down
- The underlying factor moves up by a factor $u > 1$ with probability p , or down by a factor $d < 1$ with probability $1-p$
- Assuming a price P in period 0, for the multiplicative binomial, the possible outcomes for the next four periods:
 - Period 1: Pu, Pd
 - Period 2: Pu^2, Pud, Pd^2
 - Period 3: Pu^3, Pu^2d, Pud^2, Pd^3
 - Period 4: $Pu^4, Pu^3d, Pu^2d^2, Pud^3, Pd^4$

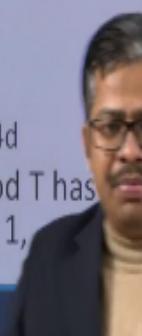
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Now in last session we discuss the binomial representation of the uncertainty and we discuss two types of binomial representation, one is multiplicative and another was additive. And we saw that how is we take one underlying factor P and it can increase in the subsequent period or it can decrease in the subsequent period. So here in that case we took price as our underlying factor P and small u is the factor by which it can increase in the next period. And we saw these type of table and this we also discuss with the help of a binomial tree.

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Binomial Representations of Uncertainty

- For the additive binomial, the states in the following periods are:
 - Period 1: $P+u, P-d$
 - Period 2: $P+2u, P+u-d, P-2d$
 - Period 3: $P+3u, P+2u-d, P+u-2d, P-3d$
 - Period 4: $P+4u, P+3u-d, P+2u-2d, P+u-3d, P-4d$
- In general, for the additive binomial, period T has all possible outcomes $P+tu-(T-t)d$, for $t=0, 1, \dots, T$



Similarly we also discussed in the last session about additive uncertainty where save you and small t r the amount of uncertainty is in the subsequent period, so if it is additive uncertainty our price will increase by $P + u$ and price will decrease by the factor of D , so similarly we can have binomial tree representation for these additive uncertainties and that we discuss that why binomial uncertainty and particularly multiplicative uncertainty is more used in the supply chain decisions.

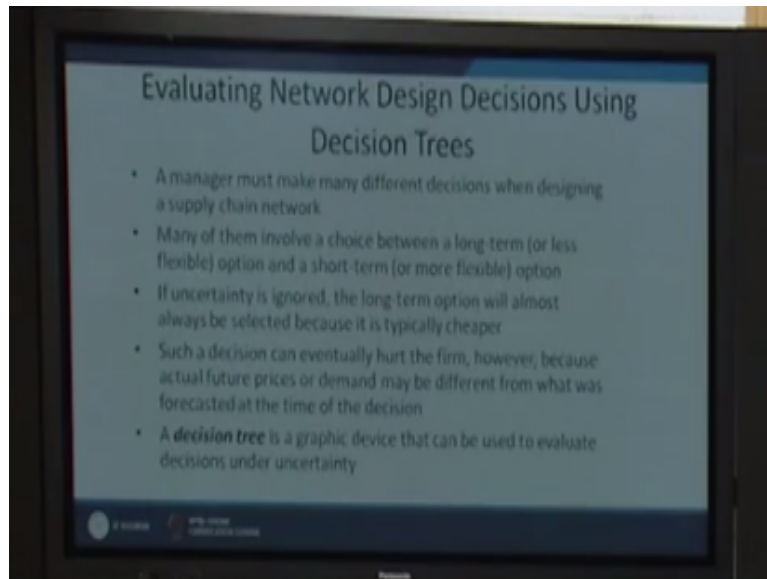
Because in case of additive uncertainties it is quite possible that you are underlined factor may get some kind of negative value, if this is price p and it is decreasing by if its value be over a period of time, so you see that in fourth period it is becoming $P - 4d$ and n th period it may become $P - nd$ and in that case this Nd factor may be much more than P and there can be a chances of negative price also which is not possible.

So in reality most of the time we will see that we will take multiplicative uncertainties in our consideration and the other reason we also discussed that many a times the size of your initial underlying value is also very important, if the initial price is 20 rupees, so 20 rupees change with the factor of 2% 3% 4% but it is difficult to say that 20 rupees with change with a factor of 2 rupees in the subsequent period.

So you cannot assign a fix value if your initial values are small, if your initial value is less 1000 rupee when the price may change by a factor of 5 rupees in the subsequent period. So that is also a limitation in the use of additive uncertainties in this case. So the example which

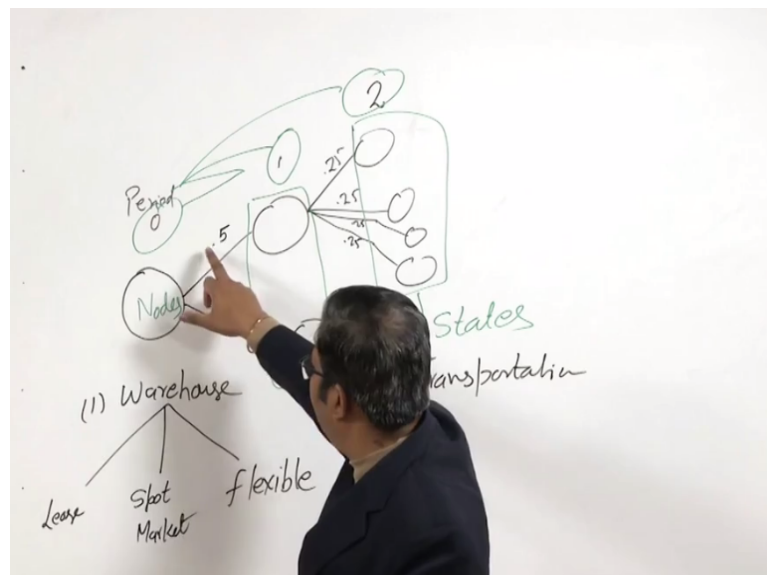
we discuss in the last case that in today session we are going to have a numerical example to discuss these representation of uncertainties in our decision making.

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Now for that purpose we will take the help of decision tree we decision tree binomial decision tree we discuss in our last session, now in today session we will see that how we can represent the decision tree for the purpose of uncertainty and with the help of probabilities associated with each node of decision tree, before I go for a specific example let me give you the idea of decision tree, that how it all evils for that involves.

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For that purpose this is a condition at state 0, now there are two possibilities this period zero and in period 1 you have two possibilities, the probabilities for the sake of simplicity we have same probability for both these states, then we can go further from this is state of time period

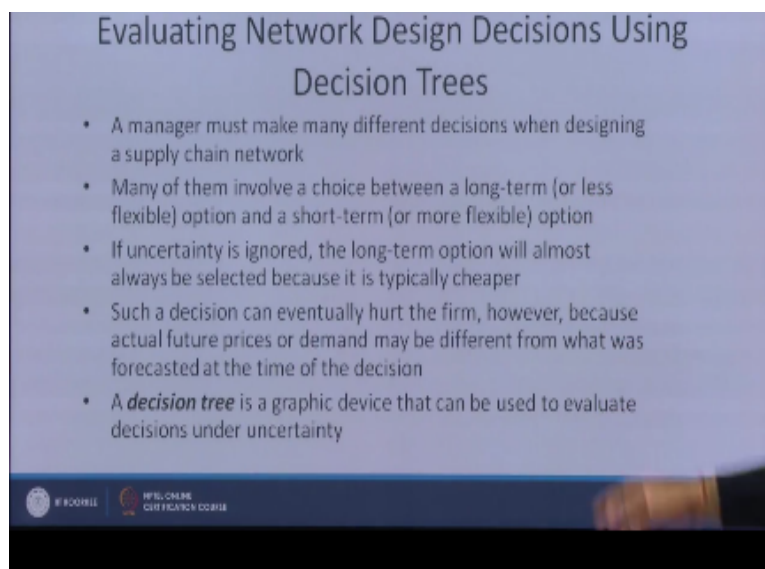
one there are further two possibilities or let us say 4 possibilities and each with the probability of 0.25.

And these are the further states, and so on with the state number 2 of time period 1 you can have another 4 possibilities each with a probability of 0.25, this is time period 2, and in the same manner we can develop this diagram, so at each these are known as nodes, these all are nodes, so all these nodes will give rise to branches, these are the branches coming out of these nodes and further you get node and then another branches are there.

And each branch with two branches these are the branches coming out of these nodes and further you get node and then other branches are there and each branch will terminate at a node and then another branches will be there, so it is like a tree type of arrangement where you have nodes giving rise to branches nodes giving rise to branches and so on know this graphic representation of decision tree is a very useful tool we already have gone through this type of decision tree discussions in the classes of statistics in mathematics also.

But here we will see a practical application of decision application of decision tree in evaluating our decisions with respect to supply chain where uncertainty is there, because all these possibilities if uncertainty is not there so there will not be multiple branches, there will only be one branch, you have one thing which is 100% sure, but since there are uncertainties, so for those uncertainty is to inbuilt in your system you have different branches with different probabilities.

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Evaluating Network Design Decisions Using Decision Trees

- A manager must make many different decisions when designing a supply chain network
- Many of them involve a choice between a long-term (or less flexible) option and a short-term (or more flexible) option
- If uncertainty is ignored, the long-term option will almost always be selected because it is typically cheaper
- Such a decision can eventually hurt the firm, however, because actual future prices or demand may be different from what was forecasted at the time of the decision
- A *decision tree* is a graphic device that can be used to evaluate decisions under uncertainty

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Now let us see what this light says that a major bust make many decisions when designing a supply chain network, so you need to make decisions like a simple example I say you that I want to develop facilities in my supply chain and I am deciding about a warehouse, this is a facility to be develop in my supply chain. Now for this warehouse I have three different alternatives, what are the three different alternatives?

One alternative that I go for a lease arrangement for certain number of periods, so there is a lease arrangement for 3 years, 4 years. So I am going for rearrangement where I have a fixed capacity dedicated capacity given to me for a certain number of period. The other option is the spot market. In spot market I go and take the space for the warehousing purpose as demand is there.

In case of lease and arrangement even if my demand is not there I will incur that much cost because I have taken it full hall for that purpose and whether I use that full hall or part of that hall, it is a material for the person who has given me that hall. So I will incur a fixed cost for all through the period. In case of spot market the chances are there that I will incur slightly higher cost.

Because whenever in demand is there I will go and take the space so slightly higher cost I will incur, but here I will not incur the cost for unused capacity. I will take only that much space for warehousing which is required, which is in my demand. So though the cost maybe higher but cost means the rate at which I will be paying the rent that maybe hire in case of spot market then the leaves maybe in the lease arrangement.

I pay just to give you an example I may be at a rate of 1 dollar per square feet that may be the rate and if I go for the spot market I may pay at the rate of 1.25 dollar for per square feet, so that is kind of difference you will normally see and but the advantages that I will only pay for the used capacity here because I will take only that much space. Here in this case I may take more space because of cheaper availability of the space.

But later on I may find that some of the space is not used and then the third option is flexible system flexible arrangement. In fragment arrangement it is possible that I take some space fixed that this much minimum space I am going to take on the lease, it is a combination of in

fact both these things lease and spot. Flexible is a combination of lease and spot, where I take some space on lease and the remaining requirement I fulfill on the spot market.

In that also there is a fear that whatever minimum I decide for lease even that may not be used, but it still be yours because of higher and used capacity can be minimise to some extent if I go with the flexible. So normally flexible is done to take the advantage of lease as well as of a spot. Both these things are combined in this flexible case. So these are certain type of decisions with a manager has to take in a supply chain network.

Similarly there are many other decisions maybe related to inventory how much you stock, want to stock, so you can follow any such practice where you have a fixed quantity and then you may order as per the requirement for some quantity for JIT and for some quantity you follow up POQ type of model. So both these types of combinations are possible, so managers need to take decision in your supply chain and these decisions are with respect to facilities.

These decisions are respect to transportation facility, I am talking of warehousing same you can take to the transportation also. The same situation we can take to transportation also where in transportation also all these three things are possible, how much trucks, how much vehicles you want to own that maybe the lease kind of treatment, you want to own some number of vehicles.

Then some number of vehicles you take from the market and then there is a possibility that you have some minimum number of vehicles with you and remaining requirement you fulfill from the market. So both all three alternatives available even in the case of transportation. So as a supply chain manager you need to make all these decisions once you go up in the hierarchy that and please see that these decisions are long term decisions, I took the example of warehouse you cannot change the capacity of warehouse in a day or two.

It is a long term decision and it can very well affect the surplus of your supply chain, so you have to be very careful about selecting these kind of options and therefore these mathematical models will help us to take appropriate decision. So as the second mention any of these decisions involve a choice between a long term or less flexible option and short term or more flexible option.

Like in this particular case if I see when I go for lease type of arrangement, so it is a left flexible I got a fix capacity with me and it is a long term kind of Regiment nobody is going to give me leave for one month or two month I need to have lease for 4-5 years. So then only management is possible, but if they left flexible if the capacity and used I have to be a the losses. So that is one option.

And another is the short term or flexible option where spot market type of options are there depending upon the requirement how many units I want to store and depending upon the space requirement for those units I can go to the market and I can take the space on rate which is highly flexible and in each period I can change my decision. So you have to trade off between these two things.

Obviously well long term decisions are there and less flexible these decisions are there, these are slightly cheaper and when you are taking short term and more flexible decisions these are slightly costlier. So you need to see the overall benefit in these two types of decisions depending upon the amount of uncertainty in your environment, if it is highly uncertain environment obviously we will like to take short term decisions, if it is less uncertainty is there you can take long term decisions.

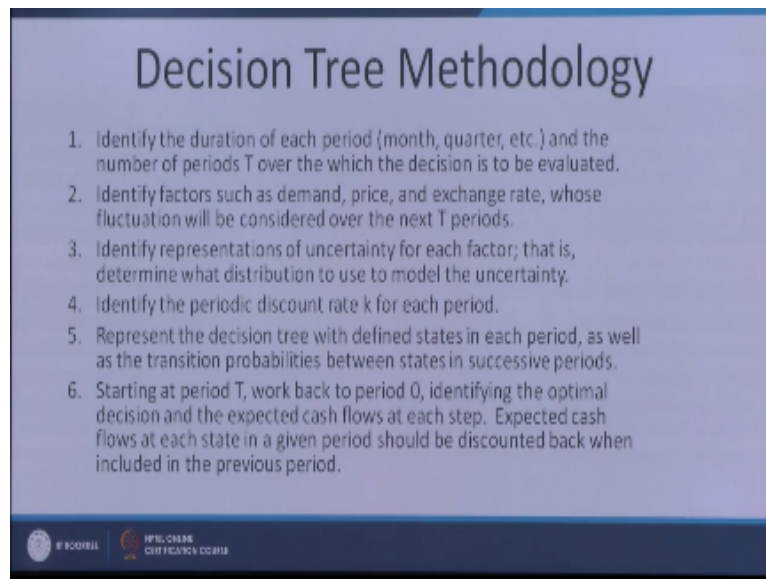
If uncertainty is ignored the long-term option will almost always be selected because it is typically cheaper so when uncertainty is less you will like to have long term decisions because in that case you will incur minimum possible cost of taking that decision, so the decision can eventually hurt the firm, however because actual future prices or demand may be different from what was forecasted at the time of the season.

Because uncertainty is going to be there, you cannot eliminate uncertainty and if uncertainty is going to be there certainly these types of decisions which initially we thought are cheaper may end up in a very costly affair for the company because there will be changes in the demand there will be changes in the price is there will be fluctuations of inflation rate, there will be fluctuations of exchange rate.

So all these things will contribute towards making our initial decision slightly costlier. So therefore it is possible that we should include elements of uncertainty in our decision making and probably in that case our decisions will be more realistic. Then finally as I was

mentioning that we will use decision tree for evaluating these decisions with the uncertainty cases.

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Now what is this decision tree methodology, so let us see that how are we going to use this decision tree methodology for evaluating decisions with uncertainty. The first thing is we need to identify the duration of each period and the number of period t over which the decision is to be evaluated. Like in this particular case there first thing is we need to decide that what is the duration of each period.

Like this is period one, this is period 2, this is period etc, etc, I am making this decision tree, but what is the duration of this period one after how much time this period is going to come, so this may be monthly that this is January 2017, this is February 2017, this is March 2017 like that, it may be a quarterly if it is January 2017 than this period one is April 2017 and so on it can be a half yearly fee also it is January 2017 then period one is July 2017 and so on.

In some cases it can be annual affair also it is January 2017 then January 2018, then January 2019, so first is what is the duration of each period, whether it is one quarter half yearly early etc, so that is one thing you need to decide. If you remember in our last session we discuss like here from node one we are coming up only with two possibilities that either your price can increase or your price decrease and price can increase by a fix value that is u and price can decrease by its is fix multiplying factor that is D .

But in reality there can be various factors by which price can increase, there can be various factors by which price can decrease. So it is advisable that it is a slightly more complicated situation. So it is advisable to keep your duration of period slightly limited, slightly smaller. so that in that small period you can come up with only two possibilities then our decision tree is more realistic and simpler also to handle.

So it is normally said that if it is a price kind of thing, demand kind of thing we can go by monthly periods in that case and then the number of periods t over with the decision is to be evaluated then 0 period first period second period third period, fourth period etc. So now these are the three periods over which I am evaluating the decision and for a bigger case when I want to take decision for slightly more longer period you need to take higher values of the

So we will see a case of 3 period but in reality you may take cases of 5 period also and because nowadays we can do all these things with the help of computers, so we can take a period of 2 10 years also or 10 period, 10 months, so all those things are possible so first we need to take decision about the duration of period and the number of periods for which we want to develop the decision tree. The second sector is identified factors such as demand price and exchange rate fluctuations are to be considered for the t period.

So like price we have discussed time and again that we are going to discuss price p , p_u , p_d and then there are different possibilities of p_u and different possibilities of p_d also. Similarly you can have demand also. Demand is increasing or decreasing, so you need to decide that what the factors you are going to consider for your uncertainty calculation. Then identify representations of uncertainty for each factor that is determine what distribution to use to model the uncertainty.

So you need to see that what type of distribution is best suited for your case of uncertainty, which particular probability distribution suits your case of uncertainty, we are taking in our class the binomial distribution of uncertainty, though you can take normal distribution also and if you remember in our last session we discuss that if we take binomial distribution for long period of time with the same values of up and down.

So it can become like a normal distribution also. So you need to see that which type of distribution you are going to take to represent the uncertainty. Then we also need to see what is the discount rate that is K for each period, it means that because we need to take decision at time = t_0 , so whatever is the outcome of a particular series of nodes and branches we need to calculate the present value of that outcome.

Whatever is the outcome coming after this period 2, so we need to calculate the present value of this and for determining the present value we need this discount rate factor k . It is possible that from period 1 to 2 you have a particular discount rate and from period 1 to 0 you have a different discount rate. In reality it is possible and it may happen also. But in our case for understanding the uses of this method we are taking a constant discount rate k for all the periods.

But please be sure that in reality it can be given k_1, k_2, k_3 etc. for different time periods. So we will see in our discussion that how to handle the variable discount rates that it is also possible. Then represent the decision tree with define states in each period. These are the states so you need to develop and these are the time period.

So you need to develop the decision tree with define states in each period as well as the transition probabilities between states in successive period. These 0.5 these 0.25 0.25 0.25, these are the transition probabilities between states in successive period, from period 1 to period 2, from this state you are getting these four news states, and the transition probabilities for these states are 0.25 each.

From this state in period zero you are getting 2 states in period and the transition probabilities for these two states are 0.5 each, so that transition probability also need to be defined for all the different nodes which you are going to get in your nodes means a different state you are going in the successive period. Then the calculation starts from the most recent period means the last period in your decision tree.

So if I consider only this much as my decision tree so my calculation will start from period 2, form period 2 I will go to this direction, first I will calculate the values here, then here and there here, so this is the way in which I will move for calculation. So starting at period t work

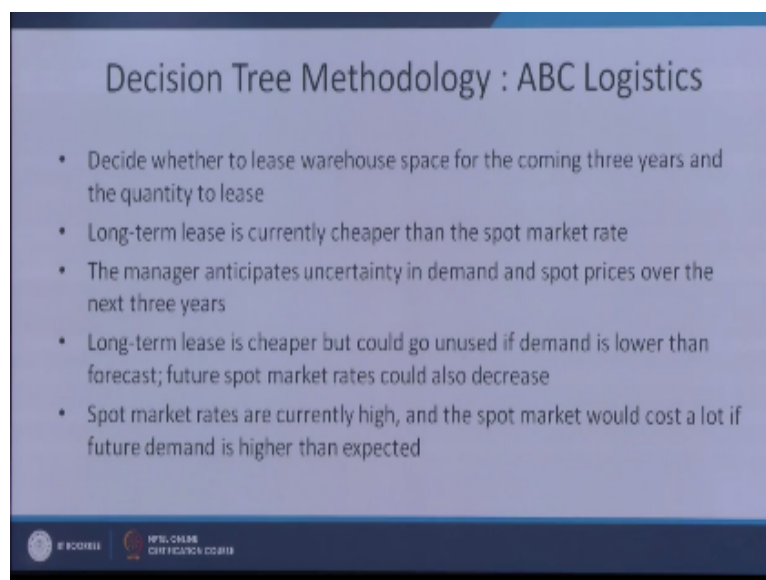
back to period zero identify the optimal decision and the expected cash flows at each step. So because all these things are probabilistic in nature.

So therefore we need to see what is the expected cash flow and therefore this world whatever cash flow is there at this stage this will be multiplied by the respective probabilities and therefore we have the expected cash flow value and then expected cash flow from period to 2 period one will come with the help of time value of money concept using the discount rate k for this period.

So effective cash flow at each state in a given period should be discounted back means we need to calculate the present value of the cash flow when included in the previous period. So you need to calculate the cash flow of this period and you also need to calculate the cash flow the present value of the cash flows of the second period, so that will be the total cash flow at state 1.

Similarly when I am talking at stage 0, so I will calculate the present value of cash flows of time period one and the cash flow at time period 0 that will be the total value of cash flow at time period 0. So this is how I will go for the backward calculation from time period t to time period 0. Now for understanding this we will use some kind of numerical example so that we can understand properly the use of decision tree methodology for handling the uncertainty. And this is what our predictive analytics will tell us.

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Decision Tree Methodology : ABC Logistics

- Decide whether to lease warehouse space for the coming three years and the quantity to lease
- Long-term lease is currently cheaper than the spot market rate
- The manager anticipates uncertainty in demand and spot prices over the next three years
- Long-term lease is cheaper but could go unused if demand is lower than forecast; future spot market rates could also decrease
- Spot market rates are currently high, and the spot market would cost a lot if future demand is higher than expected

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Now it is a situation where we want to hire some kind of warehouse and in that warehouse we have these are the three options available to macro environment, decide whether to lease warehouse space for the coming 3 years, so this coming 3 years this is the t , this is the capital T period of my decision tree and because I am talking of early basis so each period is 1 year. So now the first thing if I go back that what is the duration of each period that is one year. And the t capital T is 3 years.

Then long-term lease is currently cheaper than the spot market rate the situation is that which is very obvious that if I go for a long term lease agreement, this is cheaper than the spot market rates. The manager anticipates uncertainty in 2 things, 1 is he anticipate that there may be a fluctuation in the demand. So he is not very sure that the space which I take today the same requirement will be there tomorrow or not.

So that is one level of uncertainty and the second level of uncertainty is the price. The price of the commodity, the price of the stock market will also change. So today spot market price is different, tomorrow spot market price will be something else. As we discussed long term lease is cheaper but could go on used if demand is lower than the forecast. And spot market rates could also decrease.

Today the rate of a spot market is dollar per square feet but it is possible that tomorrow the rate may decrease to 1.25 dollar per square feet, so today it will look costlier to go for the spot market but since there is a possibility that my demand can also decrease. So if I take more space that space may go unused and another possibility is today the price of the spot market is high, but tomorrow is this market price goes down.

So that may also give me some kind of attractiveness to go for the spot market. So these are the dilemma of manager. Spot market rates are currently high and the spot market would cost a lot a future demand is higher than expected, and future demand is high so today it is already a higher price of the spot market and if more demand is there in the future and if the same price trend goes.

So it will be really very cost for me to bear the cost of this warehouse. So we will see different scenarios in this case, we will develop different scenarios and with the help of the

scenario we will try to calculate that how this type of calculation can we handle. So that calculation part entire calculation part we will see in our next class. Thank you very much.