Supply Chain Analytics
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Lecture-21
Multi Echelon Inventory Management for Four Stations (Numerical Example Continued)

So welcome back in our last session, we were discussing about multi Echelon inventory management and we discussed a problem where we have 4 installation in our supply chain and we have the cost of setup for each of these installations, we have the cost of holding at each of these installation and then we discuss the concept of Echelon holding cost and this table be prepared in the last session.

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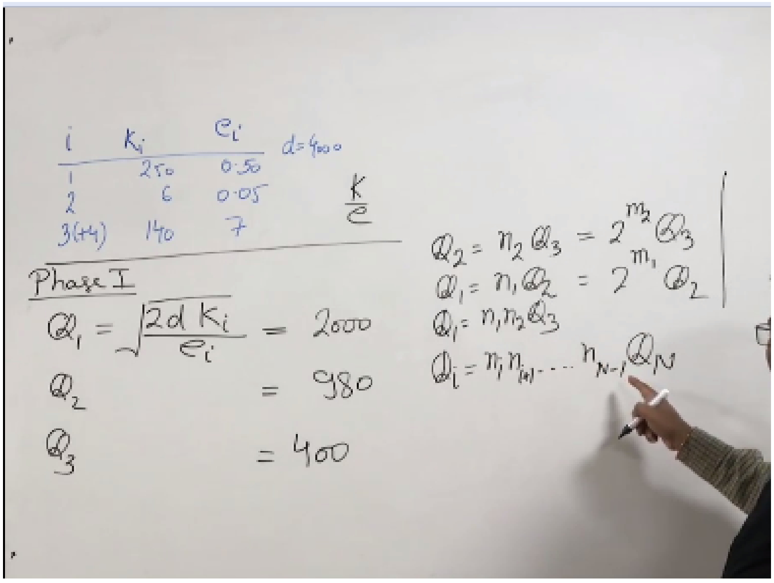

And we also discuss about the important ratios for that purpose that ratio is of k upon e. We calculated the ratio of the setup cost and Echelon holding cost and we saw that the consistency of these ratios are not there at stage 3 and 4. Because we want that the value of k upon e, value of k upon e should be lower forest successive stressed, because value of k upon e determines the quantity you are going to order at a particular stage.

Now since at stage 3 and 4 if you recall in the last session we had the value of 4 for K by A is more it was 27.5 and at that stage 3 it was 10. So what is the meaning that q4 was more than Q3, q4 was more than Q3 and we have already discussed that we will withdraw, we will take the amount of q4 from Q3. So it is never possible to give more amount from the less, you can give from a bigger lot you can take a small allowed, that is understandable.

But from a smaller lot you want to withdraw a bigger lot it is not possible and therefore for the modelling purpose we combined these two stations in our last session, station 3 and station 4 and then we rewrote this table and in that class itself we got these values that the value of holding cost at installation one and installation 2 remain as it is, but we added the values of holding cost for installation 3 and installation 4 and it became 140.

Similarly the values of a Echelon holding cost for remain same for installation one and installation 2 but for installation 3 it was 3 and installation 4 it was 4. So the total Echelon holding cost for this combined installation became 7 and now we calculated this ratio K up on e again and then we found that there is a proper consistency. The meaning of consistency again simple K upon e should be largest for installation one.

Then it should be less phone installation 2 and then it should be further less for installation 3 + 4 combined state. If this type of continuity is there, if this type of relationship is there for ratios at 1, 2 and next stage, then we say the ratio of K upon e is consistent, if it is not so then again we need to combine those installation and then proceed further, without having this consistency this is a very important issue that first we need to see whether data is consistent or not.

Without this consistency we cannot proceed further. Now when this consistency is there this problem will be handled in two phases, this problem will be handled in two phases, in the first phase of the solution we started in our last session the phase 1 of the problem we will start calculations of values of Q's, for these three different installation, now you simply consider that instead of 4 installation.

We have three stations in this problem and in the last when we will be finalizing the solution at that stage we will see that how this solution of 3 installation will be converted into a solution of 4 installation. So at the moment we will only talk of 3 installation 1, 2, 3. Now we are going to solve for these 3 installations at. So you will calculate Q1, Q2 and Q3. And these Qs will be calculated using our basic EOQ formula.

And that basic EOQ formula is 2D KI upon EI and use this formula, so KI in the case of installation 1 will be K1 up on even, hear it will be K2 upon E2 and here it will be K3 upon

E3. And when you calculate these values in Phase 1 the answer will come 400 then 980 and then 2000. These are the values you will get by putting D equals to 4000 and K upon e will change depending upon 1, 2, 3 and these are the values of for inventory stocking at different levels.

Now you see that we have already discussed at the time of model building that we want to have a particular type of relationship between Q3, Q2, Q1. And we wanted that Q2 should be explained in terms of Q3. Q1 can be explained in terms ofQ2, and here you see and already we know when we discussed about only 2 installation problem that N1 and N2 we want integer values. Now simply just by seeing here Q2 is 980, Q3 is 400. So 980 is not a integer multiple of 400.

Similarly Q1 is 2000 and Q2 is 980. Q1 is not an integer multiplier of 980. So this condition is not fulfilled here, the solution which we got is not fulfilling the required condition and this condition is only for the two installation case. In case of a multi Echelon system we already discussed in our model building that we cannot have just N1 and N2, we want to have a further relaxation of this problem.

And that for the relation we did by introducing this term 2 to the power M one, N1 is replaced by 2 to the power N1, N2 is replaced by 2 to the power N2. So that you have only even multipliers, you have only even multipliers of inventories for the previous stages. You cannot have any multiplier, you cannot have if you remember in one of the session when we were discussing about 2 installation problem at that time we had given equal to 3Q2.

So that was possible in a 2 installation problem, but here we have already discussed during the period of model building that you cannot have any values of N1, N2. So we want only integer values for N1 and N2 and for that purpose so that the very purpose of integer value was that you can have same date of starting and finishing of replenishment cycles at the successive status.

So that was the reason and for that purpose we took a further relaxation of 2 to the power M1 and 2 to the power Macro environmen2. Now this data is at all not shooting this type of relation so we want to go to phase 2 of this, we will see the phase 2 of this phone and phase 2 of the fall you can understand that right now I am explaining to 1 in terms of Q2 and Q2 in terms of Q3.

So can I explain Q1 in terms of Q3, so for explaining Q1 in terms of Q3 I will Q2 as N2 Q3 N1N2 Q3. So now here I have explain Q1 in terms of Q3. And the reason of this explanation is simple that I want to explain the quantities required at each of these installations just in terms of my last installation inventory Q3. So Q2 can be directly explain with the help of Q3 that is multiplier of N2 and Q1 can also be explained in terms of Q3.

And if there are many more stages you can see in the same way we can explain all quantities, all QIs in terms of Qn. If I consider this as Q5 and these are different values of n and then it becomes Qn. So I have this type of general phenomena that to explain to explain the inventory required at a particular stage I will have 5 this 2 can be replaced by I plus 1 and then up to n of N-1.

So with this idea I can have and this product I can represent as Pi which is the multiplier pi is nothing but the multiplier of various values of multiplication factor to get the inventory of successive stages. So you can finally summarise this discussion in the form of Qi equals to be pi qn. This becomes the inventory at any stage we want to keep Qi equals to Pi into Qn.

Now there is a small question and what will be the value of Pn just thing for minute, the value of pn will be 1, pn will always be equal to one. So please remember because in that case we are talking of the last stage and last stage the inventory itself Qn is equals to Qn. So at that stage Pn will be simply one. So now in context of this we will revise this problem. So that you can apply this formula in this particular case.

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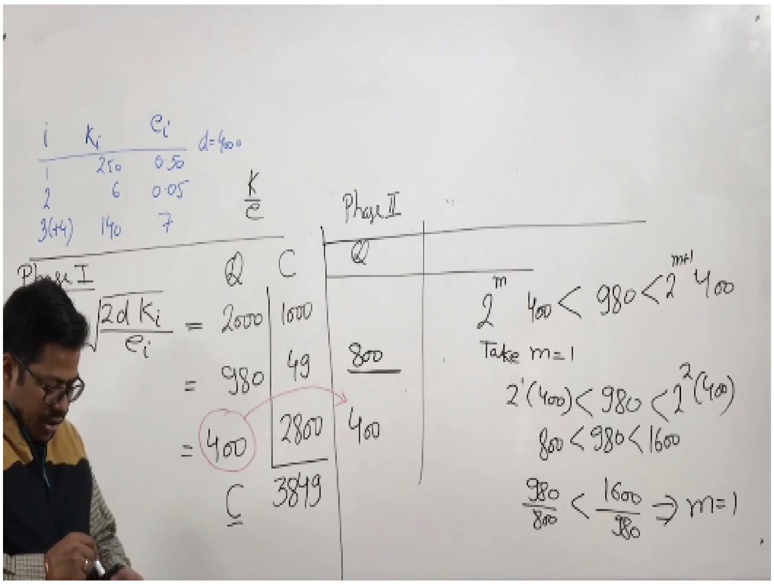

So now we move to phase 2 of this problem and phase 2 of this problem will require a bit of calculation which we will derive from the Phase 1 only and in the meantime I request to all of you to please consider the calculation of total cost of inventory what we are getting in Phase 1. In Phase 1 we are getting some values of Q1, Q2, Q3 and on the basis of these values of Q1, Q2, Q3 you will have cost of inventory associated with that.

So this is the column of queues and you take this column of cost of inventory. So you calculate cost of inventory with respect to these calculated values and then when you summarise the values which you will be getting these are 1000 units, these are 49 rupees and these are 2800. So the total value which will come of the cost in the first phase is about 3849.

This is the total cost which will come to you, now let us see the idea is can we further reduce, so this we already discussed is known as C over bound. We cannot have total cost was then this. This is the highest total cost of inventory our supply chain will incur. So idea is now to improve this cost means minimize the cost whenever in terms of cost improvement we say so the meaning of cost improvement is to minimise the cost.

So by doing some kind of mathematics in these quantities of Q1, Q2, Q3 can be minimise the cost, so we will like to see how to go ahead for that. So now we start our phase 2 of the problem solving. So now in phase 2 to initiate the problem we want to revise these values of queues, so initiate the problem I will directly take the value of Q 3 here.

And this will make my Q and as 400, but as I just explained now I want this Q2 to be some multiplier of this Q3 term. I want Q3 to be Q2 to be either 400 or I want 800 or I want 1600 or I want 3200. So I want a multiplier even multiplier of 400, not this 980, but it should be somewhere close to this 980. The idea is somewhere close to 980 which is possible here, so for that purpose what we are going to do we will see that what is the most suitable value.

What is the most suitable value of this Q2 and accordingly we will get the value of our M or you can say N which will come here N2 or M2 whatever you say and for that purpose we have this Q2 equal to 980, this is 400 and this side also you put 400. Now use this symbols here and here I will like to write 2 to the power, 2 to the power M+1. Now 2 to the power M will be selected in such a manner that it is just slightly less.

If I write 2 to the power 1, so it means 2x4 that is 800, if I write 2 to the power zero this becomes one. Then it is 400 only, so that is very less rather you have immediate less value as 800. So here I can take very conveniently take m equals to 1 and when I take m equals to one it is 2 to the power 1 into 400 less than 980 and this side it becomes M+1, so it becomes 2 to the power 2 into 400.

So this is 800 which is less than 980 and this is 2 to the power 4 into 400 it becomes 1600. Now I have two options whether to take 2 to the power M or 2 to the power M+1. I will apply the same concept which we use for deciding the integer values, which we decided to take what is the integer value of n. So same concept I will take the ratios of these quantities and the ratios will say that 1 ratio is 980 upon 800.

This is 1 ratio and the other ratio is 1600 upon 980, now when you calculate these 2 ratios you will see that 980 upon 800 is less than 1600 upon 980, this is less than 16, the left hand side is less, it is somewhere around 1.1 and this is about 1.7, so the left hand side is less than right hand side, so whenever left hand side is less than right hand side the value of m here m is equal to one.

If it would have been otherwise if left hand side would have been more than right hand side in that case M will be M+1. It is 2 to the power 1 and therefore the value of Q2 will come here 2 to the power m that is 2 into 400, so you get your new Q2 here. Now we will apply this same issue, the same process for deciding the value of Q3, what should be the value of Q 3 and for that purpose again follow the same procedure.

And here in this case now we will change these values here you will have this is to be multiplied of 800 now and in the centre you will have 2000. So this is how you have, now 2 to the power 1, if I take again the value of M=1, 2 to the power 1, 800 x 2 that becomes 1600, which if I increase M=2. So 2 to the power 4, 4x800 becomes 3200 which is not less than 2000. So this M=1 is ok, if I take M=0, so then it becomes 1, 1x800, that is 800.

So which is less and because next higher value is 1600 is available which is less than 2000. So M=1 is the right value to be used here, it is less than 2000 and here 1+1 that becomes 2, so 2 to the power 2 is 4, 4x800 that is 3200. So here 1600 is less than 2000 which is less than 3200 and again we will apply the same procedure that 2000/1600 and on the other side 3200/2000. And you will see that in this case also the left hand side ratio is smaller than right hand side.

You can do on your calculators and you will see that left hand side ratio is less than right hand side ratios and therefore in this case also we will conclude that M+1 and M=1 is 2 to the power 1 that is 2x800 that becomes 1600. So these are the new values of Q1, Q2,Q3, 400 is the Q3, 800 Q2, and 1600 is your Q1. And these values follow that issue of Q1=4 times of Q3. Q2=2 times of Q3 and the issue of consistency is very well handled here.

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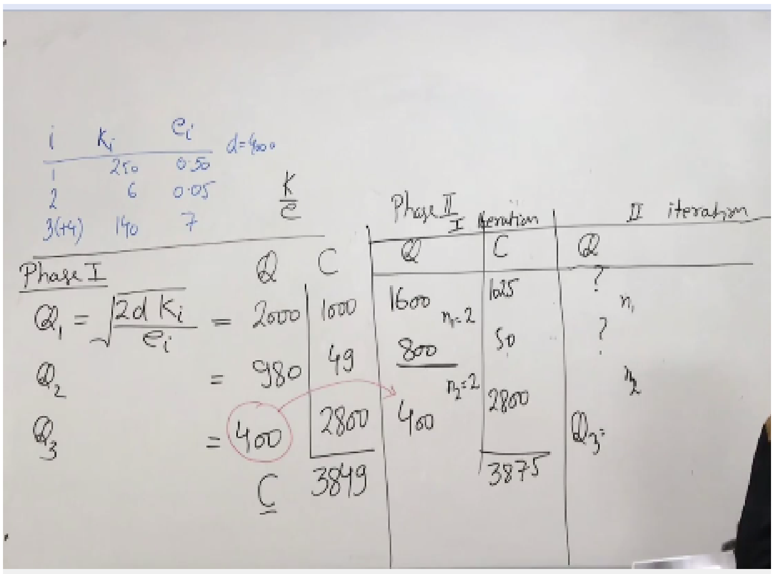

Now whether we have any advantage with this type of tissue that is known by the calculation of cost with these values of Qs. With these values of queues you are supposed to calculate fresh cost, you need to calculate the new cost and the new cost are 1025 and 2800 and total cost comes 3875, which is slightly higher 3875 because of the relaxation of the problem.

So you have the best cost 3849, but this cost 3875 is higher, but this type of arrangement 400, 800, 1600 will this gives you a lot of logistics comfort. But now the question is that whether this solution 1600, 800 and 400 is the final solution or we can have further improvement in this and for that purpose what we need to do that we will do one more level of iteration. We will do this is you can say phase 2, this is the first iteration.

Now in phase 2 will do one more iteration and in that iteration the idea is that we will like to improve the values of Q1, Q2, Q3. So that this 3875 can further reduce you had this lower bound, the best cost, but now you are trying to use the model for improvement so that you have the logistics comfort. But because of relaxation which you have used the cost has increased.

Now we want to see can we reduce this cost, can we do something with this 3875 and the idea is that here we have calculated the values of n1 which is coming to here n2, which is coming 2. Now we will in the second iteration again calculate the values of Q3, Q2 and Q1. And when we are calculating Q3, Q2, Q1 we will also determine the values of N1 and N2. Now if in this case the values of N1 and N2 remains these N1 will remain same.

There is no change in the value of n1, if there is no change in the value of n2, in that case we will say that this is the final answer. But if there is a change in the values of N1 and N2 we may need to go for a third iteration also. The point is that, the rule is that we want to have stabilization in the values of N1 and N2.

As long as the values do not stabilize there is a fluctuation in iterations to iteration between one value of N1 and another value of N1 between one value of n2 to another value of n2 we need to keep on doing the iteration. But if in this case the values remains same in that case it will be the final answer. So again the process will start by using the value of Q3. Now since we already have the values of N1 and N2 here.

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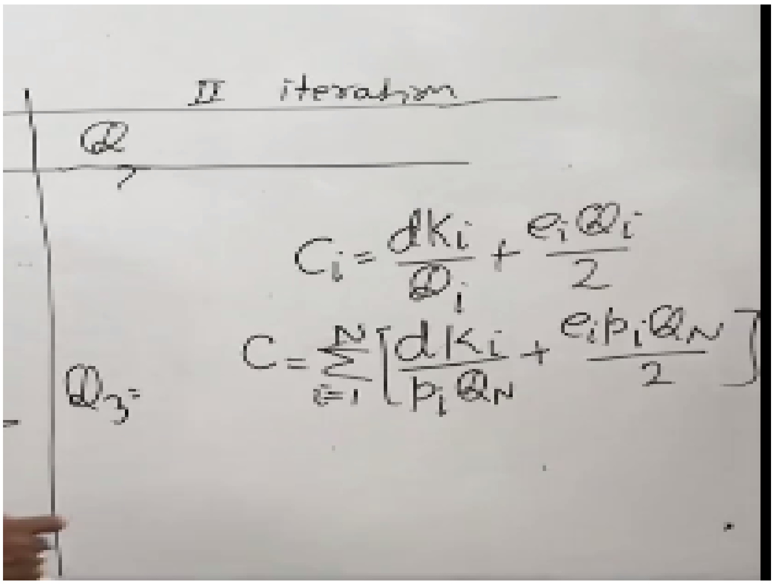

So we will use these values of N1 and N2 for determining the value of Q3 here. Now for that purpose we need to go back to this original formula of determining the inventory value POQ and using that formula we will determine the Q3 here and using that formula you will see that we are going to give you directly the formula and on the basis of that formula you can apply the calculation.

Now since you have the values of one initial values of N1 and N2 you can see that our total cost of inventory CI which is initially be represented as di qi upon Qi+Ei qi/2. Now this formula can be changed in light of qn and pi the concept which is discussed and this will become as total cost of inventory dki pi qn+ this Qi can also be written here in terms of PIQn/2 and you need to do Sigma for 1 to N.

So this is your total cost of inventory. Now when this is the total cost of inventory just by seeing you can observe that our new certain cost is ki upon pi. Our new setup cost is pi upon pi and our new holding cost Echelon holding cost is Ei x Pi. That is the unit holding cost, so using this our formula for calculation of Qn will be it is the same formula this, but since here the setup cost and Echelon holding cost or simply K and environment

So this K and E will be replaced by here this is the setup cost, so this is Ki upon ti/this Echelon holding cost that is EiPi and since I am calculating for the last stage, so you need to take summation of this i=1 and here also I=12n. And when you explain this formula let us for stage number 3, for stage number 3, the formula will become this formula is there.

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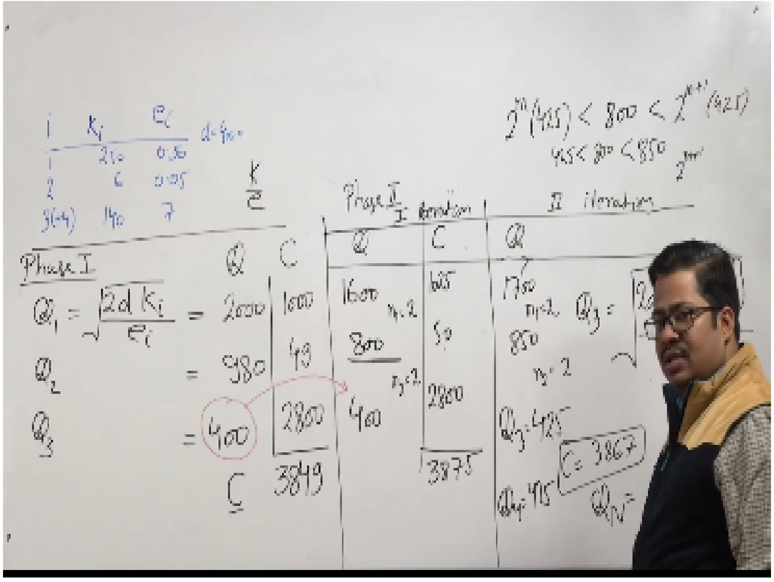

And now let us see the particular application of this formula in our this problem, so you will see that here I want to know the value of Q3. This Q3 I want to know and you have these values, so you take out 2d here, this is ki upon Pi. So one stage is Ki upon P1+K2 upon P2 and next is divided by EiPi+E1P1+E2P2. Now using this you will calculate the value Q3 and for P1,P2 you require values of N1 and N2.

So P1/P2 will come from N1, N2. P2 is simply N2 and P1 is N1 x N2. So you use this formula and then when you calculate Q3 will come using this formula we have just expanded this general formula for a specific case and when we substitute all these values Q3 will come 425. So now apply the same funda and you will see 800 coming in between from here and 2 to the power M 425 this side 2 to the power M+1 425 on the right hand side and then you take 2 to the power 1.

So this becomes 850 which is higher than 800 not possible, so you take simply 425 here. Because then you take M=0, it is 1, 1 x 425 it is 425, 800 and on this side it becomes 1, so 1 x x 425, 850. And then when you calculate the values will become N2 equals to because it is done when you compare the ratios so you will take 2 to the power n + 1 here and M+1 the value of N1 N 2 will be 2 and the Q2 will be 850.

And then use take simply before the ratio then you will see it will become 1700 and annual will become 2 and then you calculate the total cost C in this case that will come 3867. Now you see that from this stage to this is stage the values of N are not changing, here N1 remains

2, it is also 2. It was 2 N2 was 2 there, it is 2 here. So all these values remains same so there is no need to go beyond this stage, this is your final answer 425, 850, 1700.

And the total cost is 3867 which is slightly reduced from 3875 because this is a better solution and this is the final solution. In practice meaning of this is that you have Q1=1700, Q2+850, Q3+425 and Q4 also 425. Because for the modelling purpose because of the constraint we have combined stage 3 and stage 4, but actually these are two different stages, so this is the final solution.

And this way we can use the concept of multi Echelon inventory management, the concept of EOQ for optimising the inventory at each state of your supply chain. So with this example now I think the concept of handling the multi Echelon inventory is clear to us. Thank you very much.