Foundations of Accounting & Finance

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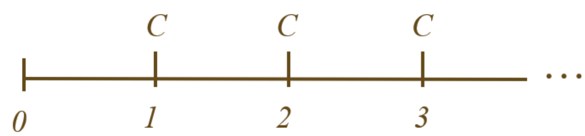
Week - 10

Lecture – 44

Simplification of Cash Flows: Annuity, Growing Annuity, Perpetuity and Growing Perpetuity

Perpetuity

Perpetuity refers to a continuous stream of cash flows that is indefinate. When dealing with perpetuities, each cash flow received in subsequent periods must be discounted to determine its present value. This process involves discounting each cash flow by the appropriate discount rate to ascertain its value as of the present moment. The procedure is depicted below:



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

For instance, if you receive a constant cash flow next year and in all subsequent years, each cash flow must be discounted by the respective number of years in the future. This process continues for each period, with the discounting rate remaining constant.

The present value of a perpetuity can be calculated by dividing the constant cash flow by the discount rate. This yields the present value of each cash flow, which is then summed up to determine the total present value of the perpetuity.

In essence, perpetuity valuation involves discounting the perpetual cash flows at the discount rate to ascertain their present worth.

Example:

What is the value of a British console that promises to pay £15 every year forever? The interest rate is 10 percent.

Solution:

To find the present value of the perpetuity, we divide the annual cash flow (£15) by the interest rate (10 percent). This yields a present value of £150.

In essence, by investing £150, one can receive £15 annually indefinitely, meeting the expected return of 10 percent. This exemplifies the concept of perpetuity.

$$PV = \frac{£15}{.10} = £150$$

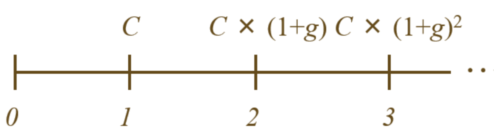
Growing perpetuity

A growing stream of cash flows that lasts forever.

In this scenario, let us examine an example where the perpetuity experiences continuous growth.

In the first year, the present value corresponds to the initial cash flow, denoted as 'C,' divided by 1 plus 'R.' Subsequently, the cash flow in the following years grows at a rate of 'g.' For instance, in the second year, it's 1 plus 'g' multiplied by the cash flow in the first year. This pattern continues, with each subsequent cash flow growing at the same rate.

To calculate the present value of this perpetuity, we use the formula following formulae:



$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots$$

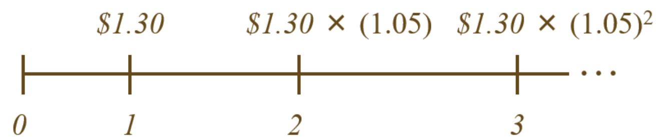
$$PV = \frac{C}{r-g}$$

This method enables us to determine the present value of a perpetually growing stream of cash flows.

Example:

The expected dividend next year is $1.30, and dividends are expected to grow at 5 percent forever. If the discount rate is 10 percent, what is the value of this promised dividend stream?

Solution



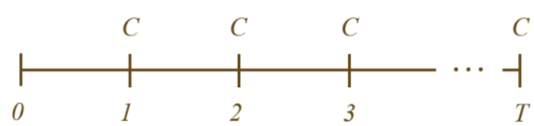
$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

Annuity

An annuity represents a steady stream of cash flows with a predetermined maturity.

In contrast to perpetuities, annuities have a defined endpoint, denoted as 't' rather than 'n.' This 't' signifies the fixed duration over which the cash flows occur, making it a standardized period.

To calculate the present value of an annuity, the following equation is used:



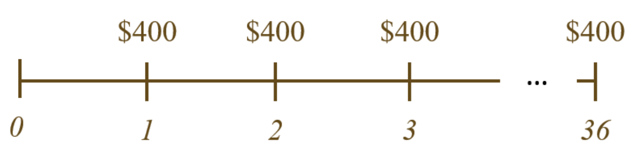
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Example:

If you can afford a $400 monthly loan payment on car purchase, how much of the loan can you afford if interest rates are 7 percent and the tenure is for 36-months?

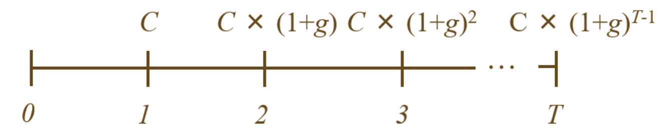
Solution



$$PV = \frac{\$400}{.07/12}\left[1 - \frac{1}{(1+.07/12)^{36}}\right] = \$12{,}954.59$$

Growing Annuity

A growing annuity represents a stream of cash flows with a predetermined maturity, where the payment amount increases over time as depicted below:



$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \dots + \frac{C \times (1+g)^{T-1}}{(1+r)^T}$$

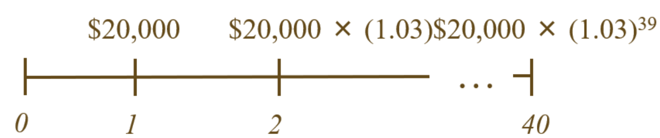
$$PV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{(1+r)}\right)^T\right]$$

In such cases, where the payment amount itself grows steadily for a fixed period, determining the present value involves accounting for this growth.

Example

A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by 3 percent each year. What is the present value at retirement if the discount rate is 10 percent?

Solution



$$PV = \frac{\$20{,}000}{.10-.03}\left[1 - \left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

Summary

We covered several key concepts in finance, including discounting, future value, and present value in both single and multi-period cases. We explored annuities, growing annuities, perpetuities, and growing perpetuities. Additionally, we learned about Net Present Value (NPV), which assesses the value of future cash inflows relative to present cash outflows. A positive NPV indicates a worthwhile investment, while a negative NPV suggests otherwise. Now let us proceed to solving few problems in this context.

In class problems: one

Stu can purchase a house today for $110,000, including the cost of some minor repairs. He expects to be able to resell it in one year for $129,000 after cleaning up the property. At a discount rate of 5.5 percent, what is the expected net present value of this purchase opportunity?

Solution

To find the present value of the sale price, we divide $129,000 by (1+0.055)^1. This yields a present value of approximately $122,274.

Now, to compute NPV, we subtract the initial investment ($110,000) from the present value of the sale price ($122,274). The result is a positive NPV of $12,274. This indicates that the investment opportunity is favourable, as Stu stands to gain this amount.

Cashout flow	110000
Expected cash inflow after on year	129000
Discount rate	5.50%
PV of cash inflow	122274.9
Expected NPV	**12274.9**

In class problems: Two

Shawn has $2,500 invested at a guaranteed rate of 4.35 percent, compounded annually. What will his investment be worth after five years?

Solution:

Using the formula for compound interest, we multiply the initial investment of $2,500 by (1+0.0435)^5 (since it's compounded annually).

This calculation yields approximately $3,093.15. Therefore, after five years, Shawn's investment will be worth about $3,093.15.

Investmemnt amount	2500
Compund interset	4.35%
Worth of investment at end of 5 years	**3093.16**

In class problems: Three

A project is expected to produce cash flows of $48,000, $39,000, and $15,000 over the next three years, respectively. After three years, the project will be worthless. What is the net present value of this project if the applicable discount rate is 15.25 percent and the initial cost is $78,500?

Solution:

To calculate the net present value (NPV) of the project, we subtract the initial cost of $78,500 from the present value (PV) of the expected cash flows over the next three years.

1. **Year 1 cash flow:** PV = $48,000 / (1 + 0.1525) = $41,648
2. **Year 2 cash flow:** PV = $39,000 / (1 + 0.1525)^2 = $29361
3. **Year 3 cash flow:** PV = $15,000 / (1 + 0.1525)^3 = $9,798

Summing up the present values of all cash flows:

NPV = 41,648+29,361+9,798−78,500

NPV = $2,309

The net present value of the project is approximately $2,309, indicating a positive NPV. Therefore, it is advisable to invest in this project since it's expected to generate a positive return.

Year	Cash flows	PVIF	PV	Discount rate	15.25%
0	-78500		-78500		
1	48000	0.87	41648.59002		
2	39000	0.75	29361.80425		
3	15000	0.65	9798.699898		
		NPV	**2309.094165**		

In class problems: Four

You have been offered a job that pays an annual salary of $48,000, $51,000, and $55,000 over the next three years, respectively. The offer also includes a starting bonus of $2,500 payable immediately. What is this offer worth to you today at a discount rate of 6.5 percent?

Solution

Firstly, let us determine the present value of each component:

1. **Starting bonus:** The starting bonus of $2,500 remains constant as its present value, considering you receive it immediately.
2. **First-year salary:** The present value of the first-year salary of $48,000 is approximately $45,070.
3. **Second-year salary:** The present value of the second-year salary of $51,000 is approximately $44,964.

4. **Third-year salary:** The present value of the third-year salary of $55,000 is approximately $45531.

Summing up these present values gives us the total worth of the job offer today, which is approximately $1,38,066.75.

Year	Cash flows	PVIF	PV	Discount rate	6.50%
0	2500		2500.00		
1	48000	0.94	45070.42		
2	51000	0.88	44964.62		
3	55000	0.83	45531.70		
		PV	**138066.75**		