Supply Chain Analytics
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Lecture-26
Network Optimization Models

So welcome back and we are already discussing about network optimisation models., in our last session we discuss the use of gravity location method where we decide coordinates of various supply and demand locations and we want to locate a warehouse, so we basically focus that we will locate warehouse from where the overall transportation cost is minimum and using the concept of graphical method where we have calculation of distances on a graphical plane.

And to minimise the total cost on the basis of that and in that we also saw simultaneously the use of Excel for particularly Excel solver that how to use Excel solver for such type of decision making with that data where one manufacturer of automobile company products wanted to discuss that where should I locate my warehouse and we had some data and we solve that.

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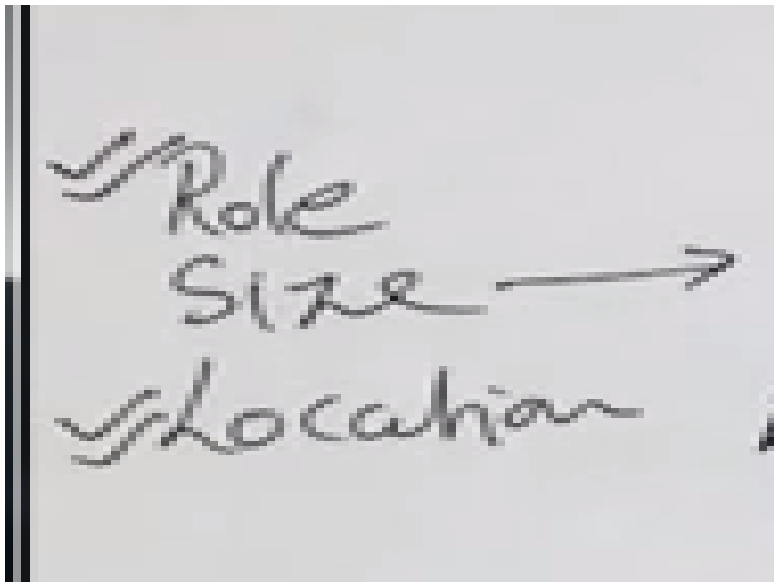

Now moving further we want to have some interesting issue related to a allocation of the facilities. In last class we were discussing about important issues one was role, one was size and one was location. So we discussed already in detail about role and locations in our last session. Now today we will discuss about this size, that what should be the capacity allocation of my facility.

And our gravity location method, the method which we discussed in the last session that was basically dealing with the locational problem, now today in this session we will discuss about the size of the facilities, the size of our capacity or the manufacturing facility, so that is what we are going to discuss.

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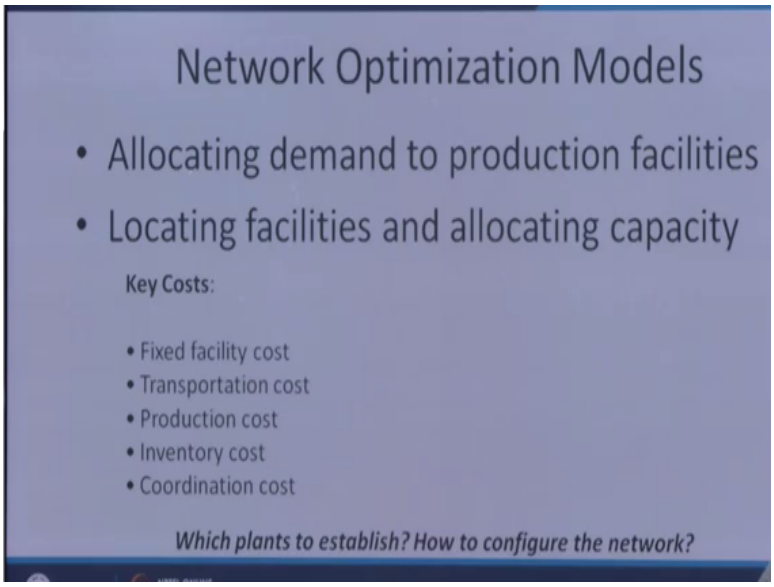

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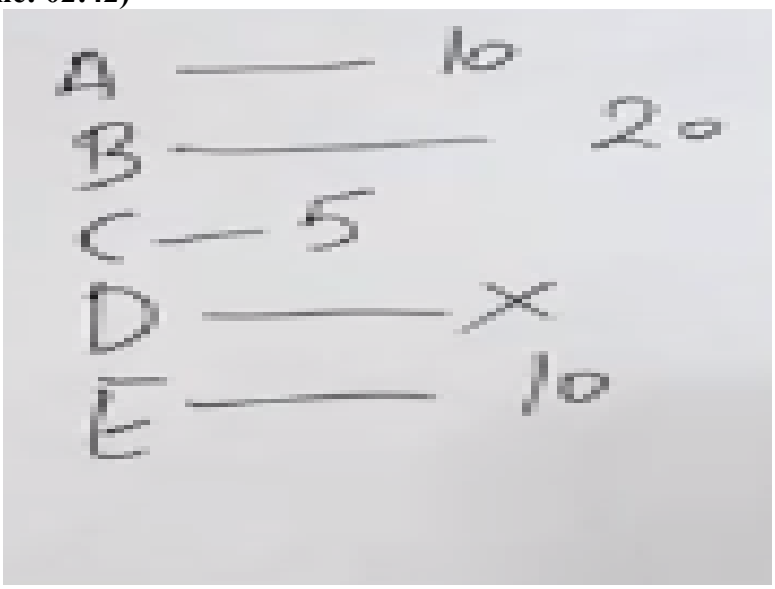

Now the allocation of capacity to a particular facility, the key cause are there is a fixed cost of the facility, so whether you see that facility for 2 products or 200 products, there is to be fixed cost of the facility that is there, then the transportation cost you are using that facility to transport some product from one end to another end, so that transportation cost will be there, the production cost obviously.

This is a variable type of cost as many number of products you will produce that was production cost you will incur, then inventory cost and the coordination cost, you need to co-ordinate with your different other facilities in the network. So depending upon your location and the size of the facilities this coordination cost will also come. Now the questions which are there in front of you that you have 4, 5 locations in your radar and you need to decide which plans to establish.

You want to establish a, b, c, d, e, these 5 location for there and out of these 5 locations you can make 10 units here, 20 here, 5 units here, you feel this plant is not suitable, this location is not suitable, so you will not decide to make here and capacity of 10 units for plant E, so this is what we want to configure the network. We need to decide which plant established and then what should be the size of each of these plants.

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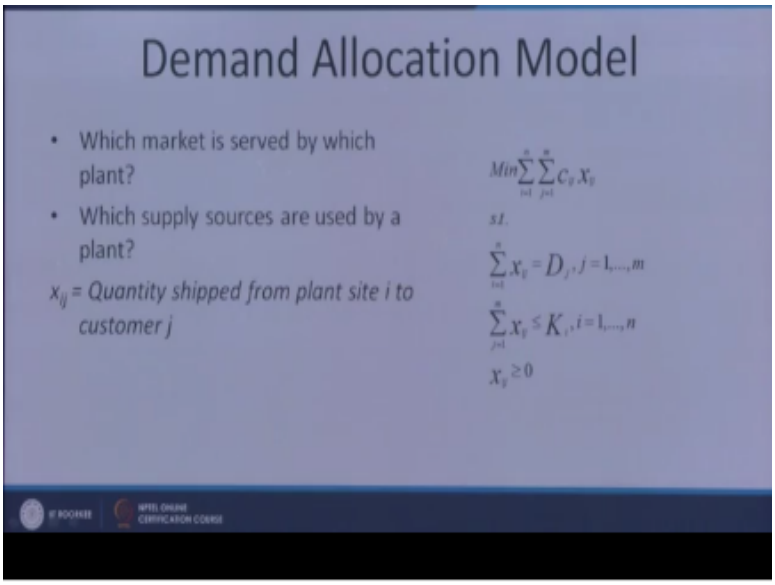

So both the issues we will like to answer in case of this network optimisation models. So for this purpose you have this type of demand allocation model. Now in demand allocation model you see that you have various facilities you will like to see which market to be served from which facility like these are the production facilities and these are the consuming area and this is a warehouse.

So you need to see that from one to one, this is one, this is 2, 3, so you are serving your area number 3 from the second facility and maybe from the warehouse also, so that decision of this type that how your different facilities will provide products to the consumer end, the

market that is very important and which supply sources are used by a plant, if this is a plant for a moment please consider that this is a plant.

Now for this plant what type of vendor distribution system 1, 2, 3 are serving this plant. So one issue is with respect to market another in with respect to plant that which distribution of vendor will supply to which plant and which plant will supply to which market. So this is the configuration of your entire supply chain network. Now for that purpose you are simply consider this is site I and this is customer J.

So quantities which shift from one source I is source and J is destination, so X, I, J representative should from I source to J destination, so we want to minimise the total cost, so CIJ, CIJ represents the cost of moving**,** cost of transportation from my Ith location to Jth destination and XIJ is the number of units move from Ith location to Jth destination.

So CIJ x XIJ, this product is the total cost, total cost of logistics involved in moving X number of units from Ith source to Jth destination and we want to minimise this and we consider that there are n number of sources and M number of destination. So therefore I varies from 1 to N and J varies from 1 to M, because Mth destinations and n sources are there.

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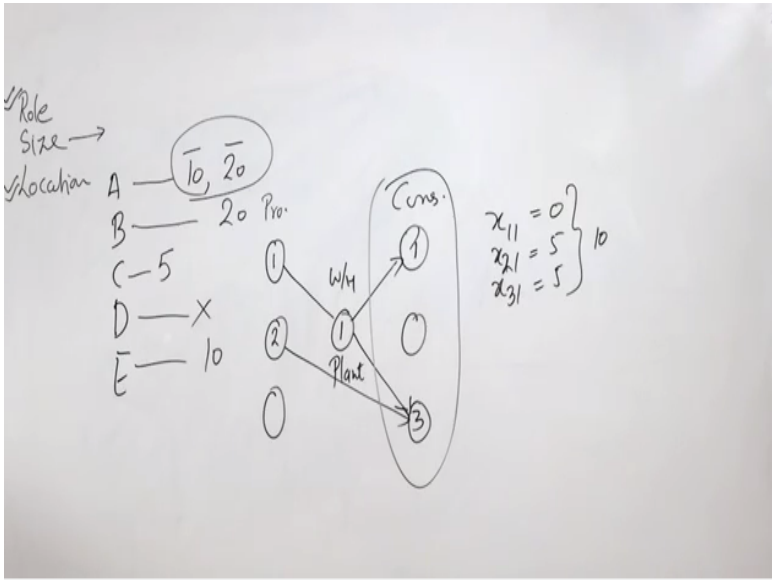

This objective function is subjected to 7 constraints, the constraints are that demand of all the destination are completely mat demand of all the destinations are completely met, demand of all the destinations are completely met, so if I have 1, 2, 3 destinations, so destination 1

requires 10 products, destination 2 requires 5 products, destination 3 requires 15 product. So my total demand is 15 + 5 + 10 is 30.

So this is DJ capital DJ present that 30, so total demand is fulfilled and the total demand is fulfilled from various units which are coming from other Ith sources, so XIJ giving to from I to N to different js, so that is one important condition that the demand of each destination is fulfilled, in demand of each destination is fulfilled in first request 10 and so I am able to fulfill the demand of 10 from all Ith sources.

If second require 5, so I am able to fulfill the demand of second destination from all the sources, if third requires 15, so I should be able to fulfill the demand of all 15 from different sources. It is quite possible once you develop this model and solve this model that if I have these are the sources, these are the destination, so if I have X11=0, **X1** X21=5 and X31=5, so this makes the demand of my destination number 1 which is 10.

So it is possible that sum of the sources not supply to some of the destination but for the sake of modelling we write in such a fashion that all Ith sources will supply to a particular destination and demand of the destination is completely better. Now there is a question to think about is that why we are fulfilling the complete demand in some cases it is also possible that you have less than equal to sign here.

You have less than equal to sign here that you cannot reply, the meaning of that is you cannot reply more than the demand of a particular destination in that case, if a destination required 10 units here so you can supply 8 units, 9 units, 7 units, but you cannot supply more than 10 units, that will signify, here what we are assuming in this model that we are fulfilling the 100% demand.

We will develop our model, we will consider our model in such a fashion that we will be able to fulfill the 100% demand of a particular destination, so in some cases you can take less than equal Dj, but in no case you can take that this is left hand side is greater than equal to right hand side, you know market, no market will accept that you supply more than its demand.

So therefore this greater than equal in favour of left hand side is never possible. Now coming to the second concern, the second concern is about the supply, a particular supply from a

particular source is not supply what it is producing, it cannot supply more than its capacity, so if you are supply from ith source and Ith source has a capacity of K units, so the total supply from Ith source to various destinations.

To all the destination from J+1 to M should be less than total availability at Ith destination, Ith source, so the second constrain represents this and obviously this XI represents the constant of non negativity that the value of XIJ cannot be negative. So these are the important condition of the model that you are fulfilling, you have the mathematical representation there, but the meaning of the mathematical representation is that we want to fulfill the 100% demand of each destination.

And it shows cannot supply more than its capacity, so these two things are written in the form of these type of mathematical expression, source cannot supply more than its capacity, we want to fulfill the 100% demand of each destination. Now moving further you see now we have another situation, we have multiple sourcing locations like this particular case where you have a, b, c, d, environment

And in this case you have multiple sources locations with different type of capacity. So here you see we want to minimise in that case in the last model we did not consider this fixed cost, we discussed about the fixed facility cost, so in that model we are only allocating the demand and we are only bothered about the transportation cost, but we did not consider this facility cost in this particular model.

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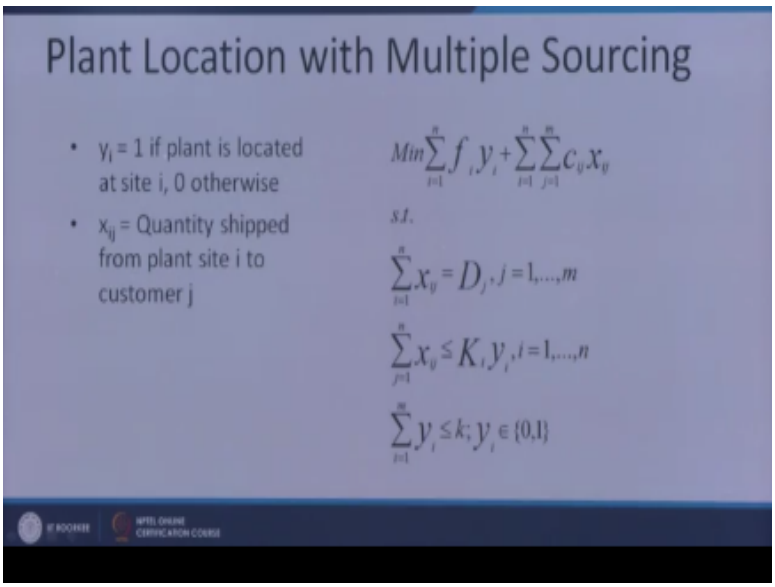

Now here in this particular case we are also considering the fixed cost of facility and that is what we are seeing here this step in this case is the same what we discussed in the previous slide , that is the cost of transportation, but out of these locations A, B, C, D, E at some location I want to have a plant and at some location I do not want to have a plant, so where I want to have a plant that is represented by YI.

I introduce a new decision variable Yi, and Yi signifies weather I want to have a plant at a particular location or not, so now Yi takes the binary variable biI is represented as a binary variable in our model, it can take only two values 1 or 0, the value of one signifies that there is a plant at a particular location and zero means we are not interested in opening the plant at a particular location.

And FI is the fixed cost of opening the plant, so the fixed cost only apply when Yi will be 1 and if Yi is 0 so this factor will become 0. So if you are not interested in opening a plant at a particular location, so there will not be any fixed cost associated at that location. So in this model you are having more comprehensive D where you are considering the transportation cost as well as the fixed cost.

And as usual this is the first constrain that you are interested in fulfilling the 100% demand of a destination, the second constrain it determined that you are interested, you can only supply, you are producing at a particular facility, at a particular plant. Now the important thing is that in the earlier case it was only Ki in the earlier days it was only Ki is this XIJ from J=1,2 M should be less than Ki.

But nowadays you have introduced Yi also because it is not necessary that there is a plant or not, so to introduce that factor that this Ki will be there only when the plant is there, a plant is not there you cannot reply anything from that location, to take care of that issue because if D there is no plant so you cannot reply anything from this location D.

So therefore to take care of this aspect we have multiply right hand side of this constant, if you see the earlier slide it is only Ki. But now in this case it is ki x Yi so that this K I will only be operative if a plant is there. Otherwise it will not and then you have the last factor that is Sigma Yi is less than equal to K is the values of Yi can only take place 0 and 1.

So you can have we value of YI only the sum of that should be less than 1, therefore you cannot think of if two different types of the meaning is that at a particular location here if I want to conceive two types of plant one is of smaller capacity, another is of some large capacity. So I will only open one particular plant at this location, so take care of this aspect this third constrain is being added that YI should be less than Ki.

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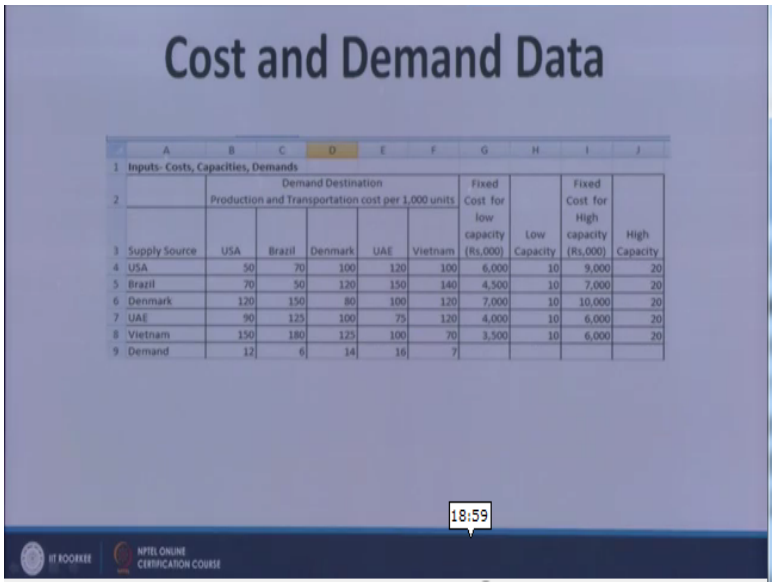

And after this we can think of developing a model so let's go for some kind of data, so this is a data available to us which we will use for developing the spreadsheet model, now here in this data I again request you all that you can open your Excel spreadsheet and you can copy this data from this table and you can use the same columns and rows, so that whatever formula we develop you can exactly follow same formula.

Now what we have, we have certain supply sources, these are available in USA, Brazil, Denmark, UAE, Vietnam and there are certain demand points. These are same USA, Brazil, Denmark, UAE and Vietnam. Now the demand of these destinations, for USA the demand is 12 units, for Brazil it is 6, for Denmark it is 14, for UAE it is 16 and for Vietnam it is 7. Now at each of these locations USA to Vietnam, you can have two types of plants, low capacity plant and high capacity plant.

Low capacity plant means of 10 units and high capacity plant means of 20 unit, the fixed cost for opening a low capacity at different location from USA to Vietnam that is available from G4 to G8, the cost fixed cost of opening a high capacity plant from USA to Vietnam is given

in I4 to I8, it is different at different location. Now coming to this data from D4 to F8, this data be B4 to F8, this data is the cost of transportation and production.

If I am producing 1000 units, so cost of production and supplying 1000 units from USA to USA is 50 rupees, then from Brazil to USA it is 70 rupees, from Denmark to USA it is 120 rupees, this includes cost of production as well as transportation and UAE to USA does 90, Vietnam to USA it is 150. So this data is available from all combinations in D4 to F data, so this is the data at USA to Vietnam you can open either in low capacity plant or a high capacity plant.

And fixed cost is given demand of various destinations are available to us, now we have to decide whether to open plant at USA, Brazil, Denmark, UAE, Vietnam, so which of these locations where will like to open the plant and what will be the size of those plants, whether these are the low capacity plans or high capacity planning and idea is same, we want to minimise the overall cost of production, transportation and facility cost. So now let us see I hope you have copied this data in your Excel sheets.

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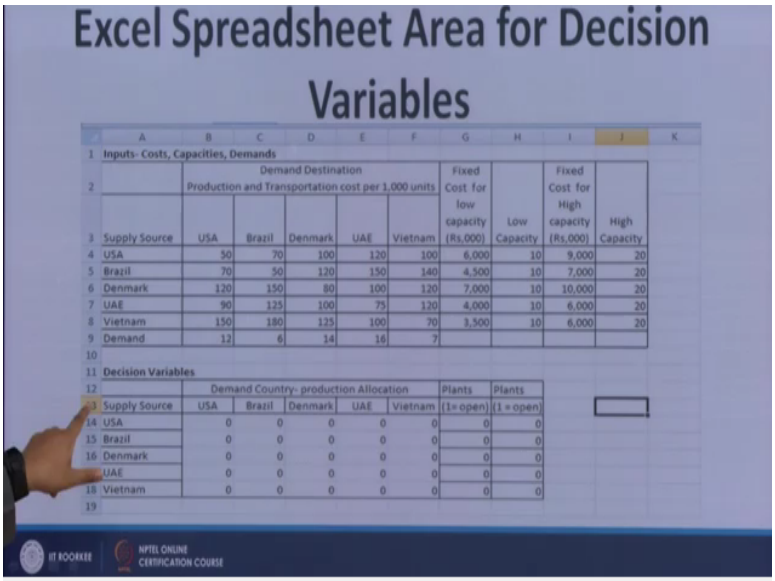

So this data once the same data which we copy in the Excel sheet, now we make a decision variable table in the same spreadsheet, just below the original data we develop a season variable table, so you can also use for making this decision variable in the same format. Here actually what you want to determine, we want to determine 2 things, 1 where to open the plant, second what will be the size of that plant.

And third when we are considering our network that from which supply stores to which destination, how many quantities, how many units will be served, so these are three important questions we will like to answer. So these 3 are our decision variables, whatever we want to determine that because our decision variable. So this is how you will make your table.

So these are almost similar kind of table as you are seeing the upper part of the spread sheet USA, Brazil, Denmark, UAE, Vietnam, these are the supply sources, these are the destination and to initiate the problem, to initiate the solution because I do not know where I will open from where how much I will shift to a particular destination, all these are marked as 0s, just to initiate the solution process.

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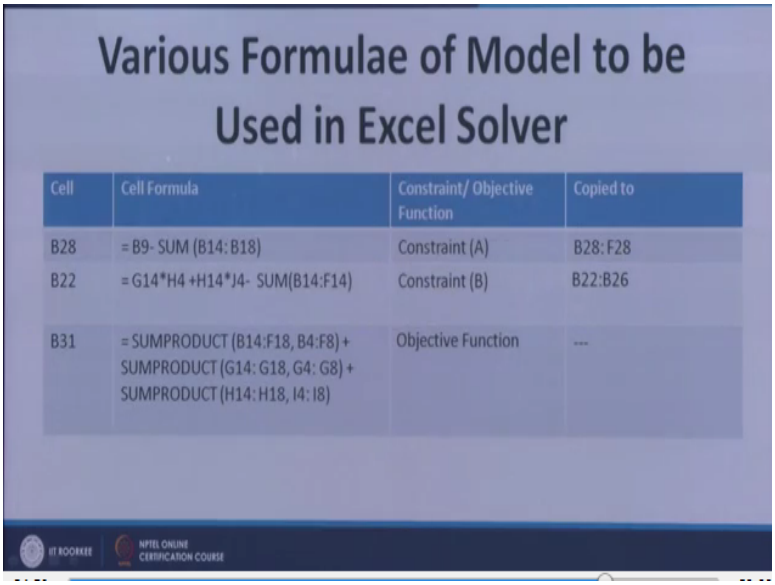

Now e will develop formulas, now formula the objective is to minimise the overall cost of fixed as well as variable, so now the formula for objective function will be because this is the representing the weather a plant will be there or not and these are representing the number of units to be shipped from my location to a destination, so one cost will come that whatever you are supplying from USA to USA.

Whatever you are supplying from USA to Brazil that units you will multiply with the cost of transportation, so this cell D4 needs to be multiplied by D40, that is and similarly for all these cells they need to be multiplied respectively in this portion of the table. So this is one type of cost which is coming because of production and transportation. The second cost will come because of the developing a facility.

Because of the opening of a facility, so this is if you open a facility here means low cost, so in those cases one will come and if you open a high capacity in this column in those places one will come, so these values of this column need to be multiplied because this is for the low capacity these to be multiplied respectively by values of this G column, and the values of this column here from H14 to H18 need to be multiplied by the values of I4 to I8 respectively.

So that is what and then you need to take the sum of all these products, so we have written that from B14 to F18 we are multiplying by B4 to F8, so that is one some product, that is for production and transportation cost. Then G14 to G18 will be multiplied by G4 to G8 that is cost of opening fixed cost of opening a low capacity plant and H14 to H18 will be multiplied by I4 to I8 that is the fixed cost of opening a high capacity plant. So that is the overall cost and this is my objective function which I want to minimise.

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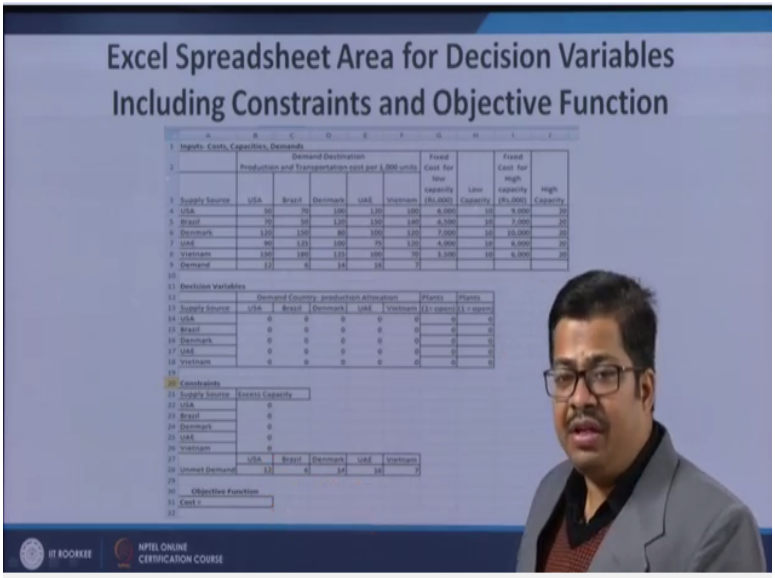

So this is the place where I will put the formula for my objective function in cell B31. This is the place where I will put my formula for B31. Now as we discussed that this will come with some constraints also what we discussed in the beginning. Now for that developing the constraints in this spreadsheet you see we have given sum rows and columns for writing the constants also.

The constants will be with respect to demand and with respect to supply, so in case of demand we will see the unmet demands are there and unmet demand means in these places you have 12, 16, 14, 16, 7, so all these places you will have the unmet demand. That in the beginning of this calculation no demand is fulfilled and finally we will see that all these

unmet demand should be fulfilled and initially the capacity of these places are considered to be 0,0,0.

So we see that where to develop the capacity, so that this unmet demand is fulfilled. Now in this session we are stopping at this point and in our next session we will see that how to build that formula into this spreadsheet, so that we can invoke the solver and we can get the result and we will also see how to analyse these results. So thank you very much.