

Supply Chain Analytics
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Lecture-12
Experimental Smoothing Method of Forecasting

So, welcome back in the previous session we were discussing about time series analysis, and we discussed one of the simplest method of time series that is moving average methods. We discussed 2 methods of moving average, the simple moving average method and the weighted moving average method. Then we will also discuss that limitations of this moving average methods are responsible for their limited application into the real field of demand forecasting.

So, we have improved method which is known as exponential smoothing method, we also discussed in our previous session different types of models incorporating various types of trends, and various types of seasonality component in to the demand data. And, now we will move further to see how we can use the exponential smoothing method for those different types of components in our demand data.

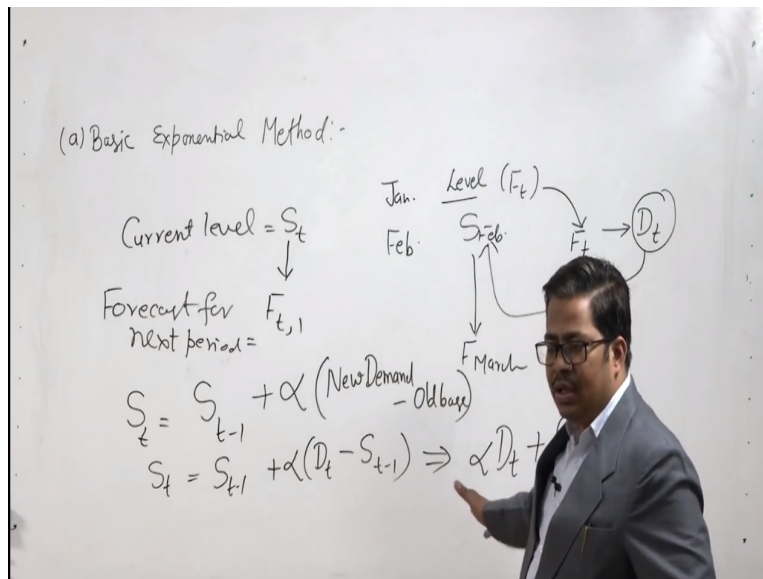
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Quarter	Demand D_t
II, 2014	8000
III, 2014	13000
IV, 2014	23000
I, 2015	34000
II, 2015	10000
III, 2015	18000
IV, 2015	23000
I, 2016	38000
II, 2016	12000
III, 2016	13000
IV, 2016	32000
I, 2017	41000

Forecast demand for the next four quarters.

For an example we have this simple data with us and we will go with the method of exponential smoothing for this simple data.

(Refer Slide Time: 01:35)



Now, to understand the exponential smoothing method, whether we have only horizontal component in our demand data, and there is no other component in the data, what is happening actually we are right now in the month of January, and February is approaching, and in the month of January we have some level or you can say the forecasted demand at on the basis of their forecast, you are doing the forecast for the February.

Now, in the forecast of February there will be some errors and the actual demand will be D_t . Now, on the basis of this actual demand of for February we will determined the level of February, and this level of February is the forecast of March. So, this is how the system will work. Now, it is up to us, that how much uncertainties how much variations of a particular period, we want to include in getting the forecast or the next period.

And therefore, if we see that there are certain changes in the external environment, which are at permanent nature, we will incorporate more deviations in to our forecast and if you see that there are certain temporary changes in the forecast, then we will not incorporate those changes in to the forecasting model. And therefore this exponential smoothing method gives us that flexibility that how much variations you want to include into the forecasting method.

Actually, as we have discussed in last sessions, that we have 2 components of forecast, one is level, and another is the random variation. So, basic exponential method we are only interested in determining the level component, and the random component cannot be determined in viable mathematical model. So, whatever is the level of the current period that is actually the forecast for the next period?

So, in the basic exponential method as I mentioned that the level of February is the forecast of the March. So, if I say if I represent label wise S_t , the current level I am representing S_t , for this current level is actually the forecast for the next period, this is the forecast for the next period. So, now how do we determine the current level that is the issue, because if I pick a point in the current level that is the forecast for the next period.

Now, the current level actually, when I am taking this exponential smoothing method so, what I am trying to do that I will take in to account some of the fluctuations of my the demand, and for that purpose this is the current level and therefore S_{t-1} represents the previous base, previous level, and some of the fluctuations of my current period, I will like to incorporate. And if, this is the new demand - old base.

This becomes my expression to calculate my updated base value or which I can also write as $\alpha \times D_t + (1-\alpha) \times S_{t-1}$ or you can write it as $\alpha D_t + (1-\alpha) S_{t-1}$, so, this becomes the expression for my new base. And now let us see that how do we use this for our this type of data, this data is there, and let us try to use this formula in this data. Here, we have a data of actual demand for the first quarter of 2017 as 41000.

(Refer Slide Time: 07:08)

(a) Basic Exponential Method:-

$$S_{t,2017} = \alpha D_{t,2017} + (1-\alpha) S_{t,2016}$$

$\hookrightarrow F_{t,2017}$

$D_t = 41000$
 $S_{t-1} = 40000$

$\alpha = 1 \Rightarrow \alpha = 0 \Rightarrow S_t = S_{t-1}$
 $\alpha = 1 \Rightarrow S_t = D_t$

$\alpha \Rightarrow 0.5 \text{ to } 30$

$$S_t = S_{t-1} + \alpha (\text{New Demand} - \text{Old base})$$

$$S_t = S_{t-1} + \alpha (D_t - S_{t-1}) \Rightarrow \alpha D_t + (1-\alpha) S_{t-1}$$

So, my current D_t is 41000, the forecast I am assuming, the forecast I am assuming for this period where I was in the fourth quarter of 2016. I did a forecast of 40000 for the first quarter of 2017. So, let us say my S_{t-1} is 40000. So, now If I determined, if I determine the base for

this period, if I determine the base for this period, if I determine the S1 2017, that is actually the forecast for second quarter of 2017.

And, be please into sure that I am not taking consideration, any kind of trans speciality in this data, though this data exhibits some kind of seasonalities, but for the say for example I just considering that seasonality in to this data right now. So, now I am determining S1, 2017 which I is if I go by this expressions is $\alpha \times Dt$, Dt is D of first of 2017+1- $\alpha \times St-1$, that S of fourth of 2016.

So, if I go with this expression I can calculate S1 2017, and S1 2017 is nothing but the forecast for the second period of 2017. So, I get the forecast of second quarter of 2017 using this method. Now here the importance is that what should be the value of this alpha, what should be the value of this alpha, theoretically speaking the value of alpha can lie between 0 to 1, value of alpha can lie between 0 to 1.

But, practically the common values of alpha which we used are from point 05 to point 30, these are the common values of alpha point 05 to point 30, but, it can be zero or it can 1 also. Now the meaning of taking different value of alpha. Let us say if I take alpha equals to point 1, if I take alpha equals to point 1 what it means. If I take alpha equals to point 1 the meaning of this alpha is that I am taking only 10 %, I am taking only 10% of my current demand.

I am taking only 10% of my current demand, and I am discounting 90% deviations, I am discounting 90% deviations and I am 90% of my previous base. So, that is the meaning of alpha equals to point 1, this means that a smaller values of alpha is also means that the smaller values of alpha has more smoothening effect. If you have a smaller value of alpha, it gives to more smoothening effect, and larger values of alpha larger values of alpha should only be taken in that case when you have a shifting base.

When you feel that there is a change which is of permanent nature, and you want to improve the change in your forecasting model, then you should go for higher values of alpha. For an example as we have in discussed in previous session also, that when a new pay commission is coming and purchasing power of people are increasing. So, this is the kind of permanent kind of change, and as a result of permanent changing you have purchasing power.

Your value of alpha met permanently shifted. So, in that case, you can go for higher values of alpha, and when you take lower value of alpha you are having more smothering effect. Now, if you take for an example two extreme cases you take extreme cases when alpha is equals to 0, and alpha equals to 1. So, if you substitute in this equation in this case where alpha equals to 0, so, it means your current base is equals to old base.

The meaning is that you do not want to include any deviation of the current period in to your current base, like the issue of a smog during October and November an NCR area, that is a temporary type of phenomena. So, because of that whatever fluctuations in demand has taken place, you do not want include the fluctuations permanently in to your model and as a result of that you will take the small values of alpha.

And, maybe you can take alpha equals to 0 in some cases, then alpha equals to 1. This will lead alpha equals to 1 will lead to $S_t = D_t$. Now S_t equals to D_t , in this particular case you see, that you have totally shifted your base, you have taken a new base, a new demand is your new base, so, you do not want to take your old base at all into your consideration, so, it is a jumping base type of scenario.

And, maybe in case of pay commissions, may be in case of rehabilitizations, large level of rehabilitation or something of that is all whenever happens. So, you can have every high value of alpha, so, these are the extreme cases, but, normally alpha lies, normally alpha lies between 0.05 to 0.30. So, 0.1, 0.2, 0.25, 0.15 these are very popular values of alpha. And on the basis of that you can do this calculation, and you can get the new base.

For this period of 201, and that will automatically become the forecast for the next period. So, this is our simple method of basic exponential method. Now as we discussed in the case of moving average method for better forecast you need to have long historical data, now that requirement is reduced here, you only need data, you do not require this long data with you. You only require data of just last 2 periods.

And, that current data helps you in getting the forecast, so, you are getting forecast with less data and which is more accurate, which is more adoptive, which is more you can say customised as per the situation whichever is happening in the market. Now once we have

understood the simplest form of exponential smoothing method. Now we go to different form of smoothing method where we will see that how you can customise your model.

To suit the requirement of trend and seasonality component in to the demand data. And, here we are using only single smothering constant alpha, in those cases you may use more than one smothering constant because this smothering constant is only smothering the fluctuations of your level data, but when you have trend. So, you require one smothering constant to smoothen the fluctuations of your base data.

And, one to smoothen the fluctuations of your trend data, when you have seasonality in to your demand data, then you require 1 smothering constant to smoothen the fluctuations of your seasonality component also. So, depending upon the type of the characteristic of your demand data, you will require as many number of a smothering constant. So, now let us move to second of smothering method.

(Refer Slide Time: 16:04)

(a) Basic Exponential Method:-

$$S_{I,2017} = \alpha D_{I,2017} + (1-\alpha)S_{II,2016}$$

$\alpha = 1 \Rightarrow F_{II,2017}$ $\alpha = 0 \Rightarrow S_t = S_{t-1}$
 $S_{t+1} = 40,000$ $S_t = D_t$
to 30

(b) Trend

- Linear
- Ratio

Period	A	B
1	20	20
2	22	24
3	26	29
4	27	34
5	29	39
6	31	44

Where we have the trend also in our data, now trend can also be of two types, one is linear trend or additive trend, and the second is ratio trend or multiplicative trend. These are the two types of trends which are possible. The meaning I show you if I have the historical data with being in that case, you can see this for this is period, this is column A, this is column B, and let me have the data for some past periods.

Now, here I am starting with 20,22,26,27,29,31 like that and in this case we have 20, 24,29, 34, 39, 44. So you will see that in both these cases you have a trend. But, here in the first case

here the demand is increased by two units, then by 4 then by 1, then by 2, by 2, so, you are having a kind of additive some constant figure or a fluctuating figure is added in to the demand of previous median.

So, it is more like a linear trend some almost constant thing is being added. Here demand increase by 4 then 5 then 5 then 5 then again 5. So here the demand is increased in a more ratio type of field that it is multiply to the previous periods. So finally from 20 to 44 it has just gambled. So that it is kind of ratio you are getting over period of time. So it is multiplying effect and here it is additive effect.

So depending upon what type of trend you have you can suit make changes in your model which we are going to discuss. Now going for there into the calculation part of this since we have in our component trend also and already that linear part is very much present. So two soles type of cases.

(Refer Slide Time: 18:53)

(a) Basic Exponential Method:-

$$S_{I,2017} = \alpha D_{I,2017} + (1-\alpha)S_{II,2016}$$

$$F_{II,2017} = S_{I,2017} + T_{I,2017}$$

(b) Trend

- Linear
- Ratio

Period	A	B	T _t
1	20	20	0
2	22	24	4
3	26	29	1
4	27	34	2
5	29	39	2
6	31	44	

$$F_{t,1} = S_t + T_t$$

$$F_{t,m} = S_{t,m}$$

Let us see if I take the first data with us first data with us if I take the first data with us. So you have this trend component initially there is no trend and then you have the trend of +2 +4 you have +1 +2 and you have again +2. So these are the trend data which you have. When we are the developing the forecast in this particular case. So now the F2 107 will be the updated base of the current period that is S1 2017 + the updated trend of current period.

That will make the forecast for the next period. So I need to make updated base for the current period and I have to make the updated trend of the current period and when I add both

this things. I will get the forecast for the next period. So if I generalised this relationship. So I will say that F_{t+1} is nothing but $S_t + T_t$ that is the forecast for the next period is the updated base for the current period and the updated trend for the current period.

Sometime it is possible but in case of trend data I will like to forecast for two three periods ahead from the current period. So in that case if I want to forecast for M period ahead. So in that case this value of trend is consider as constant value and then I will do like this. I am taking a particular case of linear trend that is why I am just adding up these things. If it is a ratio trend then the multiplying effect will come into picture accordingly. Now let us see how do we do that.

(Refer Slide Time: 21:27)

(a) Basic Exponential Method -

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + T_{t-1})$$

$$T_t = \beta (S_t - S_{t-1}) + (1-\beta)T_{t-1}$$

$$F_{t,1} = S_t + T_t$$

$$F_{t,2} = S_t + 2T_t$$

$$F_{t,m} = S_t + mT_t$$

(b) Trend

- Linear
- Ratio

Period	A	B	T _t
1	20	20	0
2	22	24	4
3	26	29	1
4	27	34	2
5	29	39	2
6	31	44	

So first we need to calculate the updated base and then with the help of updated base we will determine the updated trend also. So the updated base S_t we already know how we determine the $\alpha D_{t+1} - \alpha S_t - \alpha T_{t-1}$ that is in the simple exponential method. Now with the trend it will become $\alpha D_{t+1} - \alpha (S_t + T_{t-1})$ that is the previous forecast and that is the current demand.

And now I also need to do the updation in the trend data and for that purpose I need to calculate the T_t and T_t is nothing but the difference of my current base and the previous base. So T_t is $S_t - S_{t-1}$ and this is $\beta + 1 - \beta$. Because I am taking the second sufficient of a smothering of β which is for the trend purpose $1 - \beta$ into T_{t-1} . So this is the updated trend value and then finally the forecast F_{t+1} will be $S_t + T_t$.

So this is how we will do the forecast when trend is available, the calculation of linear trend, the additive trend, is then $S_t - S_{t-1}$ and if it is to be multiple trend, if it is to be ratio trend it will be S_t upon S_{t-1} , please be careful that if it is a ratio trend it will be S_t upon S_{t-1} . And, since I am considering the case of linear trend it is $S_t - S_{t-1}$. And, this is the old trend, then I update the $S_t + D_t$ and I get the forecast for the next period.

And, if I want to use this trend value for getting the forecast for 2, 3, 4 periods ahead then this expression will work, at F_{t+m} , and now this model is ready, and now I can use this model for getting my values for next period. Here as I discuss about alpha the same discussion apply for the beta also, the values of beta also varies between 0 to 1. The popular values of beta are from 0.05 to 0.20, because the fluctuations in trend are not much.

So, we use lesser values of beta and we want to discount maximum fluctuations of trend values and it is very rare, it is very rare that you use very extreme values of beta. So, normally beta values are less than alpha values, but, since both these are smothering constant, so, academically, theoretically their values can vary 0 to 1. So, beta can also 0 to 1, but, the popular values are from point 05 to point 20.

So, now if I apply this equation these two equations on this piece of data, so, you can see that for seventh period, for F_7 , I need to apply $S_6 + T_6$. I need to calculate S_6 and T_6 and with the help of S_6 and T_6 I can directly get the value of F_7 . And, S_6 will come from this equation for S_6 I require S_5 and for T_6 I require T_5 , and when I use these expressions, I can directly get the values of my required forecast for the next period.

Then, in case of ratio the only thing which I just told you, this value will change, this will become S_t upon S_{t-1} , in case of ratio, this calculation will change to S_t in to T_t . And, this calculation will also change to S_{t-1} in to T_{t-1} . Rest of the model will remain as it is. There will not be any change in any other component with this model. So, with this you can handle both this types of trend, you can handle the linear trend, you can handle the ratio trend.

But, now just by seeing nobody can give you the answer whether this A has linear trend, or whether B has this linear trend, you were thinking your mind that how do you can say that here you have 20 to 31, you can have a ratio of one point 5 something like that, here you are

moving from 20 to 44 you have ratio of somewhere around 2 point 2. So, why cannot we apply a ratio model in this case or why cannot we apply a linear model in this second case.

Because each time 24 to 29, 29 to 34, 34 to 39, 39 to 44, demand is increasing by very constant value 5. So, why cannot we apply a linear model in second case and why cannot we apply a ratio model in the first case, practically speaking, theoretically speaking I do not have any answer for that, only my model will tell the answer only my model will tell the answer, model means whether I am calculating using $S_t - S_{t-1}$ or S_t upon S_{t-1} .

And, after determining the forecast I will calculate the forecasting errors and whichever model, whichever model will give me minimum forecasting errors, whichever model will give me minimum forecasting error that is suitable model for my particular case. So, now our next topic of discussion is the forecasting error. So, that we can understand the meaning of selection of different type of models. Without understanding the forecasting model it is almost impossible to select the right kind of model.

Because, once you select the model the appropriateness of that model will also depend on proper selection of values of alpha and beta. So, all these things are very much you can say in a family, the kind of model the selection of alpha and beta and these models should produce minimum forecasting error. So, in our next class we will discuss more about forecasting error, and then we will take a case of third type of component that is the seasonal component in our demand data.

And, how to handle that seasonal component in the data so, that case will also take, and this data will be useful in that case also, the because, here you can see we have the seasonality in our demand data present, so, how to handle the seasonal component in the demand data, and, then we can move the most complex case where we have seasonality as well as trend, both these things in our demand data. So, how to handle such type of cases, that we will see in our next lecture, thank you very much.