

The Future of Manufacturing Business: Role of Additive Manufacturing
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Lecture - 06
Laws of Manufacturing - II

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Background of the Future

Laws of Manufacturing





Process Parameters



Inventory I
Work-in-process

Throughput R
Rate at which output
is made

Cycle Time T
"Clock time" taken
in the process



Welcome to session number six. In the previous session, we started our discussion on the laws of manufacturing and we looked what is a process. So, we have some input and that process will convert that input to an output and then there are some process parameters like inventory or mainly the work in process. The throughput the rate at which the output is made, we call it R and the cycle time (clock time taken in the process).

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Laws of Manufacturing

Little's Law
 $L = R \times T$

A PROOF FOR THE QUEUING FORMULA: $L = \lambda W$

John D. C. Little
Case Institute of Technology, Cleveland, Ohio*
(Received November 9, 1961)
Operations Research, 1961

We also discussed Little's law which connect these three parameters and we said that how much is the minimum inventory needed to maintain a throughput rate of T for a given cycle time. So, for some reason or some way you actually can reduce this cycle time T. In that case, there is a possibility you actually can reduce the inventory also. We have lived through an example.

We looked an example where if we have the inventory lower than what is needed, then the system the order process may starve, and you may not sustain that throughput rate. This is what we know is the Little's law and as I mentioned in the previous session that it is a folk law. Little only provide the proof for it in 1961.

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Laws of Manufacturing

Kingman's Law

Research Notes

THE SINGLE SERVER QUEUE IN HEAVY TRAFFIC
By J. F. C. KINGMAN
Communicated by H. F. ATTIEAT
Received 22 November 1960

Mathematical Proceedings of the Cambridge Philosophical Society

So, the connected to this law is another law which we know as the Kingman's law. I am calling it a law but normally it is in the textbooks if you see it is called as Kingman's formula. It was given by JFC Kingman in the paper The Single Server Queue in Heavy Traffic published in mathematical proceedings of the Cambridge Philosophical Society.

I think it came in 1961 again and you can actually see something that both these papers were submitted in a span of about two weeks to two different journals. Two important results more or less were derived simultaneously and in this case, what we actually see are the three things. One is utilization, the other is cycle time and the third one is variability.

You have a given capacity and there is a utilization of that capacity. It can vary from 0 to 100% and then the relationship or the law gives you what is the impact of that utilization on the cycle time. The third thing which comes is the variability. So, you can actually see that the utilization even for a given utilization, if the variability increases, the cycle time increases.

This variability could be for the processing time at a machine or it could be even the arrival. It is not necessary that all the jobs have to come at the in some kind of a so the jobs can be randomly arriving for processing and then there would be a random processing time. So, this variability will impact the cycle time for a given utilization.

You can actually see that this increases exponentially after so in this case, even if the variability may be moderate, and for higher utilization, the cycle time increases exponentially, even for moderate variability. So, this actually, mathematically gives us that old wisdom that variability inhibits speed.

If higher the variability, the cycle time would be higher and the speed would be lower. The objective of that faster may not be achieved. This gives you an idea. One way so in fact, when we talk about a typical manufacturing system, the utilization rates normally are higher, and if you actually combine it with higher variability, the cycle time rises exponentially.

One way out is either reduce the utilization or reduce the variability and we will actually see that how you can reduce variability? So, will there be some laws again, which will come for our rescue to reduce that variability. We will talk about that because one of the things which we are going to talk is about the aggregation phenomena; how aggregation reduces variability?

Is it possible for us to actually find some kind of an aggregation strategy, which allows us to reduce the variability to improve the cycle time? So, this wisdom from Kingman's law is actually very relevant, because the main idea of variability inhibits the speed comes in the form of a mathematical relationship.

If you actually see the mathematical relationship in this paper, you can easily make out that the wisdom comes in this form. We are looking only at the pictorial part of it, but the mathematical part is actually there in the, you can even check some of the textbooks. **(Refer Slide Time: 06:01)**

Background of the Future

Laws of Manufacturing

Economic Lot Sizing

The order or production in response to the high set-up cost effect is a **batch**.

The average inventory arising due to a batch activity is referred to as **cycle inventory**.

There is a **trade-off** between the **cost of carrying inventory H** and the **set-up cost S** .

If R is the *steady* outflow rate per unit time, and Q is the batch size, then




$$TC = S \times \frac{R}{Q} + H \times \frac{Q}{2}$$

This is minimized at $Q^* = \sqrt{\frac{2 \times S \times R}{H}}$

$S \uparrow \Rightarrow Q^* \uparrow, I \uparrow, T \uparrow$

$Q^* \propto \sqrt{S}$
 $Q^* \propto \sqrt{R}$

$S \times \frac{R}{Q} + \frac{Q}{2} \times H =$

With this wisdom I will go further and something which again some of you might have seen in the form of what we call as EOQ, the economic order quantity, but I want to relate it this whole idea of economic order quantity with economic lot sizing and we want to actually see what is the impact of the setup time on the inventory. So, this again relates to what we have said earlier.

But let us just see what is the wisdom which comes out of it. The story goes like this that you have a setup cost which is given by S and then there is a cost of carrying the

inventory which is given by H and what we want is we want to minimize the total cost. Now in this case if you see we are saying that, if the setup cost is high, you actually go for something called as a batch.

Order or production in response to the high set of cost is called a batch and average inventory which arises because of this batching is called as the cycle inventory and now there is a tradeoff that there is a cost of carrying this inventory, which is H. If you have high inventory, you can incur that cost of H and then there is a setup cost. So I can say setup cost could be like the setup time.

Now we make that assumption that this R which is the same as the throughput rate is the steady outflow rate per unit time. So, that is something which we have to maintain. It says steady rate which is to be maintained and then assume Q is the batch size. Now how you compute this total cost?

$$\text{Number of setups} = \frac{R}{Q}$$

Where,

R: Throughput rate

Q: Batch size

So R by Q is the number of setups which are needed and every time whenever you setup there is a setup cost which is S.

$$\text{Total setup cost} = S \times \frac{R}{Q}$$

Where,

S: Setup cost

In the steady state

$$\text{Average inventory} = \frac{Q}{2}$$

because it is steady falling and you go to Q and then you consume R. Then again you go to Q and then so it becomes like a so the average inventory is Q by 2 and you multiply it by the inventory cost.

$$\text{Total cost (TC)} = S \times \frac{R}{Q} + H \times \frac{Q}{2}$$

Where,

S: Setup cost

H: Cost of carrying inventory

$\frac{R}{Q}$: Number of setups

$\frac{Q}{2}$: Average inventory

So that becomes the total cost.

$$\text{Optimal batch size (Q*)} = \sqrt{\frac{2 \times S \times R}{H}}$$

This gives some kind of a relationship between the different cost. If H increases, you can actually see if my inventory cost is high, I want a smaller batch.

If my setup cost is high, I want a larger batch. Now this idea of setup cost you can relate to between the Ford and the Toyota examples. So, when we were in the classical mass production setting the setup time was very high and you actually were looking for larger batches.

In fact, you actually went to an extent where you do not want any variety at all, because you do not want to switch from one product line to the from the one product line to the other product line. Now when the so you can actually see that. When S is higher, your batch would be higher and \bar{I} in this case is the average inventory that would also go because that is Q by 2.

If inventory increases, the using the Little's law even the cycle time would actually go up. This relationship if you start thinking, so one way is actually to reduce the setup time. If the setup time goes down, the Q would actually be lower, the inventory would be lower. I think all that wisdom whatever we have seen till now actually can be written from this formula.

So how to reduce that S? Maybe you actually have to think about the way the Toyota Production System did that. They do lot of machining part to reduce this. You derive

different kind of dyes. So, you look for though the whole backup which comes along with that. The machine tools improve and you actually can reduce this setup time.

That will impact your batch size also. Now in this case, you can see that in fact, we have taken this path, but if the setup time reduces the cycle time also will reduce. If the cycle time reduces the inventory will also reduce. So, we already have discussed that part earlier. Now one thing which you can actually see that

$$Q^* \propto \sqrt{S}$$

It is not proportional to S.

$$Q^* \propto \sqrt{R}$$

$$Q^* \propto \sqrt{\frac{1}{H}}$$

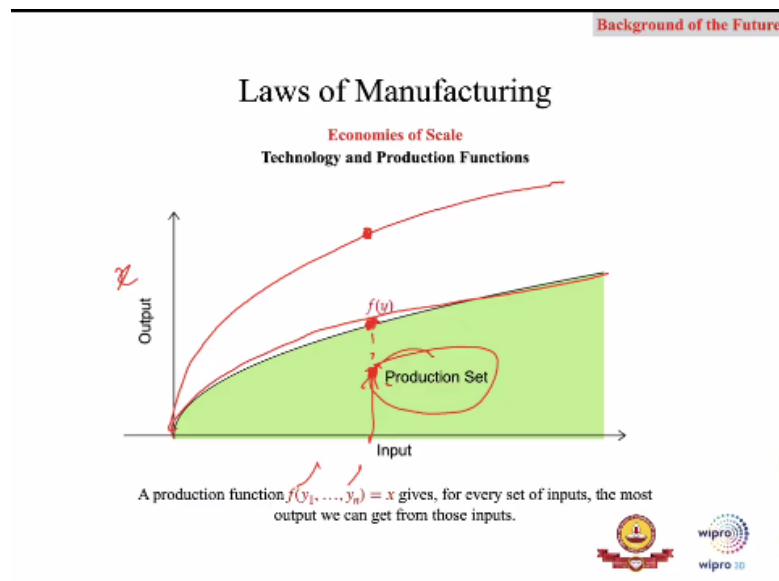
Let us stop here. Now one thing which comes very evident, is that if R increases by say, nine-fold, let us just make some number or fourfold, the Q* would increase only by say three times or two times. So, for nine-fold, it would be only three times and for four-fold, it will be only twice.

If assume that for some reason my throughput actually increases by nine times, the batch size is not linearly increasing, it just is a square root of that and this part would become relevant when we talk about the economies of scale. So, there are scale effects in this particular setting, that if R increases, the throughput increases by nine times. The Q* does not increase by nine times, it just increases by three times.

It means that, that gives you an idea about what so assume that I am running nine different plants, right now or nine parallel lines. So if each is producing something. The Q star would not increase. So from one, if I increase it to nine, the Q* would only go by three times. It will not increase by nine times.

So this wisdom is important, because what it actually says is, it is about the economies of scale, we are going to talk about that. So, I will continue.

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And so next thing which, as I mentioned that we have to look into what economies of scale mean. But before that, we have to define what is a technology and what is a production function. Technology normally is defined in the form of a production function. And what is shown in this particular figure is that you actually have a set of inputs, which is y_1 to y_n .

You have a production function, which takes all these inputs, and gives you output which is x and this is, so you can actually say this could be so it is only shown in two dimensional, but this could be a one-dimensional figure and you can actually see that. So it is possible to use a inferior technology and maybe you produce something which is lower than what is actually possible.

But in this case, we may be using the best technology and given this input, this is the output we get, which is $f(y)$ or $f(y_1, \dots, y_n)$. Now the point here is if you find a better technology than this, you may actually have a better production function. You can actually see that instead of move point here. So, one of the pursuits of the science and technology is to actually find better and better technology.

This is how it is mapped that we keep on looking for a better and better frontier. So, we keep on going, so for the given input, and when I say technology, it includes all the

processes, which comes along with it. So, it is not necessary the technology innovation has to be in only in the form of maybe the engineering side, it may be just a process innovation also.

If you just look at this particular thing that gives me an idea what actually is technology and what is a production function. So, technology means that we are defining it in the form of a production function, which takes different inputs and gives me an output and in fact, I forgot to mention that this is also called this whole shaded area is called as the production set.

It means that, that is something which is feasible for the given inputs. I can always use maybe an inefficient process, even the technology is very good, but I may not hit this I may hit something here. So, the point here is, we want to use the inputs efficiently and optimally so we can actually hit the front here, for the given available technology.

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Background of the Future

Laws of Manufacturing

Economies of Scale
Technology and Production Functions

The production function $f(\cdot)$ has **increasing returns to scale** when
 $\forall a > 1, f(ay_1, \dots, ay_n) \geq af(y_1, \dots, y_n)$

The production function $f(\cdot)$ has **decreasing returns to scale** when
 $\forall a > 1, f(ay_1, \dots, ay_n) \leq af(y_1, \dots, y_n)$

The production function $f(\cdot)$ has **constant returns to scale** when
 $\forall a > 0, f(ay_1, \dots, ay_n) = af(y_1, \dots, y_n)$

Economies of Scale

diseconomies of scale

Now this particular notion of production function is actually used to define what economies of scale mean. Now if you look at the on the slide, you can actually see that we have defined three things increasing returns to scale, decreasing and the constant returns to scale and f is the same production function.

If you just look at the increasing returns to scale, in fact I can actually call it the economies of scale. It actually captures the scale effect part. So, 'a' is some parameter and we assume that for all or for each $a > 1$. When I scale my inputs by a times. So,

earlier I assuming maybe using the say 1 unit of each one of them, and now I am just multiplying each by 2.

So I am using two units of each. The output what comes of the technology or the production function is actually more than 'a' times what I was getting earlier. It is evident that when I scale it up by 'a' factor which is more than 1 the output actually improves by more than that factor. So, $f(\mathbf{a})$ in this case could be 2, 3 whatever; a of y_1 to a of y_n . That is actually larger than 'a' times of what I was getting earlier.

So there are benefits of scaling up and that is where we call it increasing returns to scale or economies of scale. Now the same logic is applicable when we say we have decreasing returns to scale or in economics, we call it diseconomies of scale. So, there is a cost of increasing the scale and then there is a constant returns to scale when we say that, in this case, just look at this.

So 'a' is not more than 1, it is greater than 0. It does not matter whether we scale up or not. What we get is just the same thing.

$$\forall a > 0, f(\mathbf{a}y_1, \dots, \mathbf{a}y_n) = a \cdot f(y_1, \dots, y_n)$$

Where,

y_1, \dots, y_n : Input for a system

a : Scaling factor

So what we were getting earlier multiplied by 'a'. There are no benefits of scaling up. We call it the constant returns to scale.

And you already have seen right from the beginning of this particular course, what benefits these economies of scale actually brings in manufacturing.

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Laws of Manufacturing

Economies of Scale
Technology and Production Functions

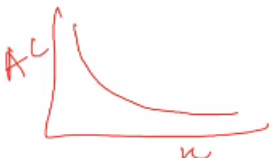


$AC(x) = \frac{TC(x)}{x}$

A firm's technology has increasing returns to scale

$\forall x, a > 1 \quad AC(ax) \leq AC(x)$

A firm with increasing returns to scale has a nonincreasing average cost function.

$TC = \frac{k + kx}{x}$
 $AC = \frac{k}{x} + k$

So, with this basic definition, I continue more into the economies of scale. We already have seen a total cost function in the form of the EOQ. Now for a given output x there would be some total cost function given as x . So what is the average cost?

Average cost,

$$AC(x) = \frac{TC(x)}{x}.$$

Where,

TC(x): Total cost

x: Output

So you can even think of say total cost would be some fixed cost plus.

Now what is the average cost in this case? Is nothing but something like this? In fact, you can actually see that if x increases the average cost is decreasing. That is a very important result, because it means that there is, so there is a fixed cost K and then there could be a variable cost small k and that fixed cost is actually getting distributed for how much you are actually producing.

So the larger the x the lower is the average cost. This idea of lowering average cost or decreasing average cost is actually related to the economies of scale or increasing returns to scale. You can actually see that for

$$\forall x, a > 1$$

$$AC(ax) \leq AC(x)$$

It means that, when I scale the things up, the average cost is actually decreasing.

In fact, this formula also does the, for this total cost function also is doing the same thing. So, a firm with increasing returns to scale we are calling it has a non-increasing average cost function. So, this becomes something which is relevant, because if I have to say plot a function like this, it looks something like this.

You can actually see that, as x increases, the average cost is decreasing and that relates to the idea of economies of scale.

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The slide is titled "Laws of Manufacturing" and includes the following content:

- Background of the Future
- Economies of Scale
- Technology and Production Functions
- Equation: $AC(x) = \frac{TC(x)}{x}$
- Text: "A firm's technology has increasing returns to scale"
- Equation: $\forall x, a > 1 \quad AC(ax) \leq AC(x)$
- Text: "A firm with increasing returns to scale has a nonincreasing average cost function."
- Three categories: Volume Effects, Capacity Effects, and Technology Effects.

Handwritten red annotations include several circled letters and symbols, such as 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N', 'O', 'P', 'Q', 'R', 'S', 'T', 'U', 'V', 'W', 'X', 'Y', 'Z', and Greek letters like α , β , γ , δ , ϵ , ζ , η , θ , ι , κ , λ , μ , ν , ξ , \omicron , π , ρ , σ , τ , υ , ϕ , χ , ψ , ω , δ , ϵ , ζ , η , θ , ι , κ , λ , μ , ν , ξ , \omicron , π , ρ , σ , τ , υ , ϕ , χ , ψ , ω .

Now the things which so when we talk about economies of scale, it is a relevant idea for manufacturing. But one thing which comes critical is, it is a very mysterious concept. It is not necessary it will be, so we still do not know why we observe the economies of a scale. There could be multiple theories around it. It is not you can even not just pinpoint to one of these things.

In fact, some time people say that even the experience curve is actually bringing the economies of a scale. But actually, these two concepts are different. So, when we talk about economies of a scale, the experience curve is an independent concept.

and that is where I think we mentioned about that, when we were talking about the bullwhip effect, we said that or we, when we were talking about the outsourcing, we

mentioned about that, when we outsource something the cost at the supplier end would actually be lower because of economies of scale and the experience curve.

But for time being, let us just look at three different possibilities, which brings these economies of scale. One is the classical volume effects. It means that when you have larger scale, you actually would have more volumes. The utilization actually improves and that would actually lower the cost.

The main idea, which we already have seen in the previous example, that the fixed cost will be distributed that also comes along with the volume effect. Now there could be capacity effects also and we already have seen that in the context of the EOQ. It means that when I scale the things up in say, from say one to nine, the batch size, the Q^* does not increase in the same form.

So, my throughput may be increasing nine folds, but my say inventory will not go up by nine folds. I need much lower inventory. So, in fact this idea can be related so if an organization has say nine retail outlets, the centralized procurement would actually bring efficiency because if they order independently, so the ordering cost may not be distributed.

Now if you actually bring all these plants together and look for centralized procurement, the transportation cost or the ordering cost will actually be distributed among these nine. That is something which is the scale effect. So, the main point is that we are still saying that economies of scale is something which we observe, but we have not fully understood it.

Because there could be multiple effects, which gives what is that economies of scale or what we know is the economies of scale.

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Laws of Manufacturing

Economies of Scale
Technology and Production Functions

$$AC(x) = \frac{TC(x)}{x}$$

A firm's technology has increasing returns to scale

$$\forall x, a > 1 \quad AC(ax) \leq AC(x)$$

A firm with increasing returns to scale has a nonincreasing average cost function.

Volume Effects

Capacity Effects

Technology Effects

Learning Curve
 $TC(x, X)$
 $X \uparrow, TC \downarrow$



I just conclude this particular part with the learning curve when we say that the total cost in this case it is not just a function of what is the output. It is also a function of how much we have produced earlier. You can actually see that if we have produced some x units earlier. So that is the experience which comes by producing something earlier.

If X increases the total cost function decreases. So, total cost is not just a function of how much output I am producing now, it is also a function of what is the learning which has come in the previous production cycles that is captured by that X . So if X increases the total cost decreases. So with this we will go further. Economies of scale we are concluding with this.

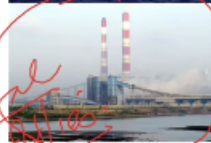
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Laws of Manufacturing

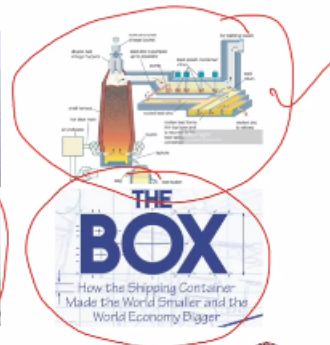
Economies of Scale
Technology and Production Functions



VLCC



Natural monopolies



But I just gives you before I go to economies of scope. These are some examples. I think, we already have talked about the steel production. In fact, right from the ancient times the scale effects were always there in the steel production. A typical blast furnace, we already talked about the logistics side the box? And you might have heard about very large crude oil containers, VLCCs.

And they actually exhibit lot of economies of scale and I think some of you maybe you can find out there is a concept called as natural monopolies, which actually means that even for very large production quantity, the average cost will always be decreasing. So you always have economies of scale. You do not have diseconomies of scale for a very large quantity.

So classical examples are electricity or water distribution. In those cases, it is always optimal to have just one firm in that market rather than multiple firms, because of the economies of scale and that is where we call it so natural monopolies. So, it is by design, that you just want one firm operating in a given market.

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Background of the Future

Laws of Manufacturing

Economies of Scale
Technology and Production Functions



The screenshot shows a news article from The New Indian Express. The headline is "Chennai Port to set record by berthing giant vessel today". The sub-headline reads: "The Chennai Port is all set to create another record in its 137 years of service to the maritime trade of the country when it berths a Very Large Crude Carrier (VLCC) vessel on Thursday." The article text below states: "CHENNAI: The Chennai Port is all set to create another record in its 137 years of service to the maritime trade of the country when it berths a Very Large Crude Carrier (VLCC) vessel on Thursday. It will be the first among the ports of the country to have such an achievement of berthing a VLCC vessel inside an enclosed harbour, a port spokesman said. According to a report on cargo traffic, operations and logistics bottlenecks prepared by the Shipping Ministry and Indian Ports Association, Chennai port could save USD 20 million per year by directly unloading oil from VLCC." There are red circles and arrows highlighting the headline and the savings figure.



With this, we will go to the, so one more example I think we talked about that VLCC is a very large crude carrier, and this particular thing actually has happened recently in Chennai and you can actually see that there, so this particular idea that you can actually bring the crude oil using this VLCC, Chennai port could save about 20 million US dollars per year by directly unloading from these VLCCs.

This is something of a scale effect because of large, your cost is actually coming down. So large is beautiful. That concept is still valid. So, I think with this, I will close this particular session. We will go to the next one when we talk about economies of scope and we will start discussing about the business models. Thank you.