

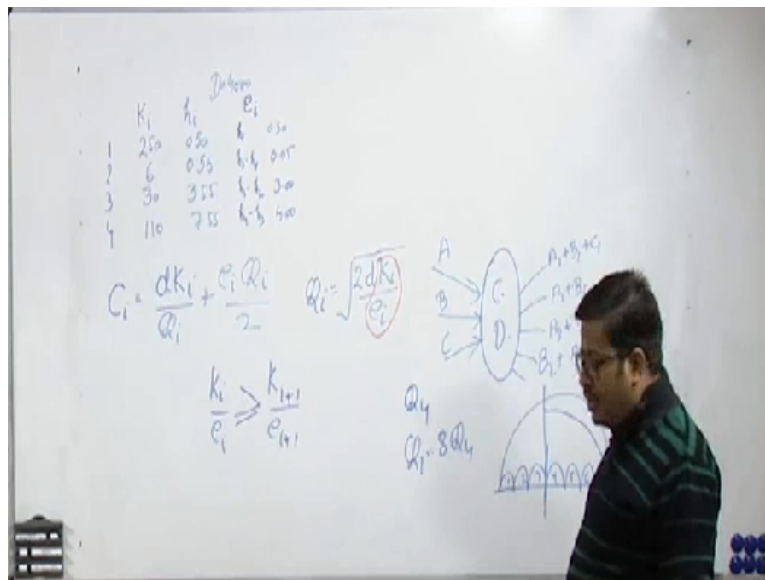
Supply Chain Analytics
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Lecture-20
Multi Echelon Inventory Management for Four Stations
(Numerical Example Continued)

So, welcome back in our last session, we were discussing about inventory management of multi Echelon systems where we have discussed a particular case of 4 installations. And we discussed that with the help of echelon inventory concept. We can apply at each of these stages, the concept of basic EOQ model.

Now going further into the discussion, and, having a particular case, now let me have some data with us and with help of that data we will like to see that how multi echelon model can be applied to that situation.

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Now we have four installations 1, 2, 3, 4 and for each of these installations the setup cost, the K_i is 250, then 6, then 30, and 110. And the holding cost which is h_i is 0.50, 0.55, 3.55 and 7.55. The demand of the supply chain D is 4000. This data is available to us. Now with the help of this data, one thing which we were discussing in the last session, that is very much visible that h_i , that is the holding cost.

As we are coming from 1 to 4, left to right in the supply chain 0.50, 0.55, 3.55, and 7.55. So you can see these holding cost are continuously increasing. So, therefore, this data is inline with what we have discussed. Now as we said that to apply EOQ model in this particular case we will consider the case of Echelon inventory, not the installation inventory. These are the holding cost given for the installation inventory. So now we will have the concept of e_i , that is the concept of Echelon holding cost.

Not the installation holding cost, and that is in case of installation 1 it will be simply h_1 , for installation 2, it will be $h_2 - h_1$. For installation 3 it will be $h_3 - h_2$. And for installation 4 it will be $h_4 - h_3$. And therefore e_i s will be 0.5, $0.55 - 0.50$ will be 0.05, $3.55 - 0.55$, will be 3.0 and, $7.55 - 3.55$ it means 4.00. So, these are the Echelon holding cost and now using the concept of echelon holding, and the setup cost.

We will have the formula for calculating the cost of variable cost of inventory, at each stage, so that is c_i . That is for a particular stage in the supply chain, the cost of inventory will be d that is K_i upon $Q_i + e_i Q_i$ by 2. This is the cost of inventory variable cost of inventory at particular stage. Now when you use this formula and apply the concept of basic mathematics for getting the maximum, minima differentiated.

Apply the maxima, minima and then calculate the value Q_i , because that is the only variable here. So Q_i will be under root $2d, K_i$ upon e_i . I request all the participants to please practise this and get this formula on your own. I am directly giving this formula here. Now in this formula, there is a term which is very important for us in further calculation. This is this K_i upon e_i , the value of Q_i will depend on the ratio of K upon e .

The ratio of k upon e will determine the value of Q . Now as we discuss in the last session, at stage 4, the quantity is which we are getting are coming from 3. At 3 quantities are coming from 2, and 2 quantities are coming from 1. So you need have has the value for Q_1 , then Q_2 , then Q_3 , and least values will be stage 4. So, that is the way. Now this ratio will play very important role in that case.

K_i upon e_i , if the ratio of K_i upon e_i , and the ratio of the next term that K_{i+1} upon e_{i+1} . If you see this comparison K upon e for 2 stages i th stage, and the subsequent stage the successive stage in that. In this case if K_i upon e_i , is more than this, what will happen Q_i will be more than the next step, that is desirable or the last possibility is it is equal to this. So, in this case Q_i will be equal to Q_{i+1} . that is also possible in cases.

That if Q , whatever is coming at stage 3, the same you are transferring to stage 4. Can you recall any supply chain situation where it happens, that whatever you are receiving the same you are transferring to the next stage. We can think of situation like cross docking. In cross docking this type of situation may emerge that whatever you are receiving at particular stage.

Because, cross docking we know we do not keep inventories. Whatever we are receiving the same just by repackaging. The size of packets may vary depending upon the requirement. So, cross docking is a kind of aggregation. So, just to take you of from this discussion. In case of cross docking, what happens this is a place, where cross docking is taking place cross docking.

So, you are getting supplies from the manufacturers, so bigger packets are available A, B, C and, then we just make much smaller packet of $A_1 + B_1 + C_1$, $A_1 + B_1$, $A_2 + C_2$, $B_2 + A_2 + C_3$ of variety of combinations. Depending upon the requirement of retailers, so we will just take the bigger supply from the manufacturer, which are more homogeneous, which are uniform, and as per the requirement of the retailer, we will mix the supplies of different wholesalers to get the order of a particular retailer.

So here whatever is coming, because stocking has the vary point of cross docking is that you do not keep the physical inventories. So total input is equal to total output, on a any particular day total input is equal to total output, no inventories are there. So, that is the situation where, you can think k_i upon e_i can be equal to divided e_{i+1} .

Because in that whatever is coming at particular stage will be given to the next stage. So, in such situations this type of calculation make em, but in most of a situation which we have discussed in

our earlier examples, we will keep some kind of physical inventory. But, now a days it is of much significance, now a days it is much significance that none of the partner.

None of the partner wants to keep the inventory, and for that purpose the systems of cross docking are becoming more and more relevant. And at the same time we also have the influence of just in time. And in just in time of systems we normally do not want to keep inventories. And therefore whether you talk of JIT, or whether you talk of cross docking. In both these situations whatever is coming the same is going.

No one wants to keep the physical inventory for a particular period at its stage. So in current example in current situations it is quite possible that we have in practical cases more and more those types of examples where $Q1 = Q2$, $Q2 = Q3$, $Q3 = Q4$ and so on. But if I talk of a traditional type of a system particularly in case like India, because many a times we see we keep inventory to avoid fluctuations of demand and supply.

But in that case inventory becomes a slight typical issue where you need to maintain a particular level of inventory, So that we can achieve the requirement of the customer and we can achieve a particular level of service, but at the same time because of new and new types of challenges coming to us. We do not know how the demand will behave in the future, and therefore we do not want to keep higher inventories also.

Like you see in our earlier case the example which we discuss in the last session at stage 1 the inventories where a times then the stage 4. A stage 4 was keeping just $Q4$ at any time. At stage1 was keeping 8 times of $Q4$ in our last example. So now the stage1 is blogged for 8 replenishment cycles of stage4. So now a stage1 the point I am trying to say the stage1 will be able to consume this much is stock of $Q1$ item.

In 8 replenishment cycles of a stage1, because in each cycle the quantities which are received by a stage4 or 1 by 8 times of $Q1$. So, it may happen that after two replenishment cycles or three replenishment cycles, demand all of a sudden changes and therefore the whole system of

inventory at 2, 3, 4 will be at loss. So, there is a challenge for the management also that how to handle this uncertainties.

And, therefore we always look for some kind of optimum values and therefore because more and more uncertainties are coming. You will find these types of cross talking examples, you will find that people believe in GIT people believe in flexibility. Here yourself can understand that I am waiting for 8 cycles of a stage 4, 8 times the order is procured at this stage 4 than a single order is procured by a stage 1.

So these stages 1, 2, 3, 4, 5, 6, 7 and 8 these 8 cycles of stage 4 represents the one cycle of a stage1. Now stage 4 after the third cycle customer says no we do not want this product, we want a different type of product which should have this much feature. But this stage1 is still having this much inventory, so because now customers are willing to purchase this product the whole inventory in the supply chain not only a stage1.

But at stage2 also there will be some inventory at stage3 also there is some inventory that complete inventory is at loss, and therefore this is a big challenge in front of us that how to address this issues and probably more real time analysis will help us to reduce this type of relationship. We need to have more relationship that we can minimise this period.

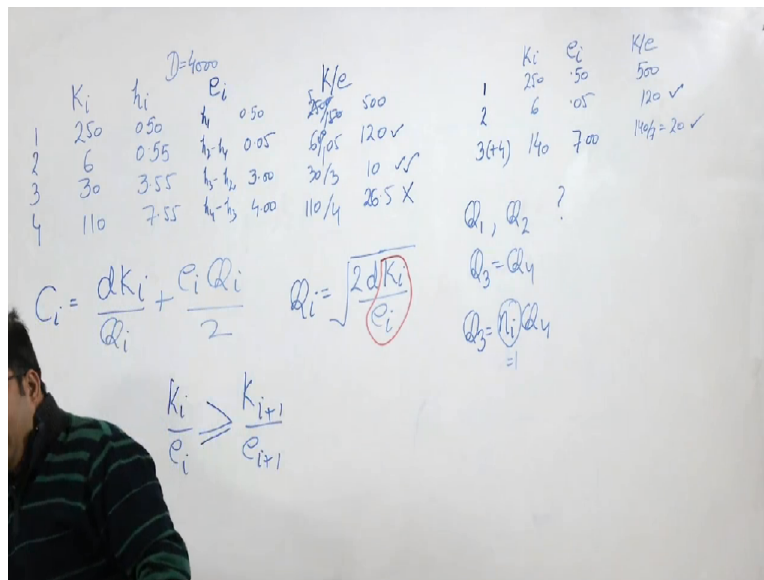
And you see this is a case where only 4 such stages were there if it is the even longer supply if chain if it is a longer supply chain. In case of India where our large number of population is living in rural areas more than 60% of population in rural areas and in rural areas the supply chains are typically very long typically we have very long supply chains in the rural areas and therefore this value may further increase it is 8 times of your Q_n .

But if I talk of a rural supply chain the values may further increase and then this problem will be even more severe in that case. So therefore it is very much required that we need to have some better models and we need to understand that what is this significance of this particular aspect that you cannot have very high values of this n factor here, because more higher values.

You have that much rigidity in your supply chain it will be difficult for your previous stages to change their products, because they have already starts that much item at their stages. So we need to have smaller values we love a smaller values of this factor, So that our supply chain is more lean there is less inventory at each stage.

And that what we are going to discuss in this particular case that the inventory should be as less as possible and the value of n is one important factor in that. Now coming further with respect to this particular problem. Now here we see we have understood the meaning of K upon E, now let us calculate K upon E for these stages.

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So, here it is 250 upon 0.5. So the value of this comes to be 500 then 6 upon 0.05. This is 120 then 30 upon 3 that is 10, and then, 110 divided by 4, so this is 26.5. Now you see the values 500 no problem we cannot say anything about here, but the next value is less than this first stage, so this is okay. The next value is 10 which is much less than 120 absolutely okay. The next value is 26.5 not possible this cannot be there.

This value is not suitable because this is more than 10 and I cannot accept because, if I take it so the Q4 will be more than Q3 and that is unacceptable Q4 more than Q3 is unacceptable. So this is a problem, now to sort out this problem we will do that we will merge a station 3 and 4, we

will merge these two installations 3 and 4. The meaning of merging is that you will have same value of Q3 and Q4 finally.

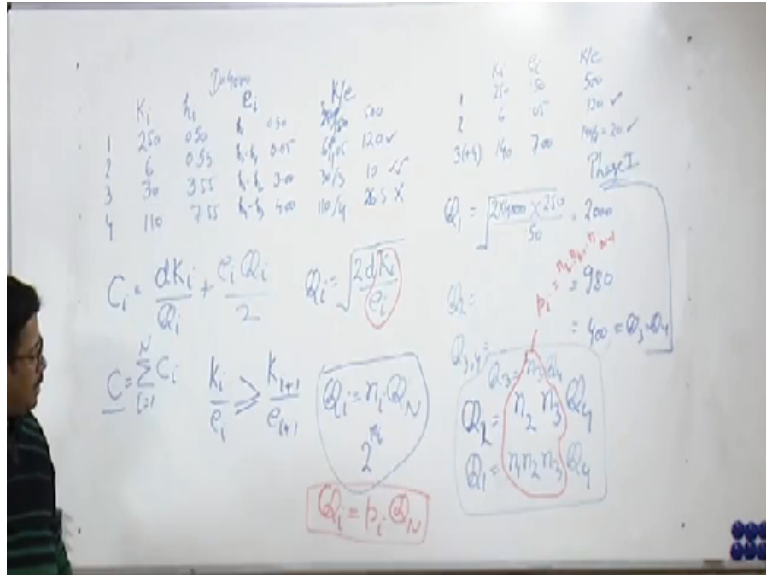
But for the sake of solving this problem we will merge a state 3 and 4, and we will recalculate the values of K upon E after merging this. So when we merging this you have a stage 1, 2 and with is you have also merge this stage 4, so you have new values of kIs, kI will be 250, 6 and now we have merge stage 3 and 4, so therefore this KI will be $30+110$ that is 140, and similarly you will merge the values of EIs also.

So, EI will be 0.50×0.05 and this $3 + 4$ will become 7, and now you again calculate K upon E. So, this is 250×0.50 same 500×6 upon 0.05 the same 120, and 140 upon 7 this is 20. So, now you see 120 and 20, so you have a pattern that the ratios of K upon E in this particular case are following the required condition, and now we can apply this model the results, because now then we have reduced the 4 installation problem into a 3 installation problem.

And for that purpose the results will be you need to calculate Q1 and Q2, but whatever we will calculate for this stage, the meaning is that Q3 equals to Q4 that is the meaning of the result there will be Q1 and Q2, and Q1 and Q2, will be the multiplier of obviously Q3 or Q4 which ever you say, but Q3 and Q4 will be same or other terms you can say Q3 is equals to here the value of NI Q4.

This is equals to 1 this is how you can understand, and then you can go for calculation of Q1 and Q2. So now let us start solving the problem using this final table and for that purpose first we will determine the starting solution to initiate the problem. We will determine the values of QIs with respect to these different 3 stations, so you this formula simple formula.

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And using this simple formula you can calculate the values of Q_1 which will be under root $2D$ is already given to us that is 4000 into KI that is 250 upon 0.50, so on for Q_2 and for Q_3 or 4, so you will have these calculations and these calculations will give you the values of this will come as 2000 the value of Q_2 when you applied K upon E as 120, so this will come as 980 and for stage 3 and 4 this will be 400. So this is the initiation of the problem.

This is the initiation of the problem the first solution which we get that is this Q_1 is 2000 Q_2 is 980 and Q_3 Q_4 is 400, but now you can see on your own once you have calculated this values and the meaning is Q_3 is also 400 and Q_4 is also 400. This means Q_3 is equals to Q_4 , now 980 is not direct multiplier of 400, and 2000 is also not direct multiplier of 980. So the condition of that Q_1 is equals to $NIQN$ is not satisfied.

This condition is not satisfied here, because you are not able to achieve N integer value of NI that is first thing and second relaxations which we discussed that it has to be 2 to the power MI that is even not satisfied, because none of them is even to closer this condition, so it means whatever we have done this is the phase 1 of the solution process or you can say this is the initialisation of the problem.

That you got some initial value, now my request is to you that my request to all participants is that you calculate the total cost you need to calculate the total cost on the basis of this, so you

need to have this CI and if you do the total cost C that is the sigma of CI I equals to $1 \ 2 \ N$, so you calculate this total cost. This is you can say this is the lower bound of our solution process.

The meaning of lower bound is that our solution cannot have a higher cost lower bound means this is the worst solution and the meaning of a solution in our cases that this is the highest cost, this is the maximum cost you will incur in managing the inventory in this particular case, but now in our second phase we will try to improve this value of see lower bound

The total inventory cost (C) (26:56) lower bound have like to improve this lower bound to a better value and for that purpose, we want to use this initial value, now we want to achieve this type of relationship. The objective is to achieve this type of relationship and when we have this type of relationship probably we will have a lower values of total cost and those lower values of total cost will be improved solutions.

We will give you the improved solution and we will see for that purpose what to do, now for that purpose to initiate our phase2 process, we will see first that this Q_i which is N_i into Q_N in this particular case we have to determine the values of N_i and N_i s are if I am at a stage2, so there will be N_2, N_3, Q_4 if I am at a stage1, so there will be N_1, N_2, N_3, Q_4 and if I am at a stage 3, so Q_3 equals to N_3, Q_4 .

So, we have this type of arrangement, now we can simplify this calculation by understanding by this particular multiplying factor, we can simplify our calculation instead of writing N_3, N_2 into N_3, N_1, N_2, N_3 . I can simply write one factor P_i , and P_i is nothing but the multiplier of these things as per the stage. P_i value can be P_i if I am talking of stage 3 it will include only N_3 , if I am talking stage 2 it will include N_2 into N_3 .

If I am talking stage1 it will be N_1, N_2, N_3 , so in general P_i will be nothing, but the N_i up to N_{i-1} this will be $P_i N_i$ into N_{i-1} , and up to N_{i-1} And, with the help of this we can generalised the relationship as Q_i equals to P_i, Q_N , so this becomes our required relationship that Q_i equals to P_i into Q_N and now we will say but how to determine the values of P_i s and where to get the optimum values of P_i s for this solving purpose

And, with the help of that we will be able to determine that yes, now the solution is being achieved, now let us stop in this class here and in our next class we will see the phase 2 of the solution process where we will be determining the values of PIs for different stages and then we will see how to stock in that particular condition and how to complete this solution Thank you very much.