

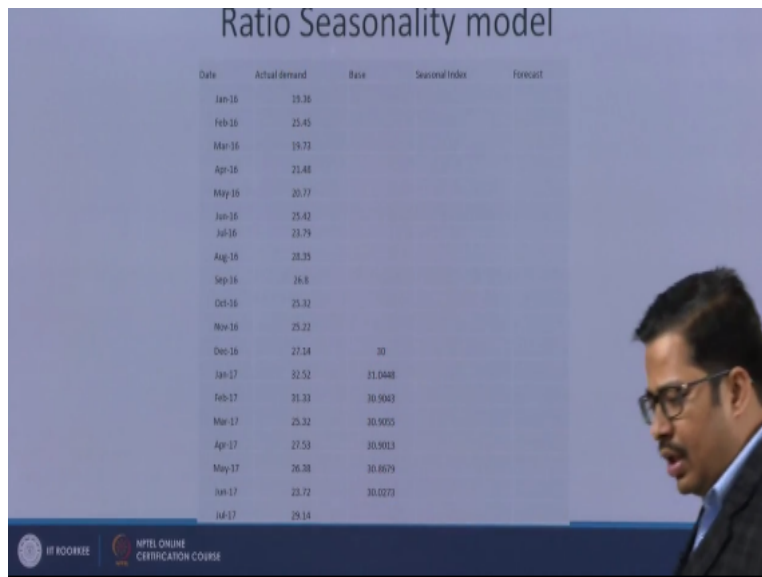
Supply Chain Analytics
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Lecture-13
Measures of Forecasting Errors

Welcome back, we are already discussing various forecasting models for the supply chain decision making. We have discussed so far, different types of time series method in which we have studied the weighted moving average methods, and then we also realised that there are certain problems with those moving average methods. And as a result of that we discussed about the smothering methods and we discussed exponential smothering methods for helping us with the problem of excessive data.

And how we can handle the fluctuations which are happening in the most recent periods. In our earlier classes we discussed about simple exponential method, and now today we are going to discuss a complex exponential smoothing method where we have some kind of seasonality.

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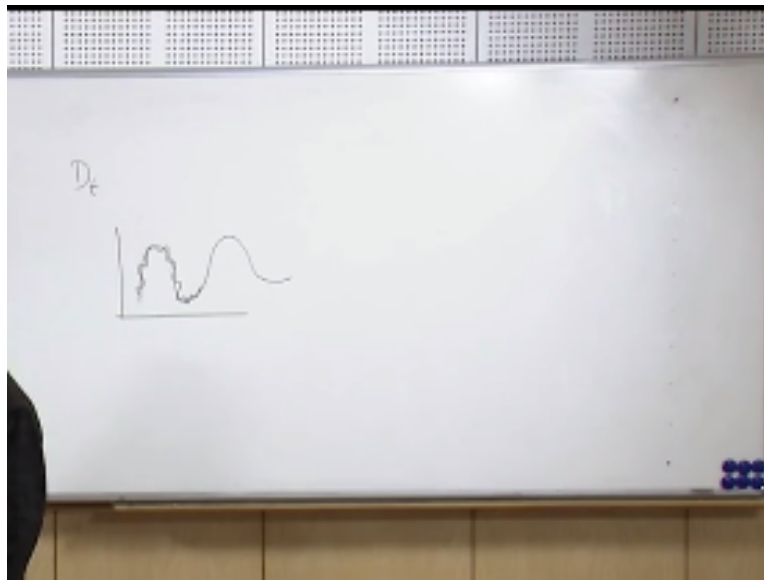
Date	Actual demand	Base	Seasonal Index	Forecast
Jan-16	19.36			
Feb-16	23.45			
Mar-16	19.72			
Apr-16	21.48			
May-16	20.77			
Jun-16	23.42			
Jul-16	23.79			
Aug-16	20.35			
Sep-16	16.8			
Oct-16	23.32			
Nov-16	25.22			
Dec-16	27.14	30		
Jan-17	32.52		31.0448	
Feb-17	31.31		30.9043	
Mar-17	25.32		30.9055	
Apr-17	27.53		30.9013	
May-17	26.38		30.8679	
Jun-17	23.72		30.0271	
Jul-17	28.14			

Under seasonality we are assuming is of ratio seasonality type. Now in our earlier two cases we have discussed the basic exponential smoothing model where there is no trend, no seasonality. Then we also discussed a case of linear trend exponential smoothing model and now we are

moving to the third category of exponential smoothing models, that is the ratio seasonality model.

Now to understand this model we have some hypothetical data available with us, and with the help of this data we will see that how this ratio seasonality model can be operated. Now the point of sales data available with us from January 2016 to July 2017. This is the actual demand, and in our notations this actual demand is represented as D_t .

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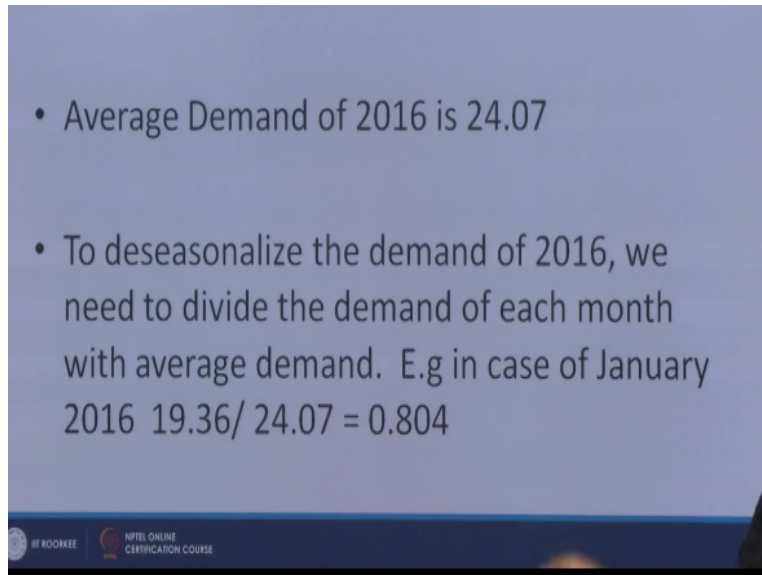


That is the actual demand which is shown in the second column of this table. Now as we have discussed in our last two classes the demand is consider to be fluctuating around a base value. So in case of a seasonal factor also there is a kind of seasonality effect, and actually the demand is fluctuating around these base values. So we will determine this base values which is rather a more smoothened curved.

And actually the demand is represented by this zigzag curve which is the fluctuation around this base value. So the third column of this table represents that this value. Now for the purpose of our understanding we have assumed we have just assumed the base value of December 2016 to be 30 and the further calculation is shown here. And, since we are talking of 2017 only so, there is no need for the base value of this period.

Now, since we already have assumed that is type of demand data has some kind of seasonality. So in this particular case we are aware that seasonality is there. So whenever seasonality is there.

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We need to de seasonalize the demand and for de seasonalisation purpose. We need to divide the demand of individual periods by the average demand of 2016. So if you see the average demand of 2016 is 24.07 that is the 24.07 that we get by summing the demand from January 2016 to December 2016 divided by 12 and you get the average demand as 24.07.

Then for de-seasonalisation purpose we need to divide the demand of individual period by this average demand, for the case of January 2016, we divided 19.36 by this 19.36 which is the actual demand of January, by the average demand of 2016 and you get the seasonal index, and by doing this calculation you have the seasonal index of all the periods 2016.

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Date	Actual demand	Base	Seasonal index	Forecast
Jan-16	19.36		0.804	
Feb-16	25.45		1.057	
Mar-16	19.73		0.819	
Apr-16	21.48		0.892	
May-16	20.77		0.863	
Jun-16	25.42		1.056	
Jul-16	23.79		0.988	
Aug-16	20.35		1.178	
Sep-16	26.8		1.113	
Oct-16	25.32		1.052	
Nov-16	25.22		1.048	
Dec-16	27.14	30	1.128	
Jan-17	32.52	30.9048		
Feb-17	31.33	30.9042		
Mar-17	25.32	30.9005		
Apr-17	27.53	30.9013		
May-17	26.38	30.8879		
Jun-17	33.72	30.9272		
Jul-17	29.34			

For each month of 2016, you have the seasonal index, so the calculation is very simple 19.36 divided by the average demand of 2016, that is 24.07, you get point 804, similarly 25.45 divided by the average demand you got 1.057, and so on for rest of the periods of 2016, that 19.73 divided by average demand of 2016, and you get the seasonal index. So now when you multiply, the actual demand of these periods, by these seasonal index.

You get the de seasonalised demands and this de-seasonalised demand will be used for our calculation to get the forecast, for any period of 2017, but here comes it question, in our mind that here I move, that I am going to discuss a ratio seasonality model. So I know that the demand is seasonal in this case, but if you see this data just 19.36, 25.45, and so on, for entire 2016.

It is very difficult from naked eye, that you can see that this is a seasonal demand data, it is almost impossible for any human being just by observing this data, you can say that this is a seasonal demand data. So, how do you need to have a de-seasonalised demand or what is the need of calculation of signal index, since, I am saying in the particular case you know that we need to calculate seasonal index.

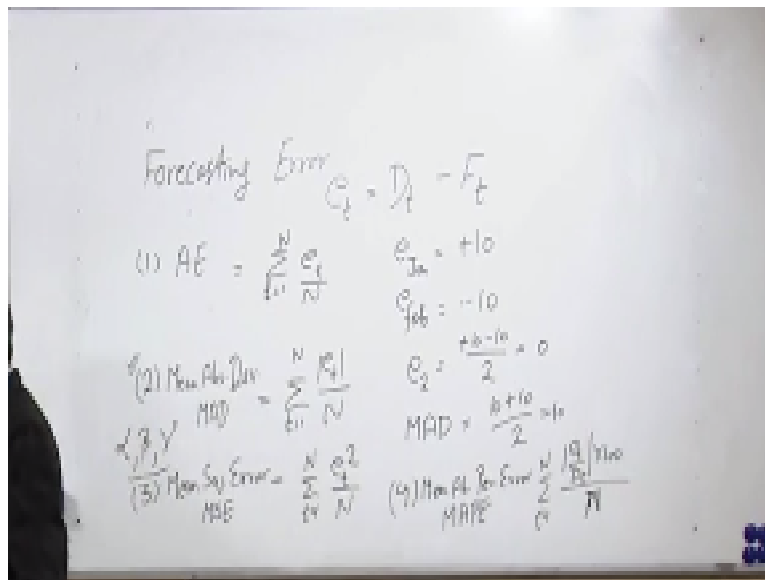
We need to determine de seasonalized demand, but actually in the reality because of our inability just by observing if it is a very clear pattern like if it is 19, 19, 19, 19 and then in the month of May, June, July demand increases to 40, 45, 50 and then again demand comes to 20, 25 like that.

So you can even by naked eye say that demand has kind of seasonality factor it increases to a high level in a particular period and then it decreases.

But here in this particular case, the data 19, 25, 19, 21, 20, 25, 23, 28, 26, 25 the data is so closely linked for each consecutive period. That the seasonality is not visible just by its observation. So, how do we decide, whether we are doing a right kind of thing. And the second thing here I am saying that we are discussing the ratio seasonality model. Now how do I say that this is ratio seasonality model?

Why doubt it is a linear seasonality model. That is also a question, so, for this purpose we must know, whether we are applying the correct model or not. And, for that purpose, we need to know about the forecasting errors. Forecasting errors will help us to determine whether we are using the right model or not. And, for that purpose we will discuss briefly not in much detail about some of the possible errors, which are there in the forecasting and whatever the measures we used for the forecasting errors.

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So in case of forecasting errors we all know that forecast are the tentative values for the future and the actual demand like this and the forecast and the forecast which we are not given. So let us say for the January 2016 if I am forecasting in the December 2015. So forecast can be lesser

21 but the actual demand is 19.36. So, the difference of actual demand and forecasted value is known as the forecasting error.

So, forecasting error let us represents for a particular period as D_t . D_t is the difference of demand and forecast for the same period. So the difference of demand value and the forecasted value for a particular period is known as forecasting error. This is very simple, no complication. But this major alone is of limited use. This definition of forecasting error has limited application. So we have developed certain other majors of forecasting error.

And, now we are going to discuss those other measures of the forecasting errors. The other majors of forecasting error that is the simplest one and the average error is defined as the average of forecasting error like in this particular case, you have forecasting error for January, February, March, April, may, June, July, and let us say if in the month of December, I am calculating what is my average forecasting error. So that is the average of forecasting error for N number of period.

So if I am doing forecast for N number of periods. So this average for errors for these N number of period, is my average forecasting error. It is normally assumed or it should be for a large emphasise, the forecasting error should be 0 or should be very close to 0. But, it is also possible we all know, that for one period there is a problem with average, you all know, but it is possible that for one period.

The error is +10, and for other period the forecasting error is -10. So, if I take the average of these two forecasting errors, so +10 and -10, the average error will be 0. But, in both the cases, in both these periods, the forecasting error is individually very high, but the average error will be 0. What I am trying to say that e-January is +10 and e-February is -10.

If I am taking the average of these two periods, that will be $e\text{-January} + e\text{-February}/2$ and $+10, -10/2$ will be 0. So, by definition it looks that average error is 0, so I am having a very good forecast. But, actually because of different signs of these forecasting errors, I am getting the 0

forecasting error. But, nevertheless in large sample it is assumed that there will be certain + values and there will be some negative values.

And with the positive and negative values you should have a 0 average error. So, that is what we expect. Now as we have discussed that there is a natural limitation of average value, and this not only in the case of forecasting error. The problem of average is always there, that because of the sign you will have this type of issue. So, there is a second measure of forecasting error, which is known as mean absolute deviation. Very popularly known as MAD.

MAD is a very popular measure of forecasting error to eliminate the problem of the sign in the forecasting error. Now, we take the absolute values of error to calculate the mean absolute deviation. So, it remains as like that only, that you are calculating the average. But, the average is of absolute values. You take the absolute values and then you determine the average of that. So, whether it is +10 or -10.

If I am calculating mean absolute deviation it will be absolute values and you get the 10 as mean absolute deviation. So, for same values we just saw that the average error was 0. But, mean absolute deviation is 10. So, mean absolute deviation is a very popular measure of forecasting error. And this mean absolute deviation helps us in determining, whether we have taking it correct forecasting model or not. Whether we are considering ratio seasonality it is appropriate or not.

If my MAD values are higher than my model is not appropriate, some other model may be more suitable in my data. So, with different type of model, I will calculate the MAD value and whichever model will give me minimum values of MAD. I will select that model for my forecasting purpose. And at the same time let me also tell you about smothering constants.

In our last classes we discussed the use of smothering constant. We will use 3 type of smothering constants alpha, beta and gamma, alpha, for smothering the fluctuations of the base, beta, for smothering the fluctuations of trend, and gamma for smothering the fluctuations of the

seasonality components. So, what should be the correct value or what should be the most appropriate values of alpha, beta and gamma.

That also can be checked, that also can be realised using this mean absolute deviation, because the selection or to take a particular value of alpha, beta, gamma, there is no mathematical calculation, there is no formula just by observation, just by practice we take alpha, beta, gamma the thumb rule, we have already discussed in our earlier class, that the smaller values of alpha, beta, gamma. The values of alpha, beta, gamma we discussed varies from 0 to 1.

The smaller values of alpha, beta, gamma these values have more smothering effect. But, in case there is a quantum jump and we want to have a different base or a different level of trend or a different label of seasonality, then you take higher values of alpha, beta, gamma. The smaller values of alpha, beta, gamma gives more smothering effect. So, that is the thumb rule. But, as we know the alpha, beta, gamma normally should be between to 0.05 to 0.30.

Though values can lie from 0 to 1. But in most practical purposes alpha, beta, gamma varies from 0.05 to 0.50. So, what should be appropriate value of alpha, beta, gamma in my case for that also we use, we take the help of mean absolute valuation. And, by putting different values of alpha, beta, gamma you calculate the MAD values and whichever set of value give you minimum values of MAD.

That is your selected value of alpha, beta, gamma for a particular set of data for a particular base of forecasting. Then there is one more method also, one more measure of forecasting error also available to us. That is mean square error and it is abbreviated as MSE-Mean Square Error. In mean square error you take the square of forecasting error and then you take the average of those forecasting square.

The purpose of MSE, because of the square factor the purpose of MSE to penalise the very high forecasting errors. So, in case because of square effect, if there is a higher forecasting error, by a square of that term it becomes even higher. And, it immediately to comes to know that there is a

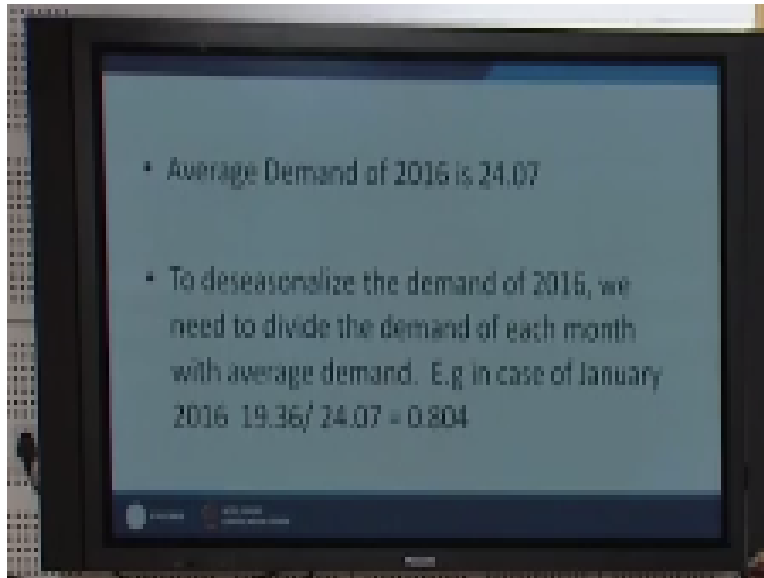
particular set, there is a particular case of data, where the forecasting error is very high. So, this gets you additional information other than MAD.

But, not very popularly used, but can be used combination with MAD. Then one more forecasting measure is there, that is known as mean absolute % error. With stands for MAPE- Mean Absolute Percentage Error. Now, mean absolute percentage error is one more additional measure of forecasting error. In which what we are doing, we see that how much the forecasting error is an upset of our actual demand.

And, that is determined by this method, and then you take the average. So, this is one more measure of forecasting error. But, out of these four the most popular is mean absolute deviation, and with the help of this mean absolute deviation we will determine that which method to use and which method is more suitable, what are the values of alpha, beta, gamma which are suitable for our particular case.

So, now we are not doing all these calculation for this given set of data. We assume that ratio seasonality is best suited for this particular case. But otherwise in all practical cases we need to do the calculation of mean absolute deviation. And, once we have done this of mean absolute deviation. Then only we can determine that this model is suitable or these values are suitable. Now, let us see once we have assumed that ratio seasonality is best suited for this particular case.

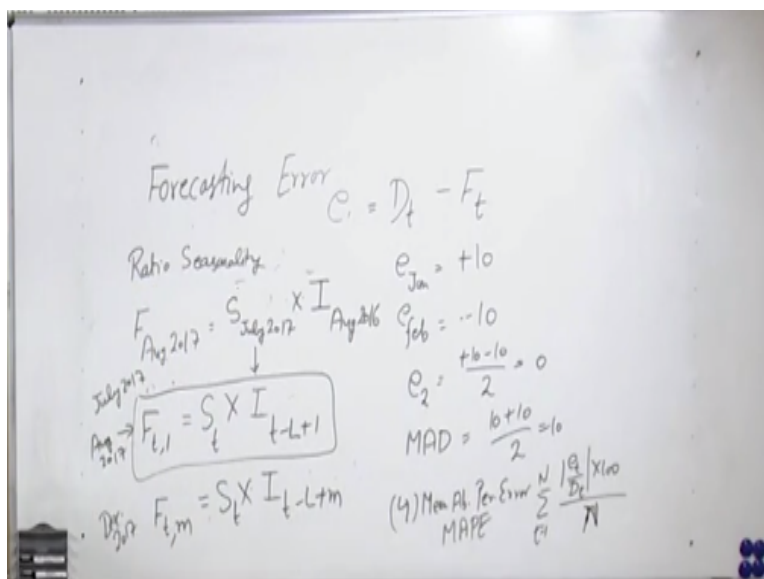
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So, then we go for de-seasonalisation. As we have already discussed by dividing the actual demand, by the average demand of a particular period, and then we determine the seasonal index of different periods. Once we have determined the signal index. And since it is a ratio seasonality model under the case of exponential smoothing, so, we need to see how to determine the forecast, and since it is a ratio seasonality case.

So, now we will go in the reverse direction like I have the actual demand up to July 17, and I want to determine what will be the forecast for august 2017.

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So, now in case of a ratio seasonality model, in case of ratio seasonality when I want to calculate the forecast for August 2017 I will use the most recent base. The most recent base is of July 2017. That is the most recent base, base of July 2017 and I will multiply since, it is a case of ratio seasonality, so actually the base gets me de-seasonalised demand. De-seasonalised forecast you can say. And, then I want the forecast of August 2017.

So, I will multiply this de-seasonalised forecast with the seasonal factor of August 2016, so that is 1.178. So, I will multiply this by the seasonal index of August 2016. Now my job here is to determine the values of base of 2017. So, I have the base of June 2017. I will use these values of 30.0273 to get my updated base for July 2017.

And, already since, I have just started this problem. So, because of the initial calculation I have the seasonal factor for August 2016. But, this seasonal factor for August 2016 will also be used to get the updated seasonal factor for August 2017. And, that August 2017 seasonal factor will be used for calculating or for determining the forecast for August 2018. So, the point which I am trying to make seasonal factor will be used for the previous years. And, the base is used of the current years.

So, if I want to write this equation in a generic term, the equation will become like the F_{t+1} . The period current and one period ahead that signifies F_{t+1} . So, if today I am in July 2017 F_{t+1} means it is August 2017. Then July 2017 is my current base multiply by I August 2016, so August 2016 will be, when I am July 2017 so, if this is a data which is presented on monthly basis.

This data is present on monthly basis. You may have data presented on quarterly basis, you may have data presented on the half yearly basis also. Here is the data presented on the monthly basis. So, you have 12 months in a cycle. So, if I am in July 2017, one cycle is of 12 months. And, cycle is represented by capital L in our case. So, if I am writing $t-L$ here. It means it is July 2017, it becomes July 2016.

And, I want the base value of August 2016. So, +1, this makes me this equation in to the general form . When, I want to determine the forecast for the next month. In case of when today I am in July 2017. If today I am in July 2017, and I want to determine the forecast August 2017. This is how I will write the equation. And, if I want to use this formula.

If today it is July 1. So, this is for August 2017. Now, I want to determine if today again I am July 2017. But, now want to determine the forecast for December 2017. If I want to determine the forecast for December 2017, or for any other month October, November, December etc. So in that case I write that I want to determine the forecast of M period ahead. And, that will be S_t , that is the base of July 2017 multiplied by I_{t-L+M} .

That signifies that I want to determine the forecast of mh period from the current period. So with this we get the most generalised forecast using the current base, and the seasonal index. Which are available to us at the moment? So, we will see now in our next class, that how to calculate this updated base value and how to calculate this updated seasonal index, and the n how to determine the forecast for the any next period in case of ratio seasonality model. Thank you very much.