Foundations of Accounting & Finance

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Week - 10

Lecture – 45

Bond Valuation - Part I

Definition of a Bond

A bond is a debt instrument, which represents a legally binding agreement between the borrower and the lender. When a government issues a bond, it acts as the borrower, while the public who subscribe to the bond become the lenders. Similarly, when an institution issues a bond, it becomes the borrower, and individuals or other institutions who subscribe to it become the lenders. Bonds can be subscribed to by various entities, including institutions and individuals, who serve as the lenders to the bond.

Features of a bond

1) **Maturity Date**

The maturity date of a bond refers to its tenure or lifespan of the bond. Unlike perpetual bonds, which have indefinite lifespans, most bonds have a specified maturity date. This date marks the end of the bond's term and represents the point at which the bond issuer is required to repay the principal amount to the bondholders. The maturity date can vary depending on the bond and may be set at 10 years, 15 years, 20 years, or any other predetermined period. Along with the maturity date, bonds typically have a coupon rate and payment schedule, except for zero coupon bonds.

2) **Coupon rate**

The coupon rate of a bond represents the interest rate that the bond pays to its holders. This rate can vary depending on factors such as market conditions, risk considerations, and the identities of the issuer and investor. For instance, investing in a fixed deposit with the State Bank of India may yield an interest rate of around 6 percent or 7 percent. This interest rate is referred to as the coupon rate of the bond. It is calculated as the annual coupon divided by the face value of the bond, expressed as a percentage.

3) **Coupon payment**

Coupon payment refers to the periodic interest payments made to bondholders based on the coupon rate of the bond. The frequency of these payments can vary, with options including annual, semi-

annual, quarterly, or even monthly payments. The terms and conditions regarding the frequency of coupon payments are typically outlined when the bonds are issued.

4) Par value

Par value, also known as face value, refers to the nominal value assigned to a bond at the time of issuance. It represents the amount of money that the issuer promises to repay to the bondholder upon maturity. Par value can vary, with bonds available in denominations such as 100 rupees, 1000 rupees, and so on. It serves as the reference point for calculating interest payments and determining the bond's pricing in the secondary market.

Example

Suppose the Beck Corporation wants to borrow $1,000 for 30 years. The interest rate on similar debt issued by similar corporations is 12 percent. Beck will thus pay $0.12 \times \$1,000 = \$120$ in interest every year for 30 years. At the end of 30 years, Beck will repay the $1,000.

- **Par (face) value** - $1,000

- **Coupon rate** - $120 / 1,000 = 12 percent

- **Coupon payment** - $.12 \times \$1,000 = \$120$

- **Maturity Date** - 30 years

Bond Values and Yields

Bonds and interest rates are related.

Example

Let us break down the bond details:

- The bond is a 5-year bond, issued on January 1, 2020, with maturity on December 31, 2024.

- It has a coupon rate of 10 percent and a face value of $100.

- As of January 1, 2023, there are still 2 years remaining until maturity.

- The bondholder has received 10 percent interest annually for the past 3 years (2020, 2021, and 2022).

Now, the bondholder wants to sell the bond on January 1, 2023, in the secondary market to obtain cash immediately.

The core questions

The key question here is at what price should one buy the bond. This decision hinges on the prevailing interest rate in the market for similar-risk instruments at the time of sale. Let us name the existing bond "Bond A" and the potential new bond with a higher coupon rate "Bond B."

If Bond B, issued on January 1, 2023, offers a higher coupon rate (let us say 12 percent) compared to Bond A's coupon rate (10 percent), it makes more sense to purchase Bond B at a lower price than its face value (100 dollars). This is because buying Bond A at its face value wouldn't yield the same return as investing in Bond B with its higher coupon rate.

Consequently, the seller of Bond A, being eager to sell, would have to offer it at a discounted price to match the market's prevailing interest rate. The buyer calculates the purchase price based on the expected return. For example, if the calculated price of Bond A is 96.61 dollars, and it yields 10 dollars annually, the return percentage is around 12 percent.

Conversely, if the coupon rate of Bond A is higher than the prevailing market rate (let's say 8 percent), Bond A would sell at a premium because its coupon rate exceeds the expected return. In this case, the calculated price of Bond A would exceed its face value (100 dollars).

The price of a bond in the secondary market depends on two key factors: prevailing interest rates and the remaining periods until maturity. These factors determine the bond's value at any given time.

Why time of period is also important?

The time period is important because it determines the number of periods over which the bond's cash flows are discounted. Whether it's 1 year, 2 years, or 10 years, the duration affects the overall gain or return on the investment. Therefore, along with prevailing interest rates, the time until maturity plays a crucial role in determining the bond's value.

Additionally, the yield of the bond is another important aspect to consider. The yield reflects the annual return on investment and is influenced by factors such as the prevailing market interest rate and the coupon rate of the bond and the terms of coupon payment. For instance, a bond may have a yield of 10 percent or 12 percent, depending on the investor's expectations and the prevailing market conditions. The yield will also vary if the interest payment is made half yearly or quarterly or yearly as compounding of the interest paid within a year happens.

Yield to maturity (YTM)

Yield to maturity (YTM) represents the interest rate required in the market for a bond and is also known as the bond's yield to maturity. In this scenario, the yield to maturity is stated as 12 percent until the bond's maturity date.\

How to Value Bonds?

To value a bond, we calculate the present value of its future cash flows, which include coupon payments and the bond's face value. The discounted value of these cash flows is determined based on the prevailing interest rates.

The value of the bond is the sum of the present values of all future coupon payments and the discounted face value. This value represents what an investor would be willing to pay for the bond in the current market.

It is important to note that bond values are inversely related to interest rates. As interest rates rise, the present value of future cash flows decreases, leading to a lower bond value. Conversely, when interest rates fall, bond values tend to increase. This relationship reflects the impact of changes in market interest rates on bond prices.

The Bond Pricing Equation

$$\text{Bond Value} = C \left[\frac{1 - \dfrac{1}{(1+R)^T}}{R} \right] + \frac{FV}{(1+R)^T}$$

The equation looks a little complex. Let us try to solve one example in order to understand the pricing of bonds.

Example

Suppose the Xanth Co. were to issue a bond with 10 years to maturity. The Xanth bond has an annual coupon of $80, implying the bond will pay $80 per year for the next 10 years in coupon interest. In addition, Xanth will pay $1,000 to the bondholder in 10 years. Assuming similar bonds have a yield of 8 percent, what will this bond sell for?

Solution:

We can infer that the bond offers an annual coupon payment of $80 and a yield of exactly 8 percent. Therefore, if the bond yields 8 percent, it should ideally sell at par value. If the yield of similar instruments is 9 percent, indicating a higher yield than the bond's coupon rate, the bond will sell at a price lower than $1,000. Conversely, if similar bonds have a yield of only 7 percent, lower than the bond's coupon rate, the bond will sell at a value higher than $1,000.

Now, let us calculate the bond's selling price.

Step 1: Present value of the face value

The face value will be received only at the end of the tenth year. To discount it, the discount rate of 8 percent will be used, as it represents the expected return on equally risky instruments. The time period (t) until the face value is received is 10 years. Therefore, the present value of the face value of the bond is $463.

Step 2: Present value of the annuity

The annuity represents the $80 interest payment received every year for the next 10 years. To determine its present value, we can utilize a simple formula. Given that the interest rate (r) is 8 percent, we substitute these values into the formula.

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

The present value of annuity is computed as 536.807

Step 3: Total Bond value

The total bond value is the sum of the present value of the annuity and the present value of the face value of the bond, which are $463 and $536 respectively. Since similar bonds have a yield of 8 percent, and this bond also yields 8 percent, the bond is valued at par. The calculations ae provided in the following figure.

Year	0	1	2	3	4	5	6	7	8	9	10
Coupon		80	80	80	80	80	80	80	80	80	80
Face value											1000
		80	80	80	80	80	80	80	80	80	1080
Similar Bond yield	8%										
First	PV of Face value		463.1935								
Second	PV of Annuity		536.8065								
	Total Bond Value		**1000**								

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$