Foundations of Accounting & Finance

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Week - 10

Lecture – 43

Time Value of Money: Present Value and Future Value in Single and Multipored case

Time value of money

In our previous session, we discussed the importance of the DCF or Time Value of Money (TVM). It is one of the fundamental concepts in finance. Understanding time value of money is crucial because it helps us grasp how money's value changes over time. This knowledge is essential for various financial analyses and decisions.

We rely on time value of money concept to compare cash flows occurring at different points in time. It aids in investment evaluations, borrowing assessments, budgeting, and other financial planning endeavors. In essence, the time value of money serves as a cornerstone in finance, guiding us in making informed and effective decisions.

Definition: The sum of money is worth more now than the same sum will be at a future date due to its earnings potential in the interim

One-Period Case - Future Value

The total amount due at the end of the investment is called the Future Value (FV).

$$FV = PV \times (1 + r)$$

PV = present value (i.e., the value today)

r = is the appropriate interest rate

Example

If you were to invest $10,000 at 12 % interest for one year, your investment would grow to ______.

Solution:

The future value of your investment would exceed $10,000 because it includes the initial investment plus the interest earned. So, $10,000 plus 12% of $10,000 equals $11,200, which is the future value. Expressed as a formula, the calculation is:

$$FV = PV \times (1 + r)$$

$$\$11{,}200 = \$10{,}000 \times (1.12)$$

One-Period Case - Present Value

$$PV = \frac{C_1}{1+r}$$

C_1 = cash flow at date 1
r = the appropriate interest rate

Example

If you were to be promised $11,424 due in one year when interest rates are 12 percent, your investment would be worth ________ in today's dollars.

Solution:

In this example, you are presented an option of investing a certain amount today and receiving $11,424 one year later and the prevailing interest rates are 12 percent.

To determine the present value of receiving $11,424 in the future, given a 12 percent expected return, you can use the formula for present value:

$$PV = \frac{C_1}{1+r}$$

C_1 = cash flow at date 1
r = the appropriate interest rate

Here, C is the future value of $11,424 and r is the expected return rate of 12 percent. Therefore,

$$\$10{,}200 = \frac{\$11{,}424}{1.12}$$

The present value of receiving $11,424 in the future is $10,200 i.e. when $ 11,424 is discounted at a 12 percent which is the expected return rate the value is $10,200.

If the expected return rate were to change, the present value would also change accordingly. For instance, if the expected return rate were higher than 12 percent, the present value would be lower than $10,200. Conversely, if the expected return rate were lower than 12 percent, the present value would be higher than $10,200.

The present value calculation is based on discounting future cash flows by the expected return rate, which serves as the benchmark discounting rate. This concept of discounting future cash flows is crucial in financial decision-making and will be further explored while discussing the concept of net present value.

One-Period Case - Net Present Value

The Net Present Value (NPV) of an investment is the present value of the expected cash flows, less the cost of the investment.

$$NPV = -\text{Cost} + PV$$

Example 3

Suppose an investment that promises to pay $10,000 in one year is offered for sale at $9,500. Your expected interest rate is 5 percent. Should you buy?

Solution:

To determine whether you should buy the investment, you need to calculate its net present value (NPV). NPV is calculated by subtracting the present value of the cash outflow (the purchase price) from the present value of the cash inflow (the future payment) discounted at the expected return rate.

Here's the calculation:

$$NPV = -\$9{,}500 + \frac{\$10{,}000}{1.05}$$
$$NPV = -\$9{,}500 + \$9{,}523.81$$
$$NPV = \$23.81$$

The positive NPV of $23.81 indicates that the investment is worth more than its cost today i.e. you will get a return higher than your expected return of 5%. Therefore, you should buy the investment at $9,500, as you stand to make a little more money than your expected return of 5 percent.

The Multi period Case FV and Compounding

Future value of an investment over many periods can be written as:

$$FV = PV \times (1 + r)^t$$

PV = present value,

r = appropriate interest rate

t = number of periods over which the cash is invested.

Example: (The Multi period Case FV and Compounding*)*

Sarla Mistry has put $500 in a saving account at the Mussoorie cooperative bank. The account earns 7%, compounded annually. How much will Ms. Mistry have at the end of three years?

Solution:

To find out how much she will have at the end of three years, we can use the concept of compounding.

In the first year, Sarla earns 7% interest on her initial investment of $500, which amounts to $35. In the second year, her capital becomes $535 (initial $500 + interest $35), and she earns 7% interest on this amount, resulting in $37.45 interest. Moving to the third year, her principal becomes $572.45 (previous year's capital + interest), and she earns 7% interest on this, amounting to $40.07.

Therefore, at the end of three years, Sarla will have a total of $612.52 ($572.45 + $40.07) in her account. This demonstrates the compounding effect, where each year's interest adds to the principal for the next year's calculation.

We can also calculate this using the formula for compound interest:

$FV = PV \times (1 + r)^t$

So, for Sarla's case:

$\$500 * 1.07 *1.07*1.07 = \$500*(1.07)^3 = \$612.52$

Example: Compound growth

Suppose a stock currently pays a dividend of $1.10, which is expected to grow at 40 percent per year for the next five years. What will the dividend be in five years?

Solution:

To calculate the future value of the dividend, we can use the formula for compound growth:

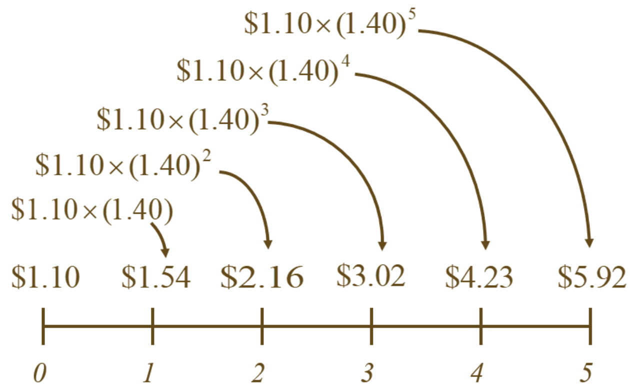
$FV = PV \times (1 + r)^t$

Substituting the given values into the formula:

$\$5.92 = \$1.10 \times (1.40)^5$

So, the dividend at the end of five years will be $5.92.

This calculation demonstrates the concept of compound growth, where the dividend grows year by year, accumulating to a higher value over time. We are not discounting here; instead, we are compounding to find the future value of the dividend as depicted in the figure below:



If we were to discount this future value to its present value, we could use a discount rate to calculate its worth in today's terms.

The Multi period Case – PV and Discounting

Example

How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15 percent?

Solution

To determine how much an investor would need to set aside today to have $20,000 five years from now, we use the formula for present value:

$$PV = \frac{FV}{(1+r)^t}$$

Where:

- PV is the present value (amount to be set aside today),
- FV is the future value ($20,000),
- r is the interest rate (15 percent, or 1.15 as a multiplier),
- t is the number of years (five years).

Substituting the given values into the formula:

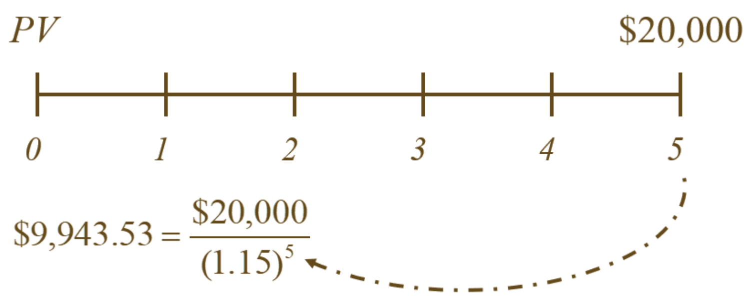
$$PV = \frac{20{,}000}{(1.15)^5}$$

$$PV \approx \frac{20{,}000}{2.011}$$

$$PV \approx 9{,}943.53$$

So, the investor would need to set aside approximately $9,943.53 today to have $20,000 five years from now, assuming a current rate of 15 percent.

Now, if the investor were to buy the investment at a price lower than $9,943.53, they would stand to gain, as their overall return would be higher than 15 percent. Conversely, buying at a price above $9,943.53 would result in a lower expected return. Therefore, $9,943.53 serves as the threshold for the investor—if their expected return is 15 percent, they should aim to negotiate a purchase price below this threshold to maximize their return. The pictorial representation of the solution is provided below:



Simplifications of cash flows

- **Perpetuity:** constant stream of cash flows that lasts forever
- **Growing perpetuity:** A stream of cash flows that grows at a constant rate forever
- **Annuity:** A stream of constant cash flows that lasts for a fixed number of periods
- **Growing annuity:** A stream of cash flows that grows at a constant rate for a fixed number of periods

1) **Perpetuity**

Perpetuity refers to a financial arrangement where a fixed payment is received indefinitely, typically at regular intervals, with no defined end date. An example of perpetuity is a retirement pension that provides a fixed monthly income for as long as the individual is alive. Regardless of how long the individual lives—whether it's 10, 20, or even 30 more years—the payment remains the same, repeating indefinitely. This continuous flow of payments, occurring repeatedly over time, defines the concept of perpetuity.

2) Growing perpetuity

A growing perpetuity is similar to a perpetuity in that it involves receiving payment indefinitely, but the payment is growing at a constant rate. The payment increases at a constant rate each period. For example, if you receive a pension that increases by 3 percent every year, it means that your pension payment will grow by 3 percent annually, ensuring that it keeps pace with inflation or other factors affecting the cost of living. This continuous growth at a steady rate distinguishes a growing perpetuity from a simple perpetuity.

3) Annuity

An annuity refers to a series of periodic cash flows that occur at regular intervals over a specified time period. For instance, if you invest a sum of money in a fund and the fund promises to pay you a fixed amount every year for the next 10 years, this series of payments constitutes an annuity. Annuities provide a predictable stream of income over a defined timeframe, making them useful for retirement planning, financial goal achievement, and other long-term financial strategies.

4) Growing annuity

A growing annuity is similar to a regular annuity in that it involves a series of cash flows over a fixed time period. However, in a growing annuity, the amount of each cash flow increases at a constant rate over time. For example, if you invest money and are promised returns for the next 10 years, but the returns increase by 5 percent each year, this series of increasing payments constitutes a growing annuity. The growth rate applies to each successive payment, resulting in a gradual rising stream of cash flows over the specified time period.

Need for the simplification of cash flows

The need for simplification of cash flows arises for making informed investment decisions. When faced with different types of cash flow scenarios such as annuities, growing annuities, and perpetuities, investors need to determine how much they should invest today to receive a certain amount of cash flow in the future. This calculation depends on factors like the expected return on investment, risk appetite and the time period involved.

For instance, in the case of a regular annuity where an investor is promised a fixed amount of money annually for a set number of years, they need to calculate the present value of those future cash flows based on their expected rate of return. Similarly, in the case of a growing annuity where the cash flows increase at a constant rate over time, investors must determine the initial investment required to receive the growing cash flows.

Moreover, perpetuities, which offer a stream of cash flows indefinitely, also require investors to ascertain the present value of the perpetuity to understand its current worth and make investment decisions accordingly.

By simplifying these complex cash flow scenarios into present value calculations based on certain formulas and assumptions, investors can effectively evaluate investment opportunities and make informed financial choices.