Supply Chain Analytics
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Lecture-18
Multi Echelon Inventory Management (Continued)

So welcome back, and in our last session we were discussing about the simultaneous optimisation of this problem when we have 2 stations and before that we already discussed about the separate optimisation separate use of EOQ formula for this 2 station problems.

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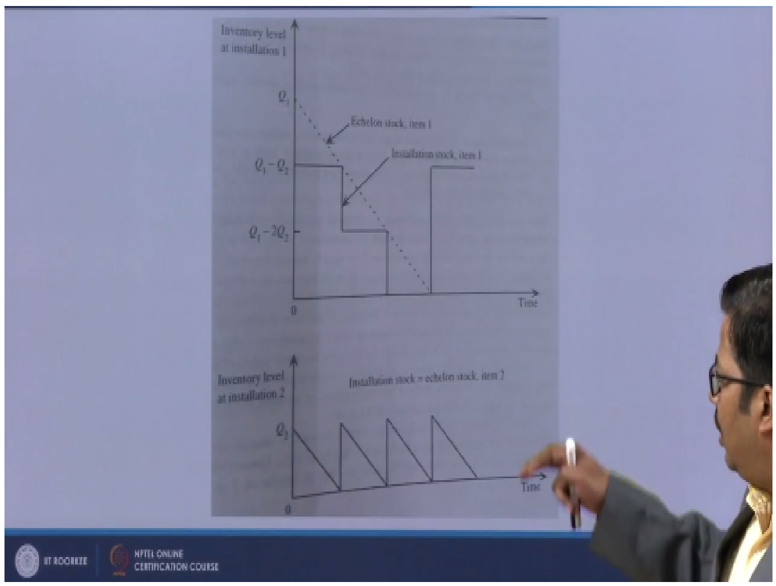

Now, in case of a simultaneous optimisation, we develop the expressions of Q. and on the basis of that expression.

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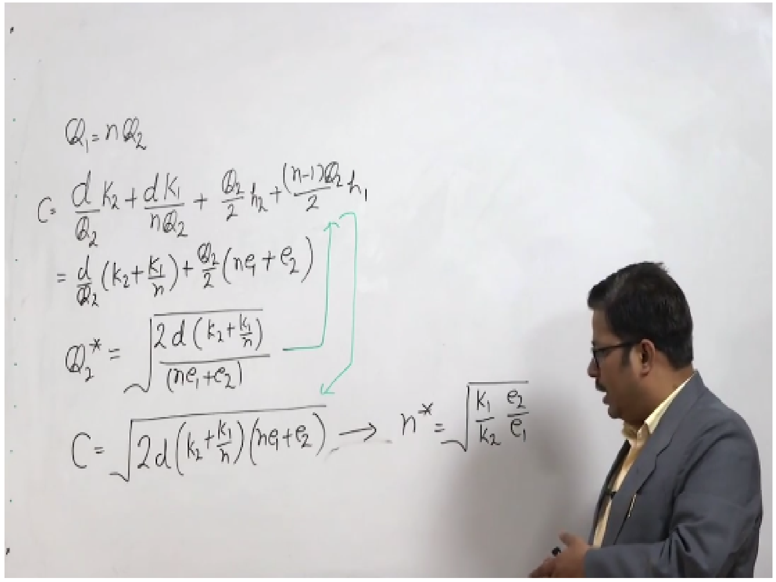

When we had Q1 equals to n Q2, and we had the total cost, which is sum of the cost variable cost as in installation 2 as well as the installation 1, and this we discussed as D upon Q2 k2 + D K1 upon N Q2 because Q2, Q1 is nQ2, + we discussed about the holding cost, that is Q2 upon 2 h2 + the holding cost of installation 1, that is n -1 Q2 by 2 h1. And, now when rearranged all these things it became d by Q2 K2 + K1 upon n, + we took Q2 by 2 out of this expression.

And then when we started rearranging the terms of h2 and n1- n- h1. We saw that it is coming nh1, and h1 we discuss is equal to e1. That is the echelon holding cost, so 1 one term came as e1. And, then h2 – h1, we discuss in the last session is equal to e2, so that is e2, this we got, and now this is our K2 + K1 by n, is the unit ordering cost. And, n1, and e1 + e2 is our holding cost for average inventory.

In a supply chain environment, particularly you remember that now we are doing the simultaneous optimisation. So, this is the effect of supply chain environment. We are trying to build into our inventory management models. And as a result if I calculate Q2*, that is the economic order quantity at stage 2 will be under root 2. This expression that is the ordering cost K2 + K1 upon N divide by this holding cost that is Ne1 + E2.

and as a result of this when we are putting this Q2 in this expression of overall variable cost C, so this expression give me the value of C and the value of C will be the minimum this value of C corresponding to the value of Q2. The value of C because of this Q2 so you see that way we are putting this here and then the value of C which we are obtaining is this, and this is under root 2 D into ordering cost multiplied by the holding cost. So, this is K2 +K1 by N into Ne1 + E2.

So this is the minimum holding cost for simultaneous optimisation. Now for this reason when we have these two expressions readily available with us. The only challenge this two determine the value of Q1, and Q1 is dependent value of this N. So the next point is what is the value of N here, for determining the value of N we suggest all our participants to do the differentiation of this minimum cost with respect to N.

And put that differentiation equal to 0 and calculate the value of N, But since the value under the right side is square root of some expressions. So, doing the differentiation of this square root term is not so easy. So advised in the literature you can go ahead with the square terms of this, so you take the square of both the sides make it C square, so this under root expression will become 2D x 2K 2+K1 by N into N even +E2.

And, then you take the differentiation and this will not change the result. At as a result of that when you calculate N from this expression when you calculate the value of N this will be, you I request all participants to do this on their own, but I writing the expression directly K1 upon K2 into E2 upon E1, and now you can compare this value of N and the value of N we got earlier in 4that case the value of N was K1 upon K2.

When we were doing the separate optimisation where we using the separate EOQ formulas at that time it was K1 upon K2 x H2 upon H1. So, from that holding cost of our instructions, now we are moving into the echelon holding cost E2 upon E1 is the echelon holding cost. So, this calculation of N text the essence of that supply chain environment.

When we are moving the purposes which are trying to say that the movement of our calculations from this installation calculations to echelon calculation is actually the capturing effect of supply chain environment, when we are not considering the echelons it means we have not captured the supply chain environment into over discussion. So, with this we can calculate the values of N also here also in this calculation of N as in previous calculation also.

These values of N can be any fractional value, but we want integer values of N and for that purpose now let us see that how do we round off these fractional values of N. We discuss in the last session that if the value of N this calculated value of N is less than 1. If it is less than 1 then you take it N equals to 1, but if it is more than that then there is process and let me give you a brief of that with the help of which we can take this discussion forward.

So, now let us have some kind of numerical data with us and with the help of that numerical data, We will try to see how do we use this whole discussion into practice and what type of benefit are we going to get with simultaneous optimisation. So, in small data we will like to discuss with you and with help of this small data, we will see that how these models can be actually practiced.

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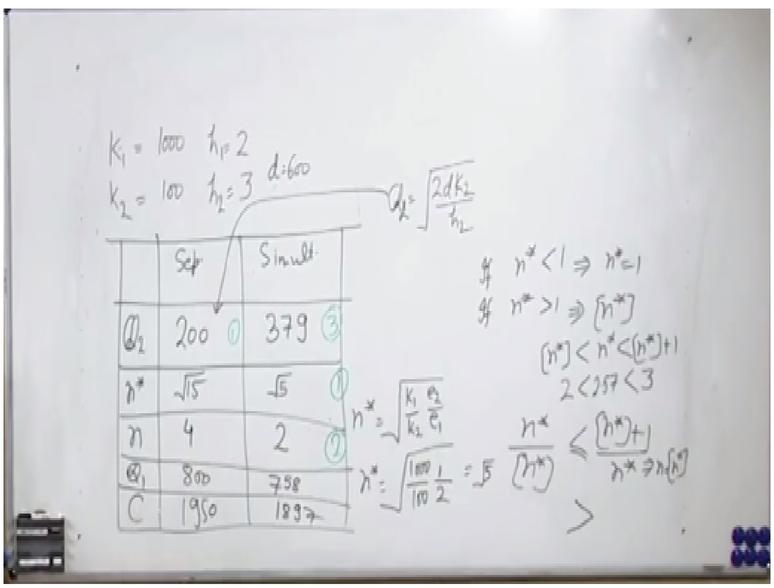

And for that purpose we have these two installations, and the cost of setup at these two installations are given to us. At installation1 the ordering cost or the set up cost is 1000 rupees and at the installation 2 this cost is 100 rupees. At installation1 the holding cost is 2 rupees per unit per year, and at installation 2, this is 3 rupees. And the annual demand is 600 units. So, this much data is available to us. Now when we have this much data with us, you can also see the values of H1 and H2.

First, before we start solving this question before we start using this data see the values of H1 and H2. H1 is a primary stage, H2 is towards the right hand side of the H1 installation1, and therefore the values corresponding to H2 are higher than the values corresponding to installation. So that is one particular thing as a student of the supply chain course, we should understand that was the what the physical significance of these things is.

Now, you see though it is not hidden first but still you see as you are moving the right hand side, your set up cost decreases if it is purely setup cost it decreases, because when you are moving towards right side not much tooling is required, because product is already finished and it is more likely to be repackaged or just the logistics loading unloading types are there. So, the setup cost is less as you move to rights and your holding cost increases.

So, this data is very particularly taken to highlight this very aspect of this supply chain, that what physical changes are taking place as you are coming left to right in the supply chain. So, now when we have the data with us so I request all my participants that use this data for separate optimisation as well as for simultaneous optimisation. You need to see that how can we have the separate and simultaneous optimisation.

And to understand the use of this data for the problem solving purpose, Let me make a table and in that tabular arrangement, you can understand what type of data you can fit it and how do you know that what type of improvement or the modification with different types of calculations are being done. So, this table can be made like this way and you have direct comparison between separate and simultaneous.

Now, the first thing is both these model how do we operate in both these models the first thing is the calculation of Q2, that is the first thing. In both these cases the first thing, the starting step is calculation of Q2. In the case of separate optimisation, the calculation is pretty simple, you just need to the old formula, which we have discussed under root 2d K2 upon h2. So, use this formula for getting your values of Q2 here.

And, use of these values will give you Q2 = 200. And, then the formula involving the higher values of Q2 we will see that how do we use that that will come here. Then, the second thing is to determine the value of n* and, the value of n* is in this particular case will be K1 upon K2 into h2 upon h1.when we are doing the separate optimisation

And that value will come under root 15. Now under root 15 is a fraction and now let us see how do we round off those fractions into our this tabular arrangement. So what we can do if this n* is a fraction, so there are three roots for that purpose. There are three roots if n* is less than 1 than you take n* equals to 1, and if n* is more than 1. So, in that case if n* is more than 1.

So in that case we consider 1n in this bracket which is the highest possible integer just a smaller than this n*. This is the highest possible integer just a small than this n*, so that you can write this expression for an example if this n* is 2.57, so this will be 2 and this will be 2+1 3. This is 2 and this will be 3, and now will have this comparison where you take the ratios of these terms.

Now, if in this ratio if left hand side term is less than or equal to right hand side term. Your round off will be on the lower side and in case of reverse if left hand side if it is this way. Here in this case n equals to this and otherwise n equals to n* bracket +1. So, by this way of rounding off you can round off the available values of n*.

Apply same in this particular case, so then you will find that round off value of n is coming to be 4. When you do this type of calculation, you will see that round off value of n is 4. And, when round off value of n is 4 then you can determine the value of Q1 Q1 is nQ2 Q1 is nQ2, so it will come 800 4 into 200 it will come 800. So now in this separate optimisation, you have calculated both these values Q1 and Q2.

And as a result of Q1 and Q2 you can also calculate total cost of inventory C, and you can use our earlier expressions which we discuss in our last session of inventory management that what will be by total cost of inventory. So, that C can be calculated here. We can have the direct calculation which will tell as C will be 1950 the value of total cost of inventory in this particular case will be 1950.

So, this is for the separate optimisation. Now when I want to do these simultaneous optimisation. Because, calculation of Q2 you see here I have done in a very sequential manner, first I calculated Q2 I determine the value of n and there is no relation of Q2 and n and then with the help of the value of n. I determine the Q1 and finally C, but now in the simultaneous optimisation you can recall just now that the calculation of Q2 involves the value of n.

So here my first step in this case will be this stage. Here it is the first and here it is the first, so first I will determine the value of n* and it will K1 upon K2 into E2 upon E1. So what is E2 and E1. E2 will be the difference of E2 is on the right hand side. So E2 is H2-H1 that is 3-2 that is 1 and E1 equals to H1 that is 2 E2 is H2-H1. So 3-2 E2 is 1 H1is equals to H2 H1 is equals to E1. So that E1 is H1 that is coming to her.

K1 is 1000 divided by K2 that is 100, so when you form this it becomes under root 5, so this is under root 5 and when you solve it under root 5 means when you round off under root 5 you will see that this will come to 2 and when now this n equals to 2 will be used for determining the value of Q2.

So, now actually the calculation of Q2 in the second case is my third step and when used I this formula for simultaneous optimisation the value of Q2 using this value of n will be 379. Value of Q2 using this expression is 379, and then again Q1 equals to nQ2, so 379 into 2 that is 758 is my Q1. And using these values of Q1 and Q2. My total cost of inventory comes to be you all can calculate and directly giving to the values that is 1897. Now you can see now this question is done.

Now, let us have some physical understanding some interpretation of these data, because that is more important for managers to analyse the data properly. So, this data this information there in front of us, Now there are certain very revelling factors out of this solution. One which is very simple to understand that because of simultaneous optimisation, I got the benefit here the cost was1950 here the very well cast is reduced and it is 1897 only.

So, that is one direct visible benefit to, but now you see another challenge I am the stage1, and as stage1 I am the owner of entire supply chain. Now, stage 2 is my retailer and in first case retailer is keeping inventory of just 200 items. But now I am moving to simultaneous optimisation the inventory level of retailer increases to 379. Now, as a manager it is very difficult for me to convince my retailer.

That you increase your inventory level from 200 to 379, and therefore many a times because a supply chain is fragmented also on one side in a supply chain we say that we all are connected. But, we all are owners of a particular entity also. So, it is a fragmented discussion also, so here you can see the retailer who is the end point in my supply chain that retailer will say that.

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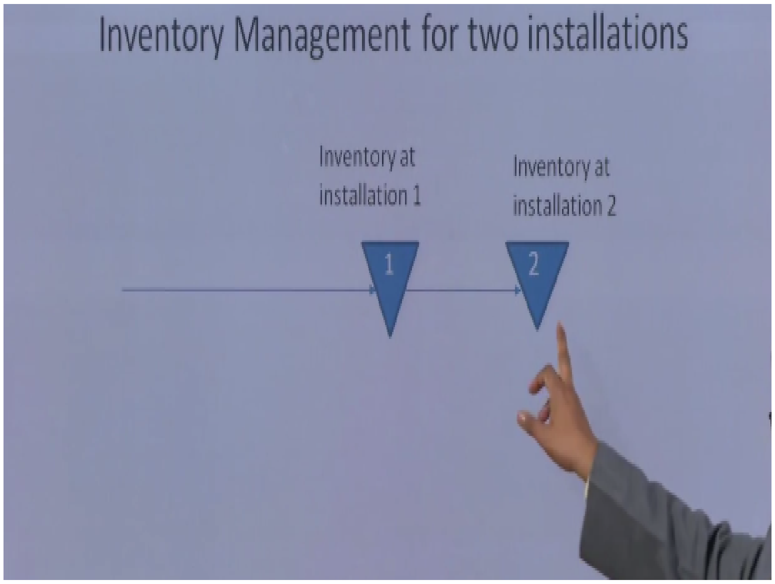

I am not ready to stock 379 units earlier my stock was only 200 unit why should I stock more. So developing this understanding in your entire supply chain that know it is beneficial for all of us, if you stock more so the overall cost of inventory is less. It is going to help the entire supply chain to increase it is circles. Because, ultimately this total cost of inventory is reducing the profitability of the entire supply chain.

So, developing this idea requires lot of efforts it is much difficult than what we have discuss in the class, so this model obviously because of this net gain gives profit, but at the same time you need to have that label of trust that label of understanding between 1 and 2 stages. So, that if these values is increasing and mostly in all most all the cases, You will find a substantial increase in Q2 at the simultaneous optimisation process, but the overall cost decreases.

Overall cost of inventory decreases and therefore this model better captures the supply chain effect and reduces the cost of inventory and therefore it is advisable that we need to build that trust we need to build that confidence, between stage 1 and stage 2. That what stage 1 or whoever is owning the supply chain means we know that this is the supply chain of dell.

This is the supply chain of Amazon, this is the supply chain of Walmart, this is the supply chain of Apple, so there somebody who owns the supply chain, and then in there supply chain there are many smaller components, many smaller installations and all those smaller installations if they are not able to understand the philosophy, the way, the owner of the supply chain owns to run it, it is very difficult to achieve these types of simultaneous optimisation cases.

And, then people will keep on doing their own individual optimisation of inventory, which may in a very catastrophic type of situation of bullwhip effect. So, we stop in today’s class at this point, where we have discussed the simultaneous optimisation with the help of one numerical problem, that how it can improve our results. In our next class we will discuss this optimisation for a multi echelon problem.

Where more then 2, it is a very simple case, only 2 stations are there, but in real life we do that there will be many, so we will take some case, where you have more than 2 installations and how do we operate with the simultaneous of those multiple installation in the supply chain. Thank you very much.