

## Foundations of Accounting & Finance

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Week - 12

Lecture – 54

### Risk and Return: Portfolio

#### Covariance and Correlation

Variance and standard deviation quantify only the variability of individual stocks. In contrast, covariance and correlation assess the connection between the returns of one stock and those of another. They gauge the relationship between these returns, providing insight into their interdependence. This is the essence of covariance and correlation analysis. The covariance and correlation can be calculated using the following equations.

$$\sigma_{AB} = \text{Cov}(R_A, R_B) = \text{Expected value of } [(R_A - \bar{R}_A) \times (R_B - \bar{R}_B)]$$

$$\rho_{AB} = \text{Corr}(R_A, R_B) = \frac{\text{Cov}(R_A, R_B)}{\sigma_A \times \sigma_B}$$

#### Interpretation of Covariance

Sign of Covariance	Relationship	Interpretation
Positive	Positive dependency Positive relationship between two returns	In any state where both returns are above their averages In any state where both terms are below their average
Negative	Negative dependency Negative relationship between the two returns	In any state where one return is above its average, and the other return is below its average
Zero	No relation ship	There will be no tendency for the deviations to be positive or negative together

Interpreting covariance involves understanding the relationship between two stocks' returns. If covariance is positive, it indicates a positive dependency between the two stocks. This means that when one stock's return is above its average, the other stock's return tends to be above its average

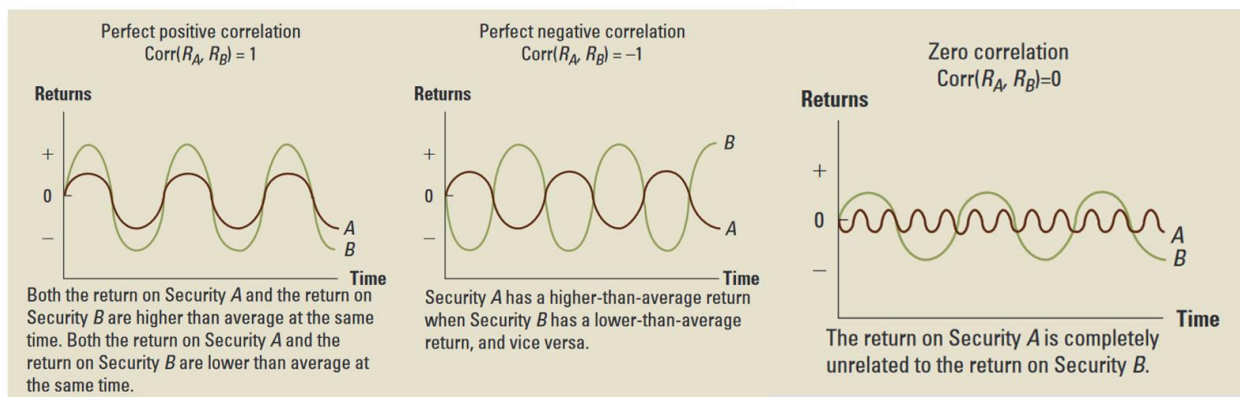
as well, and vice versa. Essentially, there is a mutual dependency between the two stocks, suggesting a positive relationship.

Conversely, if covariance is negative, it signifies a negative dependency between the stocks. In this scenario, when one stock's return is above its average, the other stock's return tends to be below its average, and vice versa. This negative dependency implies an inverse relationship between the stocks.

A covariance of zero suggests no relationship between the stocks' returns. There is neither a positive nor a negative dependency between them.

In summary, interpreting covariance involves recognizing the extent of dependency between the returns of two stocks. Positive covariance indicates a positive relationship, negative covariance indicates a negative relationship, and zero covariance suggests no relationship between the stocks' returns.

### Interpretation of Correlation



Correlation provides insight into the relationship between the returns of two stocks, indicating whether they are positively correlated, negatively correlated, or uncorrelated.

As indicated in the figure above, positive correlation suggests that the variables, in this case, the stock returns, move in the same direction. When one stock's return increases, the other stock's return also tends to increase, and vice versa. This positive correlation is represented by a correlation coefficient close to +1. Graphically, it is depicted as overlapping or parallel movements of the stock returns.

Conversely, negative correlation indicates that the variables move in opposite directions. When one stock's return increases, the other stock's return decreases, and vice versa. This negative correlation is represented by a correlation coefficient close to -1. Graphically, it is illustrated as opposite movements of the stock returns.

Zero correlation implies that there is no relationship between the stock returns. The returns of one stock are unrelated to the returns of the other stock. This is represented by a correlation coefficient of 0. Graphically, it is shown as independent movements of the stock returns.

In summary, correlation analysis helps determine the extent to which the returns of two stocks move together. Positive correlation signifies movement in the same direction, negative correlation indicates movement in opposite directions, and zero correlation implies no relationship between the stock returns.

### *Example*

#### **Correlation between General Motors and Ford**



#### **Correlation between General Motors and IBM**



Here I have analyzed some correlations. The correlation between General Motors and Ford is expected to be high because they operate in the same sector. Conversely, the correlation between General Motors and IBM is expected to be low due to their presence in different sectors. They may not move in tandem because of the distinct nature of their businesses. Therefore, we are examining co-movement. The correlation between General Motors and Ford is significantly higher than the correlation between General Motors and IBM.

### **The Return and Risk for Portfolios**

So, moving forward, let us get into the return and risk aspects of portfolios.

When it comes to portfolios, we are no longer focusing on individual stocks but rather on a combination of them. Investors often have varying estimates of the returns, deviations, standard deviations, and correlations among securities. This diversity reflects the expectations of investors.

Now, how do investors select a combination of securities for their portfolio? They aim to construct a portfolio with a high expected return and a low standard deviation in returns. Essentially, they seek a portfolio where the combined expected return of all stocks is high while maintaining a lower overall standard deviation. This balance is crucial when assessing the expected return of a portfolio.

## The Expected Return on a Portfolio

So, what exactly is the expected return of a portfolio? It is simply the weighted average of the expected returns of the individual securities within the portfolio. This weighted average takes into account the percentage of funds allocated to each instrument in the portfolio.

$$\text{Expected return on portfolio} = X_A \bar{R}_A + X_B \bar{R}_B = \bar{R}_P$$

## Variance and Standard Deviation of a Portfolio

$$\text{Var}(\text{portfolio}) = X_A^2 \sigma_A^2 + 2X_A X_B \sigma_{A,B} + X_B^2 \sigma_B^2$$

$$\sigma_P = \text{SD}(\text{portfolio}) = \sqrt{\text{Var}(\text{portfolio})}$$

- First term- Variance of Security A (variability in individual security)
- Second term- Covariance between the two securities (relationship between the two securities)
- Third term - Variance of Security B (variability in individual security)

Sign of Covariance	Relationship	Impact on Variance
Positive	Positive relationship between two returns (Both securities rise and fall together)	Increases the variance of the entire portfolio
Negative	Negative relationship between the two returns (one security to go up when the other goes down, or vice versa)	Decreases the variance of the entire portfolio (Hedge)

A positive relationship between the returns of two securities increases the variance of the portfolio. Conversely, a negative relationship between the two securities decreases the variance. This decrease occurs because when one security moves up, the other moves down, offsetting each other's impact. However, when both securities move in the same direction (either up or down), as in the case of positive correlation, the variance of the portfolio is expected to be high.

## Example

Construct a portfolio of Supertech and Slowpoke with 60% investment in the portfolio from Supertech and 40% from Slowpoke based on the calculation we did in the previous sessions. What is the expected return of the portfolio?

Relevant Data from Example of Supertech and Slowpoke		
Item	Symbol	Value
Expected return on Supertech	$\bar{R}_{\text{Super}}$	.175 = 17.5%
Expected return on Slowpoke	$\bar{R}_{\text{Slow}}$	.055 = 5.5%
Variance of Supertech	$\sigma_{\text{Super}}^2$	.066875
Variance of Slowpoke	$\sigma_{\text{Slow}}^2$	.013225
Standard deviation of Supertech	$\sigma_{\text{Super}}$	.2586 = 25.86%
Standard deviation of Slowpoke	$\sigma_{\text{Slow}}$	.1150 = 11.50%
Covariance between Supertech and Slowpoke	$\sigma_{\text{Super,Slow}}$	-.004875
Correlation between Supertech and Slowpoke	$\rho_{\text{Super,Slow}}$	-.1639

### *The Expected Return on a Portfolio*

To calculate the expected return of the portfolio, we will convert the percentages to decimals: 60% becomes 0.60 and 40% becomes 0.40. Then, we will multiply each investment proportion by its respective expected return:  $0.60 * 17.5\%$  for Supertech and  $0.40 * 5.5\%$  for Slowpoke. Adding these together gives us the portfolio return, which is approximately 12.7%.

Security	Expected Return	Amount in Portfolio	Expected return on Portfolio
Supertech	17.50%	0.60	10.50%
Slowpoke	5.50%	0.40	2.20%
	Total	1	<b>12.70%</b>

### *Variance and Standard Deviation of a Portfolio*

To calculate the variance of the portfolio, we use a formula involving the probabilities, variances, and covariance of the individual securities. The first term is the square of the probability of the first investment (0.6 in this case) multiplied by the variance of Supertech. The second term involves twice the product of the probabilities of both investments (0.6 and 0.4) multiplied by the covariance between Supertech and Slowpoke. The third term is the square of the probability of the second investment (0.4) multiplied by the variance of Slowpoke.

Adding these terms together gives us the portfolio's variance, which is approximately 0.02. To find the standard deviation, we take the square root of the variance, resulting in a standard deviation of about 15.44%. The detailed calculation is provided below:

Expected return on Supertech	17.50%
Expected return on Slowpoke	5.50%
Variance of Supertech	0.06688
Variance of Slowpoke	0.01323
Standard deviation of Supertech	25.86%
Standard deviation of Slowpoke	11.50%
Covariance between Supertech and Slowpoke	-0.004875
Correlation between Supertech and Slowpoke	-0.1639
Investment in Supertech	0.6
Investment in Slowpoke	0.4
	1
<b>Variance of Portfolio</b>	
First term	0.02408
Second term	-0.002340
Third Term	0.00212
<b>Variance of Portfolio</b>	<b>0.02</b>
<b>Standard Deviation of Portfolio</b>	<b>15.44%</b>

### The Diversification Effect

Diversification is a key strategy for reducing risk in an investment portfolio. By spreading investments across different assets, investors can potentially lower their overall risk. In our case, the standard deviation of the portfolio, representing its risk, is 15.44%. This diversification effect is evident because the two companies, Supertech and Slowpoke, are slightly negatively correlated. When one company's return is above average, the other tends to be below average, and vice versa. This negative correlation helps balance out the portfolio's performance.

However, if both securities were positively correlated, the benefits of diversification would be limited. In fact, if they were perfectly positively correlated (correlation coefficient of +1), diversification would offer no benefit at all. This is because both securities would move in the same direction, amplifying risk rather than mitigating it.

As long as the correlation between securities is less than 1, the standard deviation of a portfolio of two securities will be less than the weighted average of the standard deviations of the individual securities. This underscores the importance of considering correlation when building a diversified portfolio.

### What is diversification?

Diversification refers to the practice of spreading investments across different assets or securities within a portfolio. By doing so, investors aim to reduce the overall risk of their portfolio because individual assets may react differently to the same economic events or market conditions.

For example, if one asset in the portfolio performs poorly due to a specific event, another asset may perform well, thus offsetting potential losses. This variation in the performance of individual stocks helps to reduce the overall volatility or risk of the portfolio.

Overall, diversification is considered a fundamental strategy for managing risk and enhancing the potential for consistent returns in an investment portfolio.

### **Risk: Systematic and Unsystematic**

There are two different kinds of risk; systematic and unsystematic risk.

#### **Systematic risk**

Systematic risk, also known as market risk, pertains to the broader market environment. It encompasses factors such as GNP, interest rates, and inflation, which affect a wide array of assets to varying degrees. This type of risk impacts stocks within the same sector in a uniform manner. It is important to note that systematic risk affects stocks in the same direction consistently and cannot be diversified.

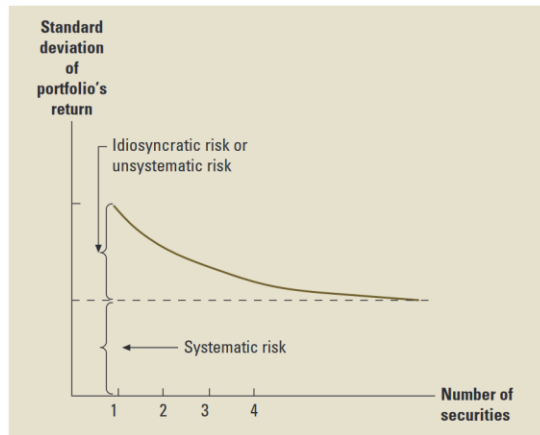
#### **Unsystematic risk**

Unsystematic risk is specific to individual stocks or entities and is not related to broader market factors. For instance, issues within a particular company, such as internal conflicts among promoters as in the case of Raymond's, recently (2024) may affect only that company's stock and not the entire industry. This type of risk, exemplified by the case of Raymond's, is termed unsystematic risk.

To mitigate unsystematic risk, investors can diversify their portfolios by investing in a variety of stocks across different sectors. By spreading investments across multiple stocks, the impact of adverse events specific to one company can be minimized. While unsystematic risk can be reduced through diversification, it cannot be completely eliminated. In contrast, systematic risk, which is influenced by market-wide factors, cannot be mitigated through diversification alone.

#### **The essence of diversification**

The essence of diversification lies in its ability to mitigate unsystematic risk, which is specific to individual stocks or entities. When combining stocks in a portfolio, the effects of positive and negative events on one stock may offset those on another. By diversifying investments across multiple stocks, investors can spread their risk and reduce the impact of adverse events specific to any single company. In essence, diversification minimizes unsystematic risk and helps protect investors from the vulnerabilities associated with individual stocks, as exemplified by the case of Raymond's.



At the minimum level of risk, which you can see in the above figure, systematic risk prevails uniformly across all firms. Beyond that lies unsystematic risk, which varies. For instance, investing solely in one stock increases unsystematic risk. However, when investing the same amount in a basket of stocks, unsystematic risk diminishes significantly, nearly to the point of elimination. Though it may occasionally remain slightly above zero, it is greatly reduced. Thus, in this scenario, I am left only with systematic risk, as depicted here.