Foundations of Accounting & Finance

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Week - 11

Lecture – 52

Risk and Return - Part II

Holding Period Returns

The holding period return is the return that an investor would get when holding an investment over a period of T years, where the return during each year i is given as Ri.

$$HPR = (1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_n) - 1$$

For example, consider an investment in a national savings certificate. After making the investment, one must wait for 6 or 7 years, depending on the scheme, before receiving a lump sum payment. Therefore, the typical holding period return is the return realized over the entire holding period of 6 or 7 years. This concept allows investors to assess the overall return on their investment over a specific time frame.

Example

If the returns were 11 percent, -5 percent, and 9 percent in a three-year period, an investment of ₹1 at the beginning of the period would be worth

Solution:

=(1 + R1) *(1 + R2) * (1 + R3)

= (₹ 1 + .11) * (₹ 1 + (-.05)) * (₹ 1 + .09)

= ₹ 1.11 * ₹ .95 * ₹ 1.09

= ₹ 1.15

Holding Period Return = ₹ 1.15 - ₹ 1.00 = .15 or 15%

** Reinvesting the first-year dividend in the stock market for two more years and reinvesting the second-year dividend for the final year*

Input area:

Year	Returns
1	11.00%
2	-5.00%
3	9.00%
4	25.83%
5	-9.17%

Output area:

Holding period return	15%

Another example

Consider investing in an instrument for 5 years, where you make the investment today and receive the returns only at the end of a 5-year period. Let us assume the rate of return is 6 percent annually.

Here is how it works:

- If you invest $100, at the end of the first year, it would earn 6 percent interest, making the capital for the second year $106.
- In the second year, the $106 would earn 6 percent interest, resulting in a capital of $112.36 for the third year.
- This process continues, with the capital getting compounded each year, earning interest on interest.

This cumulative return over the entire 5-year holding period is what we refer to as the holding period return. It illustrates how an investment grows over time due to compound interest, providing a simple example of how holding period returns work.

Example

A stock has had returns of 14.38 percent, 8.43 percent, 11.97 percent, 25.83 percent, and −9.17 percent over the past five years, respectively. What was the holding period return for the stock?

Solution

Let us calculate the holding period return for this stock. Using the formula provided, we will multiply the returns for each year and subtract 1.

Holding Period Return = (1 + 0.1438) * (1 + 0.0843) * (1 + 0.1197) * (1 + 0.2583) * (1 - 0.0917) - 1

After performing the calculations, we find that the holding period return is approximately 0.5871, or 58.71 percent.

It is important to note that this is not an annualized return; rather, it represents the total return over the entire 5-year period. This figure indicates the overall performance of the investment during the holding period.

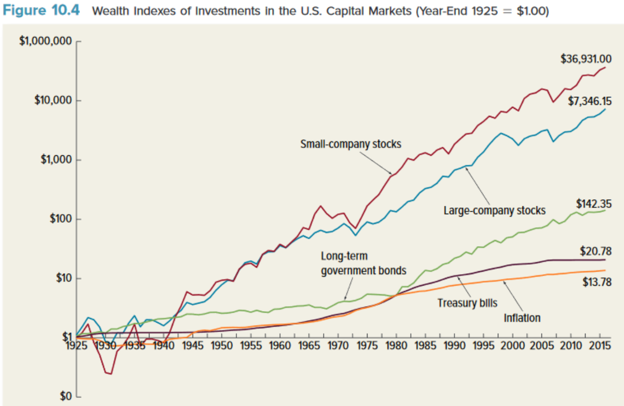
Input area:		
	Year	Returns
	1	14.38%
	2	8.43%
	3	11.97%
	4	25.83%
	5	-9.17%
Output area:		
Holding period return		58.71%

$HPR = (1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_n) - 1$

Historical Returns

Let us focus on historical returns, using an example sourced from the textbook. This data sheds light on the performance of the US market from 1926 to 2015. Each line in the graph represents a different asset class: red for small company stocks, blue for long-term government bonds, green for treasury bills, and orange for inflation.

Figure 10.4 Wealth Indexes of Investments in the U.S. Capital Markets (Year-End 1925 = $1.00)



SOURCE: *Morningstar,* 2018

From this data, we gain insights into the returns achieved over the decades. It is noteworthy that the year-on-year returns have fluctuated significantly, showcasing a mix of positive and negative returns, as well as periods of high and low returns. This volatility underscores the dynamic nature of the US market throughout this extensive time frame.

Year	Large-Company Stocks
1926	11.14%
1927	37.13
1928	43.31
1929	−8.91
1930	−25.26
1931	−43.86
1932	−8.85
1933	52.88
1934	−2.34
1935	47.22
1936	32.80
1937	−35.26
1938	33.20
1939	−.91
1940	−10.08
1941	−11.77
1942	21.07
1943	25.76
1944	19.69
1945	36.46%
1946	−8.18
1947	5.24
1948	5.10
1949	18.06
1950	30.58
1951	24.55
1952	18.50
1953	−1.10
1954	52.40
1955	31.43
1956	6.63
1957	−10.85
1958	43.34
1959	11.90
1960	.48
1961	26.81
1962	−8.78
1963	22.69
1964	16.36
1965	12.36
1966	−10.10
1967	23.94
1968	11.00
1969	−8.47
1970	3.94
1971	14.30
1972	18.99
1973	−14.69
1974	−26.47
1975	37.23
1976	23.93
1977	−7.16
1978	6.57
1979	18.61
1980	32.50
1981	−4.92
1982	21.55
1983	22.56
1984	6.27
1985	31.73
1986	18.67
1987	5.25
1988	16.61
1989	31.69
1990	−3.10%
1991	30.46
1992	7.62
1993	10.08
1994	1.32
1995	37.58
1996	22.96
1997	33.36
1998	28.58
1999	21.04
2000	−9.10
2001	−11.89
2002	−22.10
2003	28.68
2004	10.88
2005	4.91
2006	15.79
2007	5.49
2008	−37.00
2009	26.46
2010	15.06
2011	2.11
2012	16.00
2013	32.39
2014	13.7

You can calculate holding period returns for any combination of years

Now, we can calculate the holding period return for any combination of years from the above table.

Holding period return during dotcom bubble and financial crisis

Let us examine two notable periods in financial history: the dotcom bubble from 2000 to 2003 and the financial crisis from 2007 to 2008.

During the dotcom bubble (2000-2003), the returns were as follows: -9.1% in 2000, -11.89% in 2001, -22% in 2002, and +28% in 2003. Calculating the average return for this period, we find it to be -3.54%, indicating an overall negative return.

Shifting our focus to the financial crisis (2007-2008), the returns were: 5.49% in 2007, -37% in 2008 (a significant downturn), 26.46% in 2009, and 15.06% in 2010. Despite the positive returns in 2007, 2009, and 2010, the drastic negative return of -37% in 2008 heavily impacted the overall return, resulting in a negative return of -3.3% for the entire period.

	Year	Dot com bubble	Year	Financial Crisis
	2000	0.0910	2007	0.0549
	2001	(0.1180)	2008	(0.3700)
	2002	(0.2210)	2009	0.2646
	2003	0.2868	2010	0.1506
Holding Period Returrn		-3.54%		-3.30%

Return Statistics

Analyzing the history of stock market returns can be complex and unpredictable, often lacking clear patterns. However, when seeking a single measure to summarize past annual returns, the average return emerges as the most reliable estimate of the return that an investor could have realized in a specific period or year.

Average Return

$$\text{Average Return} = \frac{\text{Sum of Returns}}{\text{Number of Returns}}$$

The average return serves as a single measure to assess the overall performance of an investment over a specific period. It represents the typical return that an investor could have realized, capturing both the high and low ends of the spectrum.

To calculate the average return, simply sum up all the returns observed and divide by the total number of returns. This straightforward calculation provides a clear indication of the investment's performance over the given period.

Example

Suppose the returns on common stock from 1926 to 1929 are 0.1370, 0.3580, 0.4514, and −0.0888, respectively. The average, or mean return over these four years is:

Solution:

Using a simple average calculation, we add up the returns and divide by the total number of returns. In this case, the average return is approximately 21.44 percent. This represents the typical return over the specified period.

	Year	X
	1	0.1370
	2	0.3580
	3	0.4514
	4	-0.0888
Average Return		21.44%

Average Stock Returns and Risk- Free Security

Average stock returns of large stock around 12.1 percent, provide a benchmark for comparison with other securities like T-bills. However, it is essential to recognize that as risk levels vary, so do returns. Comparing returns across different types of securities, such as long-term government bonds versus small company stocks, illustrates these points clearly. The historical returns data showcases these differences, with small stocks yielding around 16.7 percent compared to the 12.1 percent of large company stocks.

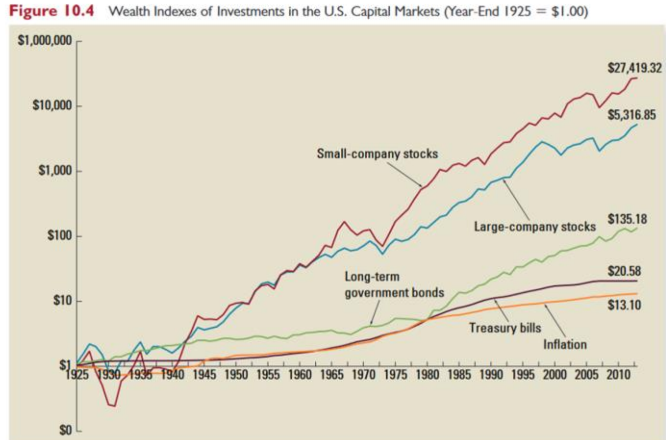
Series	Arithmetic Mean (%)
Small-company stocks*	16.7
Large-company stocks	12.1
Long-term corporate bonds	6.4
Long-term government bonds	6.1
Intermediate-term government bonds	5.4
U.S. Treasury bills	3.5
Inflation	3.0

One crucial concept in this analysis is the idea of excess return, which measures the additional return gained from investing in stocks compared to risk-free securities like T-bills. For instance, if T-bills offer a return of 3.5 percent and large company stocks yield 12.1 percent, the excess return is 8.6 percent. This excess return reflects the additional compensation investors receive for taking on the added risk associated with stocks.

The risk-return relationship is fundamental in finance, highlighting the trade-off between expected return and risk. While returns are not guaranteed, investors expect to be rewarded for bearing additional risk. Variability in returns, captured by metrics like variance and standard deviation, further illuminates the intricacies of this relationship.

Variation in the returns

Figure 10.4 Wealth Indexes of Investments in the U.S. Capital Markets (Year-End 1925 = $1.00)



Redrawn from *Stocks, Bonds, Bills and Inflation: 2015 Yearbook,*™ annual updates to the work by Roger G. Ibbotson and Rex A. Sinquefield (Chicago: Morningstar).

Variation in returns is a critical aspect to consider when assessing risk in investments. As illustrated in the figure above, common stocks often exhibit fluctuations, with returns fluctuating between positive and negative values over time. On the other hand, T-bills typically show a stable, positive return pattern without significant negative fluctuations.

Understanding the extent of variation from the average return is crucial as it provides insights into the level of risk associated with an investment. High variability indicates a higher level of risk, suggesting that the investment's returns can deviate significantly from the average. In such cases, there is a higher probability of both substantial gains and losses. Conversely, investments with low variability from the mean pose lower risk, as the likelihood of experiencing significant losses is reduced.

By analysing the variability in returns, investors can gauge the potential risks involved and make informed decisions about their investments. It is essential to consider both the potential for gains and the possibility of losses when evaluating investment opportunities.

Risk Statistics

Risk statistics, such as variance and standard deviation, play a crucial role in quantifying uncertainty and measuring variability in investment returns. Variance provides insight into the extent of variation in returns, while standard deviation indicates the degree to which returns deviate from the mean.

Let us discuss these measures with a couple of examples to illustrate their significance.

Variance and Standard Deviation

$$\text{Var} = \frac{1}{T-1}[(R_1 - \overline{R})^2 + (R_2 - \overline{R})^2 + (R_3 - \overline{R})^2 + (R_4 - \overline{R})^2]$$

Variance and standard deviation are essential measures of dispersion, indicating the extent of variation from the mean within a dataset. If we have a set of values, variance quantifies how much these values deviate from the mean, taking into account both positive and negative deviations. The standard deviation, which is the square root of the variance, provides a more interpretable measure of dispersion by expressing it in the same units as the original data.

Example

Suppose the returns on common stocks are (in decimals) 0.1370, 0.3580, 0.4514, and 2.0888, respectively, calculate the variance and standard deviation.

Solution:

Let us calculate the variance and standard deviation for the given returns on common stocks.

First, we calculate the average (mean) return, which is approximately 0.2144 (or 21.44%).

Next, we compute the squared deviations of each return from the mean. Squared deviation is the difference between each return and the mean, squared. For example:

- For the first return: (0.1370 - 0.2144)^2 = 0.00593536
- For the second return: (0.3580 - 0.2144)^2 = 0.02058496
- For the third return: (0.4514 - 0.2144)^2 = 0.05617
- For the fourth return: (-0.0888 - 0.2144)^2 = 0.09193

Then, we sum up these squared deviations, which results in approximately 0.17471

The variance is calculated as the sum of squared deviations divided by the number of observations minus 1 (n - 1). Since we have 4 observations, the variance is 0.17471/ (4 - 1) = 5.82

Finally, the standard deviation is the square root of the variance. So, the standard deviation is approximately √5.82≈ 24.13 %.

X	Actual Return	Average Return	Deviation	Squared Deviation
1	0.1370	0.21	(0.08)	0.00599
2	0.3580	0.21	0.14	0.02062
3	0.4514	0.21	0.24	0.05617
4	-0.0888	0.21	(0.30)	0.09193
Total				0.17471
Average return	21.44%			
Variance	5.82%			
Standard Deviation	24.13%			

Example

Using the following returns, calculate the average returns, the variances, and the standard deviations for X and Y

	Returns	
Year	X	Y
1	9%	12%
2	21	27
3	−27	−32
4	15	14
5	23	36

Solution:

Let us calculate the average returns, variances, and standard deviations for assets X and Y.

- First, we calculate the average return, which is the sum of all returns divided by the number of observations. The average return for asset X is 8.2% for Y is 11.40%
- Next, we calculate the deviations of each return from the mean.
- Then, we square each deviation to get the squared deviations.
- After that, we sum up all the squared deviations.
- The variance is calculated by dividing the sum of squared deviations by the number of observations minus 1 (n - 1).
- Finally, the standard deviation is the square root of the variance.

By following these steps, we can determine the average returns, variances, and standard deviations for both assets X and Y.

X	Actual Return	Average Return	Deviation	Squared Deviation
1	9%		0.008	0.000
2	21%		0.128	0.016
3	-27%		(0.352)	0.124
4	15%		0.068	0.005
5	23%		0.148	0.022
Total				0.167
Average return	8.20%			
Variance	0.041720			
Standard Deviation	20.43%			

Y	Actual Return	Average Return	Deviation	Squared Deviation
1	12%		0.006	0.00004
2	27%		0.156	0.02434
3	-32%		(0.434)	0.18836
4	14%		0.026	0.00068
5	36%		0.246	0.06052
Total				0.27392
Average return	11.40%			
Variance	0.068480			
Standard Deviation	26.17%			

Example

You have observed the following returns on SkyNet Data Corporation's stock over the past five years: 21 percent, 17 percent, 26 percent, 27 percent, and 4 percent

a. What was the arithmetic average return on the company's stock over this five-year period?

b. What was the variance of the company's returns over this period? The standard deviation

Solution:

a. To find the arithmetic average return, we sum up all the returns and divide by the number of observations. So, for Skynet Data Corporation's stock, the arithmetic average return over the past five years is:

Average Return= (21+17+26 + (-7)+(-4))/5 = 12.20%

b. Next, we will calculate the variance of the company's returns over this period.

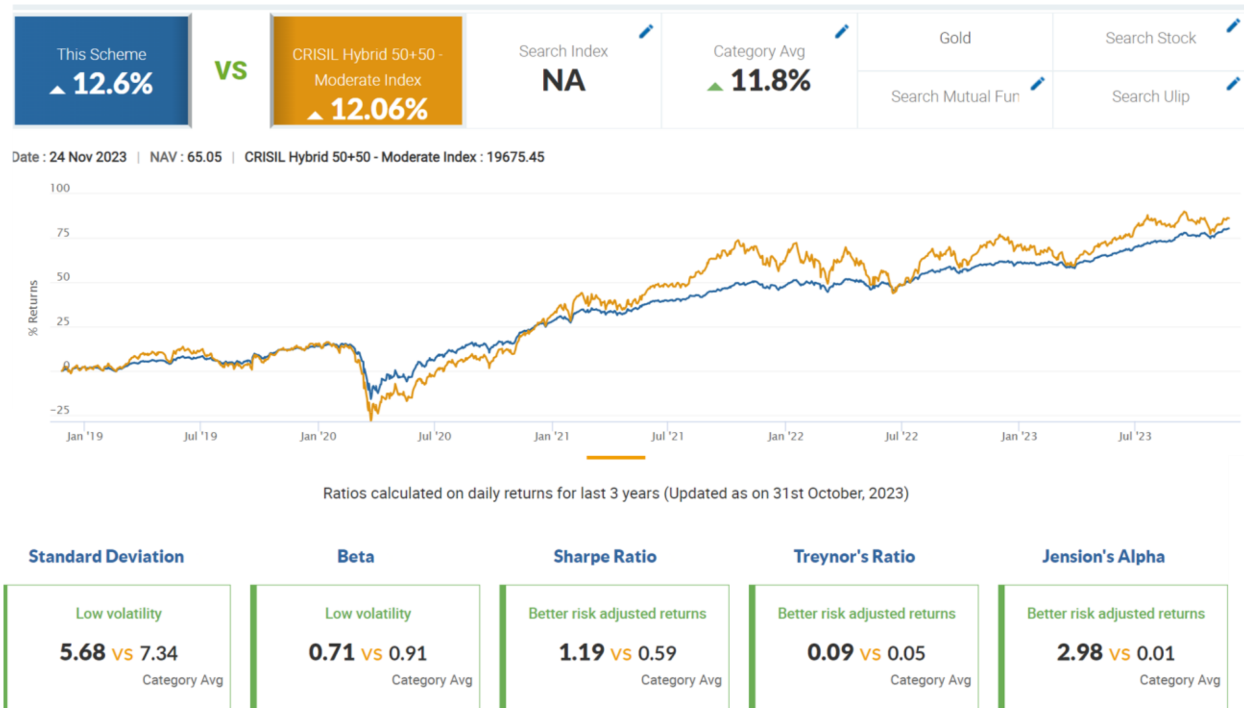
We start by finding the deviations of each return from the mean (average return). Then, we square each deviation to get the squared deviations. After that, we sum up all the squared deviations. Variance is calculated by dividing the sum of squared deviations by the number of observations minus 1.

For Skynet Data Corporation's stock:

X	Actual Return	Average Return	Deviation	Squared Deviation
1	0.21	0.12	0.09	0.00774
2	0.17	0.12	0.05	0.00230
3	0.26	0.12	0.14	0.01904
4	(0.07)	0.12	(0.19)	0.03686
5	0.04	0.12	(0.08)	0.00672
Total	0.61			0.07268
Average return	0.1220			
Variance	0.018170			
Standard Deviation	13.48%			

So, the variance of the company's returns over this period is approximately 1.817%, and the standard deviation is approximately 13.48%.

Stock market reporting



When you are analysing stocks, it is common to refer to various ratios and metrics provided by financial platforms such as Moneycontrol.com. These metrics offer insights into the performance and risk profile of a stock or fund. For instance, the mentioned standard deviation, which measures the volatility or risk associated with an investment. A lower standard deviation indicates less volatility.

Similarly, beta reflects the sensitivity of a stock's returns to market movements. A beta of less than 1 suggests that the stock is less volatile than the overall market.

The Sharpe ratio, Treynor's ratio, and Jensen's alpha are all measures of risk-adjusted returns. They assess how well an investment performs relative to its risk. A higher Sharpe ratio or Treynor's ratio suggests better risk-adjusted returns, while positive Jensen's alpha indicates that the investment outperformed its expected return based on its risk.

Comparing these metrics with industry averages or benchmarks can provide valuable insights into the relative performance and risk profile of a stock or fund. The explanation of each matrices are provided below:

Average Returns	• Over the last three years, investors in this fund earned an 12.6 percent return per year
Standard Deviation	• Standard deviation of the returns is 5.68 percent • Relatively well diversified portfolio
Beta	• How volatile fund performance has been compared to similar funds in the market
Sharpe Ratio	• Risk Premium of the assets divided by standard deviation • Measure of risk to reward ratio (1.19)
Treynor's Ratio	• How much excess return was generated for each unit of risk taken • Risk Premium of the assets divided by Beta of return
Jension's Alpha	• How much own fund generated additional returns compared to a benchmark