Foundations of Accounting & Finance

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Week - 10

Lecture – 46

Bond Valuation - Part II

Variant of the previous example

Let us consider a scenario where market interest rates have risen to 10 percent after one year of issuing the bond. However, the bond will continue to pay a coupon of $80 for the next 10 years. With nine years remaining until maturity, the bondholder wants to sell the bond for immediate cash.

The bond will provide $80 annually for the next nine years, with an additional $1,000 payment in the ninth year. Now, with a remaining life of nine years, the bond's selling price needs to be determined. We will use a similar approach as before to calculate the bond's value under these new market conditions.

Step 1: Present value of the face value

To determine the present value of the face value of the bond, the value of the $1,000 payment at the end of the ninth year is calculated. Since the expected return is 10 percent, and the face value is received after one year, it is discounted accordingly to find its present value. After computation, the present value of the face value is obtained as 424.

Step 2: Present value of the annuity

To calculate the present value of the annuity, which is the $80 received annually for the next 9 years, the formula is applied. Given an annual coupon of $80 and an expected return of 10 percent, the annuity is discounted over 9 years. After computation, the present value of the annuity is found to be $463.

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Step 3: Total value of the bond

The total value of the bond, taking into account the present value of the annuity and the present value of the face value, is approximately $884.82. As the market interest rates rise, the bond sells at a discount. This implies a higher expected yield on the bond.

Interest rate in the markrt raisen to 10% and nine years to maturity											
Year	0	1	2	3	4	5	6	7	8		9
Coupon		80	80	80	80	80	80	80	80		80
Face value											1000
		80	80	80	80	80	80	80	80		1080
Similar Bond yield	**10%**										
First	PV of Face value			424.0976							
Second	PV of Annuity			460.7219							
	Total Bond Value			**884.8195**	***discount bond***						

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Variant of the same example

Suppose the interest rate drops to 6 percent instead of rising to 10 percent as previously mentioned.

Step 1: Present value of face value

The present value of the face value is calculated by using the same equation used in the previous example. So, the present value is obtained as 591.

Step 2: Present value of an annuity

To calculate the present value of the annuity, which is the $80 received annually for the next 9 years, the formula is applied. Given an annual coupon of $80 and an expected return of 6 percent, the annuity is discounted over 9 years. After computation, the present value of the annuity is found to be $544.

Step 3: Total bond value

After calculating the present value of the face value and the annuity, we sum up these values to find the total bond value. In this case, the total bond value is $1136.03. Therefore, when the interest rate drops to 6 percent, you will buy the bond at a premium.

Interest rate in the markrt dropped to 6% and nine years to maturity

Year	0	1	2	3	4	5	6	7	8		9
Coupon		80	80	80	80	80	80	80	80		80
Face value											1000
		80	80	80	80	80	80	80	80		1080
Similar Bond yield	**6%**										
First	PV of Face value			591.8985							
Second	PV of Annuity			544.1354							
	Total Bond Value			**1136.034**	***premium bond***						

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Bond Concepts

- Bond prices and market interest rates move in opposite directions.
- When coupon rate = YTM, price = par value
- When coupon rate > YTM, price > par value (premium bond)
- When coupon rate < YTM, price < par value (discount bond)

Computing Yield to Maturity

Yield to maturity (YTM) is the rate implied by the current bond price. To compute yield to maturity, we can utilize financial calculators or employ a trial-and-error method if such tools are unavailable. Let's illustrate this with an example.

Example

Suppose we are interested in a six-year, 8 percent coupon bond. A broker quotes a price of $955.14. What is the yield on this bond?

$$\text{Bond Value} = C\left[\frac{1 - \frac{1}{(1+R)^T}}{R}\right] + \frac{FV}{(1+R)^T}$$

$$\$955.14 = \$80 \times [1 - 1/(1 + r)^6]/r + 1{,}000/(1 + r)^6$$

r is the unknown discount rate, or yield to maturity

Given the bond price of $955.14, we can plug this value into the bond pricing equation and solve for r. Through a bit of trial and error, we can narrow down the range of possible values. Since the bond is selling at a discount, we know that the YTM must be higher than the coupon rate of 8 percent.

By iterating through different values of r, we find that the YTM for this bond is approximately 9 percent. This process may require some trial and error, but it allows us to determine the yield to maturity even without a financial calculator.

Current Yield

Current yield is calculated by dividing the annual coupon payment by the price of the bond. For example, if a bond with a face value of $1000 pays an annual coupon of $80, the current yield would be 80 divided by 1000, that is 8 percent.

In the case where the bond is priced at $955.14 and still pays the same $80 coupon annually, the current yield would be 0.0838, or approximately 8.38 percent. This is slightly lower than the yield to maturity (YTM).

Why is the current yield lower than the YTM? This is because the YTM represents the total return from the date of purchase until the date of maturity, taking into account both coupon payments and any price discount. On the other hand, the current yield only considers the coupon payment relative to the current price of the bond.

For discount bonds, the current yield may seem low because it only reflects a portion of the return, ignoring the potential gain from the price discount. Conversely, for premium bonds, the current yield may appear higher as it disregards built-in losses from the price premium.

Pure Discount Bonds / Deep discount bonds /Zero coupon bonds

Pure discount bonds, also known as zero coupon bonds or deep discount bonds, differ from traditional bonds in that they do not pay periodic interest payments, or coupons. Instead, they are issued at a deep discount to their face value and pay out a single lump sum at maturity.

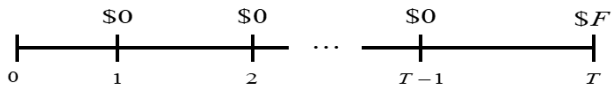
With pure discount bonds, investors purchase the bond at a price below its face value and receive the face value of the bond at maturity. The difference between the purchase price and the face value represents the investor's return, with no additional interest payments received during the bond's tenure.

Due to the lack of periodic interest payments, pure discount bonds are typically sold at a significant discount to their face value. Investors are attracted to these bonds because they offer the potential for substantial capital appreciation as they approach maturity, providing a fixed return if held until maturity.

Information needed for valuing pure discount bonds:

- Time to maturity (T) = Maturity date - today’s date
- Face value (F)

- Discount rate (r)



Present value of a pure discount bond at time 0:

$$PV = \frac{FV}{(1+R)^T}$$

Example

Let us consider a deep discount bond offering $1000 at the end of 10 years. Given the risk profile, let us assume a market interest rate of 7 percent.

Solution:

To determine the price at which you would buy this bond, you would discount the $1000 payment at a rate of 7 percent for 10 years, reflecting the current market interest rate. Essentially, you are calculating the present value of the future payment.

Since the market interest rate is 7 percent, you would discount the $1000 payment at this rate for the 10-year period. The resulting present value will be the price at which you would buy the bond.

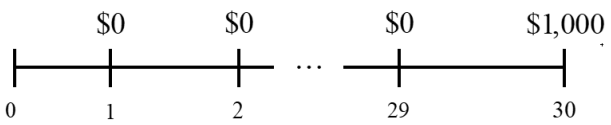
It is important to note that a zero coupon bond, or deep discount bond, cannot sell for more than its face value. These bonds are also known as pure discount bonds, as they do not pay periodic interest.

Factors such as time to maturity, face value, and discount rate are crucial in determining the price of a deep discount bond. By discounting the future face value at the specified discount rate for the given time period, you can ascertain the bond's price.

Example

Find the value of a 30-year zero-coupon bond with a $1,000 par value and a YTM of 6%.

Solution:



$$PV = \frac{FV}{(1+R)^T} = \frac{\$1{,}000}{(1.06)^{30}} = \$174.11$$

Calculations using excel functions

Bond Value in Spreadsheet

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet to calculate bond values**							
3								
4	Suppose we have a bond with 22 years to maturity, a coupon rate of 8 percent, and a yield to							
5	maturity of 9 percent. If the bond makes semiannual payments, what is its price today?							
6								
7	Settlement date:	1/1/15						
8	Maturity date:	1/1/37						
9	Annual coupon rate:	.08						
10	Yield to maturity:	.09						
11	Face value (% of par):	100						
12	Coupons per year:	2						
13	Bond price (% of par):	**90.49**						
14								
15	The formula entered in cell B13 is =PRICE(B7,B8,B9,B10,B11,B12); notice that face value and bond							
16	price are given as a percentage of face value.							

YTM in Spreadsheet

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet to calculate bond yields**							
3								
4	Suppose we have a bond with 22 years to maturity, a coupon rate of 8 percent, and a price of							
5	$960.17. If the bond makes semiannual payments, what is its yield to maturity?							
6								
7	Settlement date:	1/1/15						
8	Maturity date:	1/1/37						
9	Annual coupon rate:	.08						
10	Bond price (% of par):	96.017						
11	Face value (% of par):	100						
12	Coupons per year:	2						
13	Yield to maturity:	**.084**						
14								
15	The formula entered in cell B13 is =YIELD(B7,B8,B9,B10,B11,B12); notice that face value and bond							
16	price are entered as a percentage of face value.							
17								

Summary

In summary, we covered various aspects of bond valuation, including yield to maturity (YTM), current yield, and the relationship between bond price and market interest rates. We discussed how to calculate the present value of bonds and how bond prices change based on prevailing interest rates. In the next session, we will get into stock valuation.