Lecture-19
Multi Echelon Inventory Management for Four Stations

Welcome back, we are discussing about inventory management in the supply chain. And in the inventory management discussions in last 2 sessions we have discussed about very simple case of inventory management in supply chain where we had only 2 installations in the supply chain, and we used our unique conventional EOQ model to handle those 2 installations.

And we saw with the help of an example also in the last class that how when we do the simultaneous optimisation even in the case of just 2 installations we have some type of cost saving. The very well cost saving in a supply chain environment. And when we do the individual optimisation it is not possible. Now the same discussion in this class we will take to multi echelon systems, where in a supply chain more than 2 installations are possible.

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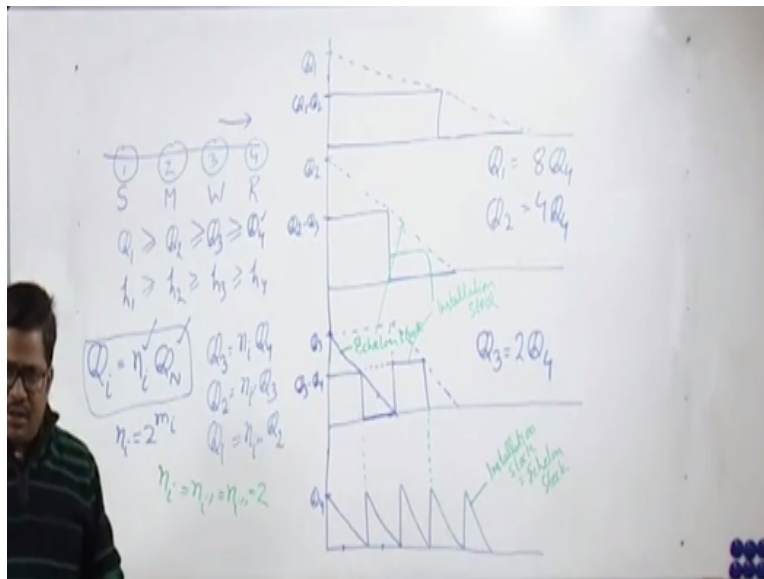
And when more than 2 installations are possible you can have now, a supply chain where we can think of 4 stations, 1, 2, 3, 4, and as we are moving from this side to this side we know that it can be a supplier, it can be a manufacturer, the wholesaler, and the retailer. So, these are normally the 4 components which are available in a supply chain, the supplier of raw material, supplier of the

components, your other type of part suppliers. Then some kind of manufacturing or assembly of those components may take place here.

And, these are the wholesalers or the dealer, which are taking products from the manufacturer, and finally customers got their products at the retailers end. So, this is a popular supply chain, generic supply chain model. And, we have discussed in our first sessions about this type of generic supply chain. Now, considering this type of generic supply chain, and using the points which we have already discussed in earlier 2 sessions.

About 2 installations of inventory management, now here retailer requires $Q4$ item, the wholesalers requires $Q3$ item. The manufacturer require $Q3$ item, and supplier has demand of $Q1$ item at particular time. Now, it is very well understood that the retailer is getting it supplies from wholesaler. Wholesaler is getting it supplies from manufacturer, and manufacturer is getting it is supply from the supplier or the vendor.

Therefore, $Q3$ is responsible out of $Q3$ items a part of that will go to retailer in the form of $Q4$. From $Q2$ items a part of that will go to wholesaler in the form of $Q3$. And from $Q1$ a part of that will go to manufacturer in the form of this supplier. So, therefore $Q1$ needs to be bigger or at best equal to $Q2$, and so on for other stages also, $Q2$ needs to be greater than or equal to $Q3$, and $Q3$ needs to greater or equal to be $Q4$.

So, this type of relationship is to be there, between various quantities which you are procuring at a particular stage. Because, a particular stage is responsible to supply products to the next stage in the supply chain. That is one very important thing when we will develop the model for inventory management. In this type of situation you always need to see that $Q1$, $Q2$, $Q3$, $Q4$ or if I take it to Qn to this level.

This type of relationship that the quantities which you are procuring which you are receiving at predecessor stage, should be more or equal to the quantities which are require at the receiving end. And now the other important thing which we have already discuss in the 2 installation system. The holding cost, because as I moving from 1 to 4 I am doing the value addition, so this

is the direction of value addition. So, therefore my holding cost which is proportional to the these value addition H1, h2, h3, h4.

So my holding cost because of increase in value, because of more value I am adding as I am moving from left to right. So therefore holding cost will also increase as I moving from this to this station. Now with this if I go to develop the model for inventory management. Let us see whether we have sufficient conditions to apply our conventional EOQ model. Now, at this point I will like to recall what we discuss in the last 2 sessions.

That to apply EOQ model the inventory consumption, and replenishment at a particular stage must follow, if you recall you can tell me the saw teeth pattern, saw teeth pattern must be available for applying the EOQ model of inventory management at a particular stage. So, now will let us see whether the saw teeth pattern is available in this particular case or not. And, then only the basic EOQ model can be applicable for inventory management.

In this particular case now, let us that at stage number 4, the last stage where we have procuring because this knowledge we have already from our earlier discussion. That in a 2 stage model we are procuring inventory at other stages in relation to our last stage, so here the last stage is fourth one. So, I will procure inventory at other stages, in relation to this fourth stage. So, what I am trying to say that Q3 should be sum multiplier of Q4.

Q2 should be sum multiplier of Q3 and so on Q1 should all be sum multiplier of Q2. so, all these inventories at other stages are multiplier of their successive stage, and if I see If I substitute the value of Q3 here in this case and Q2 here in this case, I can see that I can represent Q1, Q2, Q3 all three in terms Q4. So in this case also my complete management will written with respective quantities which I am receiving at my fourth stage.

Now, at the fourth stage I am requiring this Q4 items. Now these are the Q4 items, and I receive Q4 items from stage 3, and I start consuming these Q3 items. Then after sometimes I will consume these, and I have adjusted my lead time, I have adjusted my order point. In such a fashion that as soon as I am touching this 0 level. I get a fresh supply of Q4 items.

And, then I will start consuming these Q4 items, and this way my second cycle will be completed at fourth stage. Again, I will receive a new supply, and this will go on. So, you see we have a very clear saw teeth pattern, these vertical lines are representing the replenishment of my stocks. And these slant lines are representing my consumption. So, at stage 4 you have a very perfect case for applying the basic EOQ model of inventory management.

Now, let us come to stage 3, in a stage 3 you require Q3 items now, what we have done here, a very specific case now, what we have done, just to make it simpler, I have taken the values of ni, ni dash, and ni double dash, equal to 2. Just for the purpose of our understanding, all these values are 2, 2, 2 each. So Q3 is actually 2Q4. So, all these values I have taken ni equals to ni dash equals to ni double dash equals to 2.

So, these things make me Q3 equals to 2Q4. Now, I am receiving Q3 supplies here. And, out of Q3 supplies I will immediately supplied, because I have adjusted the receiving cycle in such a manner that on day 0, when Q4 is supply to stage 4, the same day I receive Q3 supplies at stage 3. And out of Q3 since Q3 is 2 Q4, a part of that part of Q3 will go to stage 4. So, what is remaining with me Q3-Q4, that is with me, and that will go with me till this particular time.

And, you can see that by this time I will be carrying this Q3 –Q4 stock in my warehouse, and I will be continuing with this, and then at the second replenishment cycle at stage 4. When again to 4 items will be required these Q4 items will be shift at this point, and then I will not have any inventory for this period. Then again I will receive a new supply of Q3 items.

But, out of that I will immediately supply this is Q3, this point represents Q3, but out of that again I will immediately supply that Q3, Q4 portion to the stage 4, and I will be left with Q3- Q4 here again. And, I will carry this inventory for one more cycle of stage 4 and then this inventory will come to 0. So, with this way you will see that I have these types of step curves. This is Q3 level, immediately Q3 comes to Q3-Q4.

I will carry this Q3-Q4, for some time, and then my stock will come to 0 level, and then I will be having 0 stock, at stage 3 or this much period. This much period is represented by one cycle of stage 4. And, then again when fresh supply is required at stage 3, I receive Q3 items here, and out of Q3 items Q3 – Q4 remains in the physical stock at stage 3. Because, Q4 has gone to stage 4, and I will carry with Q3-Q4 supplies, for one cycle of stage 4.

And, then when the next cycle requires the fresh supply of Q4 items, so Q4 items are available with me, and supply these Q4 items to stage 4. And, the stock will again come to 0. So, I will remain 0 stock for one more cycle of stage 4. And, then again Q3 supplies will come, and this type of step pattern, will keep ongoing. You will see that this type of step pattern will keep on going.

Now, these lines green line represents the installation stock, last class also we discussed this that this is the installation stock. So on here also this is installation stock. That what is available at a particular stage 3 or stage 4. But, we already introduced a concept known as echelon stock in our last class. So now echelon stock just to revise that echelon stock represents the amount which you have at that particular stage at stage 3 and at stage 4.

So whatever you have, so if I talk of echelon stock of stage 1, so whatever physical inventory you have at stage 1 + 2 +3+ 4 at a particular time whatever inventories you have, physical inventories you have at stage1, 2, 3, 4 that is the echelon stock for stage 1. For stage 2 what physical inventories you have at stage 2, 3, 4. That determines the echelon stock for stage 2 and so on for stage 3 and 4.

So, if I talk the echelon stock for stage 2, so you will see that here it is Q3, and here it is 0 initially, so the Q3 is the total stock, which is available with me, and at a particular time if I talk of this particular time, if I talk let us say a particular point here. So, at this particular point here, you see the physical inventory at stage4 is this much. This is the physical inventory available at stage 4.

And, this is the physical inventory available at this stage. So, I want to see the total inventories of this + this, this point will come somewhere here. And so on, if I want to see the total inventory at this particular point, so here it is 0 already. And, this much inventory is available at stage 4. So, this point will come here. At this point you see the total inventory at stage 4, as well as at stage 3 is 0.

So, if I combine all these points of echelon stock I will get a line like this, this is the line which represents the echelon stock at stage 3. The stock, which is the physical inventory at stage 3 + stage 4. So, it is the echelon stock, in case of stage 4, since there is no stage, after stage 4, so whatever inventory is there at stage 4. That is the installation stock as well as the echelon stock. So, in case of stage 4 this curve is making you installation stock, as well as echelon stock.

So it is same here, but in case of stage 3, these step lines are representing the installation stock. But, this type of curve where you have a slant line of consumption, this represents the echelon stock. And, then if you see for at any cycle, you can have this type of curve represented by the dotted lines. This represents the echelon stock. So, you can see that we can apply now EOQ model for stage 3 also.

If we consider it is echelon stock we can apply the EOQ model here also. And, now take this case forward you have stage 2 now, now you take the stage 2, in stage 2 you have Q2, which is 2Q3. Q2 is 2Q3. So, now out of Q2, whenever you procure Q2, so to initiate we have develop the system in such a manner, that whenever you require Q3 on time t0. The same day you have procured Q2 also.

So out of Q2 half of Q2 that is Q3 will go to stage 3. So, you are left with Q2-Q3, and you will continue with this Q2-Q3 for this much period. The time is second requirement of Q3 comes from stage 3, and then it will come to 0, and then again you remain at 0 levels the same discussion which we have applied for stage 3 will be applicable to stage 2 also.

So, again you can think of this type of echelon stocks at stage 1 also. You can think of these slant lines which are representing the consumption of echelon inventory, these slant lines represents

the consumption of echelon inventory. Echelon inventory again is the inventory available at stage 2 + stage 3 + stage 4. So, actually this slope is coming because of this consumption at stage 4. At stage 2 and stage 3 you have the horizontal lines. But, the angular lines are available at stage 4.

So, because of the angular lines at stage 4, you get these slopes in stage 2 also. And, therefore, these installation stock which is represented by these step lines. And, the echelon stocks are represented by these slanted lines. So, this is again showing this case, that this slant lines make us possible the use of EOQ model, at stage 2 also. Now, come to finally a stage 1 also.

The same explanation can be applied to stage 1 also where you are procuring Q1 items, in such a manner that Q1 is equals to 2 Q2. And we have adjusted the procurement cycle of stage1 in such a manner that whenever there is a demand of Q2 items. The Q1 items come on the same day. So, on time t0, you receive Q1 items at stage 1, and out of that immediately Q2 items are shift to stage 2. So, you are left with Q1 – Q2 items.

And, you will continue with Q1-Q2 item for one complete cycle of stage 2. And, then you will be with 0 inventory for another complete cycle, and again by using the concept of echelon stock, you can have this type of dotted lines to represent the echelon inventory at stage 1. So, now you see by considering the concept of echelon stock at stage 1, stage 2, stage 3, and obviously in stage 4. Echelon and installation stocks are same.

So, here we have that because of these echelon stock concepts in these echelon stock concepts in these 4 stages. You can very well apply the concept of EOQ model. So, we will see the model with the help of echelon stocks the help of echelon stocks at all these stages, and that will help us in using the basic EOQ model. Here if you see that we have used the values of ni, ni dash, ni double dash, all two.

So, that will help us in very uniformly completing the replenishment cycles. The 2 replenishment cycles of stage 1, makes one cycle of the stage 2. 2 replenishment cycles of the stage 3 makes one replenishment cycle of the stage 2. And, 2 replenishment cycles of stage 2 will make one cycle of the stage 1. And, therefore this will also enable us that I can write all these terms, all

these Q1, Q2,Q3 in terms of my Q4. So, here you see that Q3 can be represented that we have already done that.

It is 2 Q2, and because 2Q2 is 2 Q3, so I also write Q2 equals to sorry this is Q4, here it is Q4. Q2 is equals to 2 into 2 Q4 that is 4Q4. And, 2 cycles of Q2 represents one cycle of Q1. So, Q1 is actually 2Q2. So, here I can also write Q1 is equals to 8Q4. So, I have represented all these Q1, Q2, Q3 in terms of Q4. Q3 is 2Q4, Q2 is 4Q4, and Q1 is 8Q4. So, therefore the point is that in case of I want to generalise this discussion.

I can write Qn as my last stage of supply chain. This is the (()) (26:19) stage of my supply chain. And, I can write all the stages, let us say it is Qi in terms of my values of so, this is how I can represent all my stages with the help of the last stage. All my stages with the last stage, so this is very important that here, when we are going to develop the multi echelon inventory system. In this multi echelon inventory system we will be determining these 2 things only.

We will determine the value of Q1, the value which is required at the last stage and then for the subsequent stages. Here we have assumed for the sake of simplicity we have assumed that all Ni s are 2. So, that is why have this type of very uniform representation. But in our other cases we will determine that whether it is 2 or 4 or 6 or something else.

And accordingly Q1, Q2, Q3 can be determined directly determining the value of Qn only. So, now one more thing before we close this session, one more thing will like to discuss. Now Qi can be Ni into Q1. Now, Ni as we have discussed earlier also in the case of 2 installations, should be any integer value. So, it can be 1, 2, 3, 4, 5, 6, 7 anything, and if you remember the example which we took t understand the case of 2 installations.

In that we took deliberately the value Ni as equals to 3. Q1 was 3Q2, in that case. But, here we have a limitation and for that limitation we will use a kind of relaxation of this problem. We will like to use a relaxation of this problem. Now, the limitation is that if for the purpose of this problem. Here Ni is 2, and here Ni dash is 3, and here Ni double dash is 2. Let us say these are the values, 2, 3, 2.

In that case it will not be possible for me to make this type of groupings of these cycles. Because, of odd and even values of Ni s I cannot make these type of cycles of my supply chain. That all these cycles are starting and ending, in the same manner. You have the same starting date of all the cycles, and 2 cycles of this will make 1 cycle of stage 2. And, 2 cycles of stage 2 will make 1 cycle of stage 1.

But, if it is not so, in that case it will be difficult for me to develop these types of relationships 2 cycle will make 1 cycle of this. But 3 cycles of stage 2 will make 1 cycle of stage 2 and then it will be difficult for me to have same starting and ending date. And, as a result of that either I will be having more inventories at this stage or I will be running out of stock, for 1 complete cycle if I want to match this type of analysis.

So, for that purpose we have done a bit of relaxation in this relationship. Where Qi equals to Ni Qn we have used. Instead of that we will be writing Ni as 2 to the power Mi. And, Mi can take any value, Mi can take any integer value 0, 1, 2, 3, 4, 5. So, what will happen because of this relaxation, because of this additional condition we have imposed, you will get the values of Ni, only in the even values.

If Mi is 0, 2 to the power 0, it is 1. Then 2 to the power 1, it is 2. 2 to the power 2 it is 4. 2 to the power 3 it is 8. So, only even values will be there. 1, 2, 4 like that and, that will make our life very simpler, and therefore this type of analysis will be possible to handle the supply chain more effectively. We can go to a bit of mathematics and Roundy was one of the professor who suggested that if we use this type of relaxation.

There will be slightly higher values of the overall cost, but those values of higher cost will not be more than 2 %. So, that is very well justified, because of lot of comfort we are going to receive by doing this type of simplification. So, here we stop in today's lecture, and we will see the application of this type of model in our inventory management. So, in the next class we will see that now we are clear that we are going to use this type of relaxation.

And will have application of EOQ model at all 4 stages. So, now with the help of some data, with some practical exercise, we will see in our next session. That how do we apply this model in a multi echelon situation. Thank you very much.