Supply Chain Analytics
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Lecture-17
Multi Echelon Inventory Management

Welcome back, so we were discussing about the inventory management in our last session, and we started discussions about 2 stage inventory management in the supply chain. And, we discussed that how directly at stage 2.

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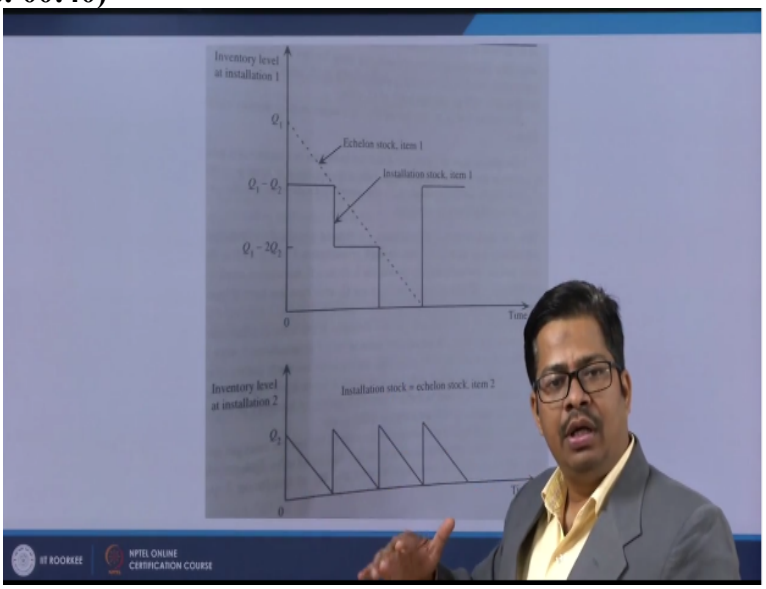

We can apply EOQ model because of this saw teeth pattern, but directly, we saw that we have this type of step pattern at stage 1. And, therefore we cannot that EOQ model at stage 1 in this supply chain. But, then immediately it was told to us, that there is a concept known as Echelon inventory stocks. The Echelon inventory stocks where, you have physical inventory at stage 1 + physical inventory at stage 2.

And, when we used this Echelon concept then we saw in the last session, that we can have these types of dotted slanted lines. And as usual what we discuss in the last class, that these vertical lines are representing the replenishment stocks. The stocks when it is coming to you, the replenishment stocks. And these slanted lines, the line with slopes these are representing the consumption lines.

So, same way, now we have in our stage 1 also, these vertical lines as our replenishment lines, and these virtual lines, these dotted lines are the consumption line. And, therefore you have the saw teeth pattern at installation 1 also, and now because of the saw teeth pattern and installation 1, you can apply EOQ model at installation 1 also. So, now this agreement is being made.

That we can apply EOQ model at both these installation 1, and installation 2. Now, we have told that how to apply EOQ for a single stage, for a single installation. So, now let us start doing that only, and as we have discussed that all the assumptions which we take for developing basic EOQ model will apply here also. The only additional assumption, the only additional condition which we discussed in the last session.

That, because of the value addition as we are discussing in a supply chain environment the h2 the holding cost which we are incurring at our right hand side of the supply chain, will be more than the holding cost which we incur at the left hand side. So, as we are coming from left to right, the holding cost values keep on increasing. We will discuss one more concept about handling this increasing holding cost because, now we have discuss about the echelon stocks.

So, the concept of echelon stock will help us in understanding the combined optimisation also. But, before we start this combined optimisation let me go with the separate optimisation. When, we handle the inventories at installation 1 and installation 2 in separate manners.

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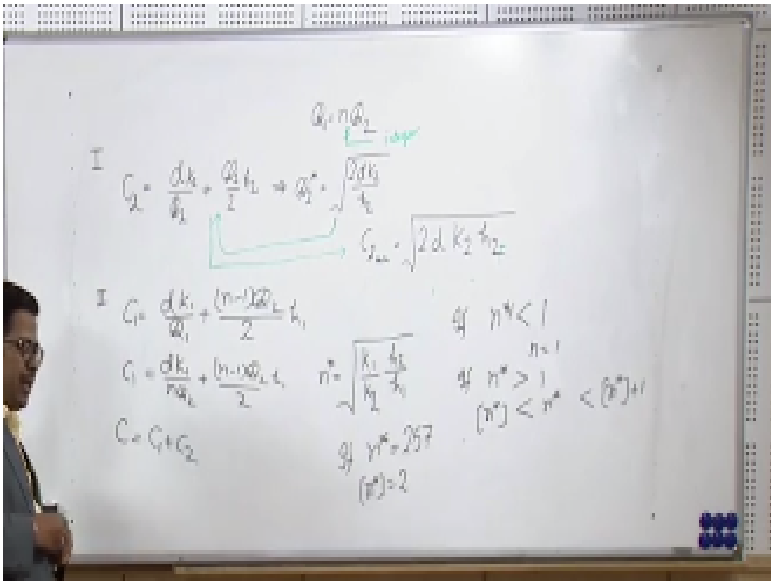

And that is how we do the inventory management, when we are handling 2 cases separately, then at installation 1 let us say the total cost of inventory, total variable cost of inventory is being represented by C1, and now C1 is composed of ordering cost or the setup cost and the holding cost. So, the ordering cost is the number of times you give the order. Yours total requirement is d, which is at the end of the supply chain.

This is the total requirement d, and each time okay, let us discuss first for installation 2, so C2 is our, because we have discussed that our Q1 means nQ2. So, right now we are discussing the inventory management for our second installation that we are starting this process.

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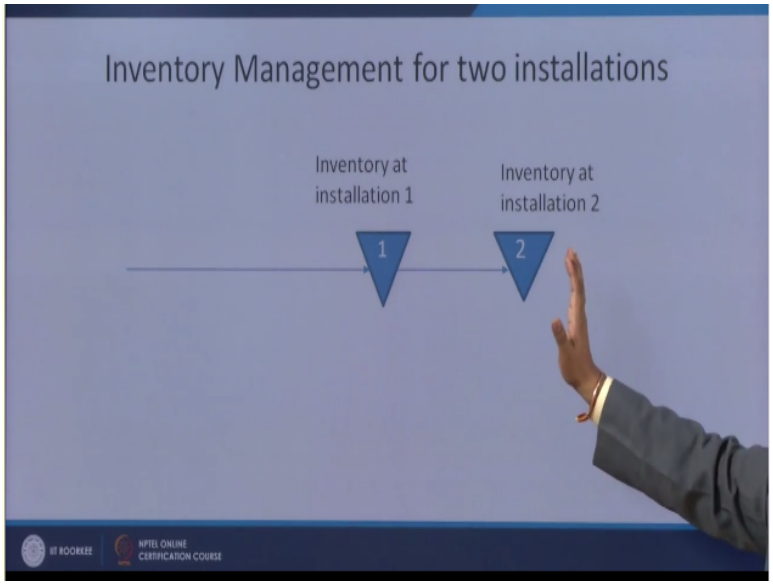

Because, here it was very clear that how we can apply that basic EOQ model. So, for that purpose we are starting with our installation 2, so, at installation 2, C2 the cost is d by Q2. And, the cost of order per order is K2. So, dK2 upon Q2, is the ordering of setup cost. Then the second cost is the holding cost. And, EOQ development we have discussed that it is always paid on the average inventory stock you have.

So, Q2 is the total, 0 is the least, so, Q2 by 2 is the average inventory and the holding cost is h2 per unit per year. So, this becomes the expression for calculating the total variable cost, and we already know to determine the value of Q2. You need to differentiate it and the apply principles of maximum (()) (06:59), equate that differential equals to 0. And, then you get the value of Q2.

So, this will result in to Q2*, that is under root 2dK2 upon h2. This is the economic order quantity value, which we should have at installation 2 and, this Q2 value if I substitute back into this. If I put Q2 back into this. This will result the C2 minimum. Which will be I request all participants that they should do this, and put this value of Q2 in this equation. And, then see that what C2 minimum is coming.

So, I wait here for a minute, and you can practice, and you can check that your result should be under root 2dK2, h2, so I am giving you directly the results. But, I request all the participants that if you substitute these values of Q2, you get this C2. This is at stage 2. Now coming to second level, we want to determine the value of Q1 also. The Q1 is nQ2 and, for that I write expression for C1 also. That is the total variable cost at installation 1.

D because, that is the rate of consumption of your supply chains. Or d will remain as it is. K2 will be change by K1, and Q2 will be change by Q1. So, this is the ordering cost or setup cost at installation 1. Now, comes the part of holding cost. Q2 by 2 h2, now when you see you are procuring the inventory immediately you are left with Q1-Q2. In this case we took Q1 equals to 3Q2.

In this particular case we have taken Q1 equals to 3Q2. So, as soon as you procure the inventory your 1 Q2. You say if it is nQ2**.** So, out of nQ2, your 1Q2 is gone to second stage. So, you are

left with nQ2 –Q2. You are left with at stage 1. You are left with nQ2 – Q2. And, then finally you reaches to the 0 level. So, the average inventory is at stage 1 is nQ2 - Q2 divided by 2 that is the average inventory you have.

And, that we will write here that is nQ2-Q2 divided by 2.this is the average inventory we are going to have at stage 1 multiplied by the holding cost of stage 1 h1. And, again now you see, the first expression is in Q1, the second step of this expression is in Q2. So, we can convert the whole expression in terms of Q2 only. So, I want to remove this Q1 from my expression. So, it can be written as dK1upon nQ2 + n -1 Q2 by 2 h1.

So, this is the expression I have for the total variable cost at installation 1. Now, out of this total variable cost Q2 I have already determined here, so I need to determine only unknown quantity. So, can you have a guess, the only unknown quantity you can see in this expression is this F. So, I will differentiate this total variable cost at installation 1 with respect to n.

Because, I want to determine the value of unknown, and this when I am putting the value of Q2 what I have received from here, that I am going to put here. So, n will come, if I calculate the n will come, and after rearranging my various mathematical terms. N will come K1 upon K2 into h2 upon h1. So, this value of n will come, and here though in this expression and in this model everywhere.

We have assumed that the value of n will be an integer 1, but here When I am doing this differentiation, when I am doing this calculation. I am not taking that integer aspect into consideration. Here, at this moment or I can write this n as n *. So, when I am writing this n*, this n* is any kind of fractional values. This can be any kind of fractional value.

And, there is a procedure we can follow, since we have taking this assumption that n will be exactly integer. Now, why n should be in n integer, so that is every obvious question, and it must come to our mind, that why we need to have n exactly an integer. The reason for n to be an integer is that if n is an integer so, our procurement cycle at stage 1. Can be match with procurement cycle at stage 2.

If n is not an integer what may happen that the procurement cycle at stage 1, will differ in it is timing, then the procurement cycle at this stage 2. What may happen that either this right now, you see this straight line is exactly matching with this straight line like this. So, you see as soon as you receive this stock here, you pass Q2 portion to stage 2, and, this is happening only because n is an integer in this case.

N is an integer in this case only because of that it is happening here, but if n is not an integer, what will happen in that case, if N is not an integer in that case, this line will not match with this line. Either this line will be slightly ahead or it will be slightly backward. That is the problem with this. How, hat is be going to the problem. Because, if n is not an integer, let us say it is 1.5, so you have 1.5 inventory.

Which you are ordering if n is 1.5, so you will order Q1 which is 1.5 times of Q2. And, for that reason if this is how your Q2 cycles are moving, and we have this other pen to show our Q1 cycles. So, here Q1 is 1.5 times of Q2. So, you received Q1 and out of that a portion of Q2 is gone, so, you are left with Q1-Q2. Which is equals to 0.5 Q2.So, you are left with only 0.5 Q2 here, you are going here.

Now, by the time you order next round of inventory, so you will go up to 2 Q2 this level. Then, you will given of that to the installation 2, and therefore you will be either having more than required inventory yet sometime, and sometime you will have less than required inventory. So, either you will fulfil the demand of this by just 0.5Q. Half the requirement of second stage is being made, by the available stock at installation 1.

Or, if you are not willing to do that, then you will have excess inventories at your installation 1. So, because of these problems of mismatch of inventory management at 1 and 2. We always keep n as integer , n has to be integer this sometime we call as relaxation of original problem. A language of mathematics we say that we have done the relaxation of the original problem.

Original problem means by doing the differentiation doing the maximum, minimum, and * can be any fractional value, any continuous value. But, since we require only integer value, so we will see that how do we follow a process to convert these n stars in to a proper integer. So, now let us see that part where we will convert n* into a proper integer, so that this model can be properly used. Now, if the value of n*.

If **if** n* is less than 1, so your n is 1. But, if n* more than 1, in case of n* is more than 1 than how to select the values of how to round off, the meaning is how to round off, the n. So, that we can get the proper scientific answer of doing the integer values. And, for that purpose we take this value. We consider this value, which is the most possible, the nearest highest integer which is less than this which is less than this is the highest possible integer which is less than this.

The meaning is if the value of n* if n* let us say is 2.57 if n* is 2.57, so in that case n* in this bracket becomes 2, and then we also have this type of relation. So you conceive this type of relationship between these three values and with the help of comparison of these three values, you decide whether we are going to have higher integer values or lower integer values. So, with this you will come to know.

We will do the comparison with the help of some numerical data in our next class and with that, we will come to know with that how this model can be used. Now this is the case when we have done separate optimisation, this is the case where we have done this separate optimisation and in this separate optimisation we have only considered one factor that Q1 is dependent on calculation of Q2, and then we determine the values of n.

And, by doing a proper procedure of rounding off of which we will discuss. We will have Q1 equals to Q2 and Q2, and that is and of the separate optimisation separate calculation of inventory values. But you will see we have not taken the benefits of supply chain environment in this particular case. We consider two entities separately, and for any reason there can be a relation of Q1 equals to NQ2 and this by doing this mathematical jugglery we found out this particular process.

Now when I go for simultaneous optimisation I will use the concept of this echelon stock to determine whether I can use this supply chain environment for my inventory management or not. The point I am trying to say the total cost, total variable cost inventory. In case of this inventory is C that is C1+C2.

This is the total cost of supply chain in a separate optimisation, separate calculation of inventory values. Now, for simultaneous optimisation. I will take you slightly different calculation which is embedded in the concept of this echelon stocks. So, now we are going for that simultaneous calculation.

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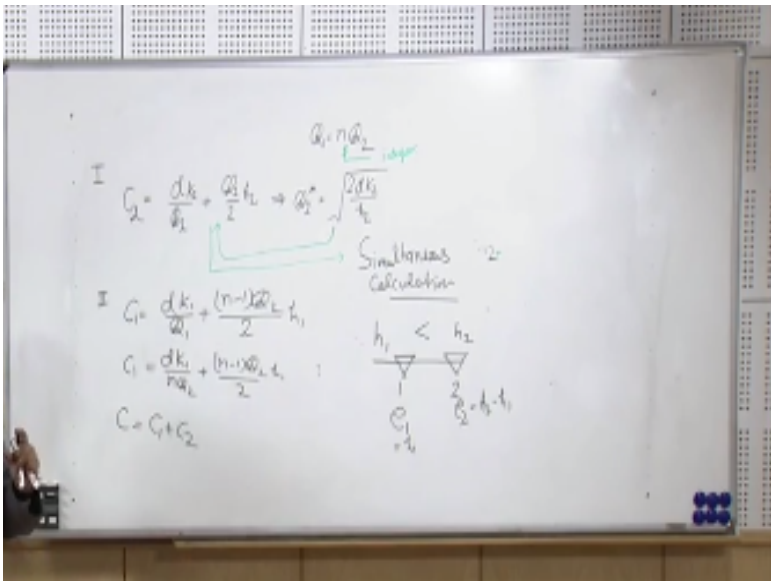

And in this simultaneous calculation, you will see that we will define earlier the holding cost H worse defines for the installations. Now, we will define the holding cost for the echelons, so now the holding cost as we are moving in the supply chain from stage 1 to stage 2. We said the inventory holding cost H1 and H2, so H2 will be more than H1, because you are doing some kind of value addition. Now we are defining echelon holding cost which is represented by E.

Now, when I am talking of H2, so already I had some value addition up to stage 1 and on that value addition up to stage 1. I have already paid holding cost H1, so it looks logical that I should pay at stage 2, the holding cost only for the amount of value addition which is being done from

the stage 1to stage 2, So this concept will help me in giving the concept of echelon holding cost and echelon holding cost E1 since this is the starting stage of my supply chain.

So, even is equals to H1, the holding cost H1 is coming because of all the values which are added at stage1, but the echelon holding cost at installation 2 E2, will be the result of difference of value you are adding from 2 to 1. And therefore E2 will be H2-H1, so this new cost E2 and E1 will help me in getting a more robust model where I will use supply chain environment for the inventory management. So, now when I have this concept of E1 and E2.

I will like to rewrite the total cost of inventory in a single expression, I wrote cost of inventory very well cost of inventory in two separate expressions C2 and C1 and then finally I combined as C equals to C1 + C2. Now, when I have define this two things I am doing the simultaneous calculation simultaneous optimisation, I will like to write total C in a single expression and let us write that total C.

When we write total c, so obviously there will be 2 expressions for the holding cost, and there will be 2 expression s for the ordering cost, and let us see what happens, and how these types of results will give much. Now, when we are doing that, you will see that we have total cost

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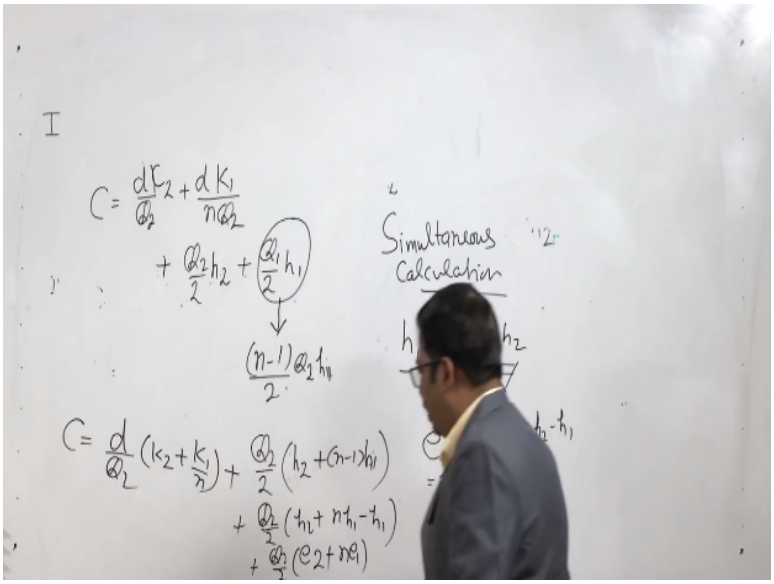

Which, is the result of whatever, you are ordering at installation 2, d by Q2 and K2 +dK1 upon nQ2. So, this relation will be from the ordering cost. + The holding cost. Holding cost now will not be paid for the installation. Rather we will have the holding cost given by the echelon holding concepts. And, for that purpose Q2 by 2 that is e2 + Q1 by 2 e1. Now, Q1 by 2 can be replaced. One it is n Q2, so this can be written as n-1 by 2 Q1, e1.

So, this is the or I can write in a slightly arranged manner, that it becomes d by Q2, from these 2 expressions. K2 + K1 by n + from these 2 steps, you see you can take sorry, this is Q2 here. This Q2 by 2 you can take out, and this remains e2 + n -1 e1. So, this is the expression, and when you see this expression you can very well understand.

That this holding cost right now is being written as e2 and e1 and e1 here. It is actually h2 and h1. And, it will h2, h1. So, it will like this, and then you will have slightly rearrangement of this expression, it will be Q2 by 2. H2 + n h1 – h1. And, this can be rewritten as Q2 by 2, h2 – h1, will account for e2. So, these 2 expressions will give me my e2, and h1 is same as e1.

So this will become n e1, so my expression if I simply summarise will be if I just for the sake for clarity, remove these 2 lines. You can combine the expression like this.

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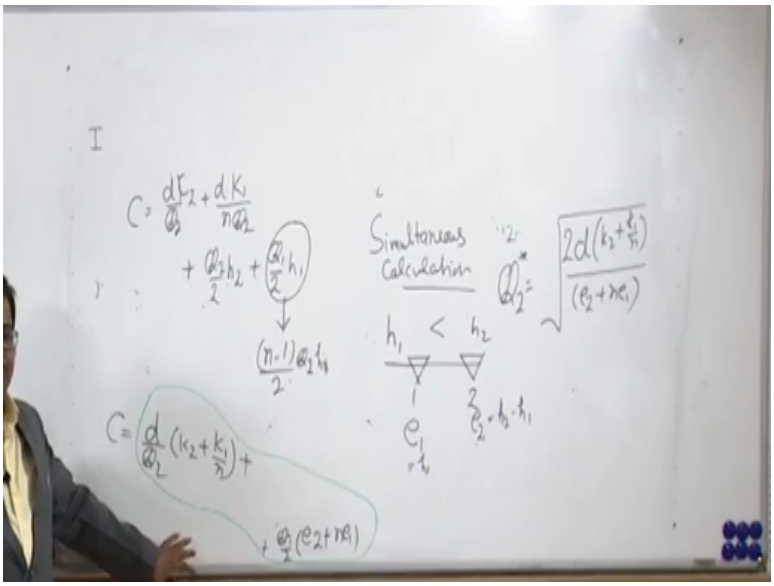

It is d by Q2, K2 + K1 by n + Q2 by 2 e2 + ne1. Now, if I see any of my total cost expressions, and you see this expression. So, this d by Q2 in the number of orders. I am placing again here, this becomes cost of each order. This expression with in bracket K2 + K1 by n. This becomes the cost of placing an order. Similarly, Q2 by 2 is the average inventory level,

And, the expression with in this bracket e2 + n e1. This is actually the holding cost for which I am taking, so now based on our knowledge of EOQ models which we discussed, now if I rearrange these to get the value of Q2. So, the final expression for my Q2 in case of simultaneous calculation will be you all can also practise with me, you all can also write with me, that Q2 will be now under root to d.

Then here comes the cost of ordering that is K2 +K1 by n divided by the holding cost. That holding cost is this e2 + ne1.so, this is the new economic order quantity at Q2 level, and this takes here, because of the echelon concept this takes here, of the supply chain environment. So, we will see the use of this formula with the help of a numerical data, in our next class, and I request you to please do the differentiation of this expression, those who are interested in mathematics.

Do the differentiation of this expression with respect to Q2, and try to get this formula of Q2.though we have used our past one is of EOQ write in this formula, but you can try a fresh, so that you can get this formula. Thank you very much.