Decision Making Under Uncertainty

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Lecture – 05
Discrete Random Variables

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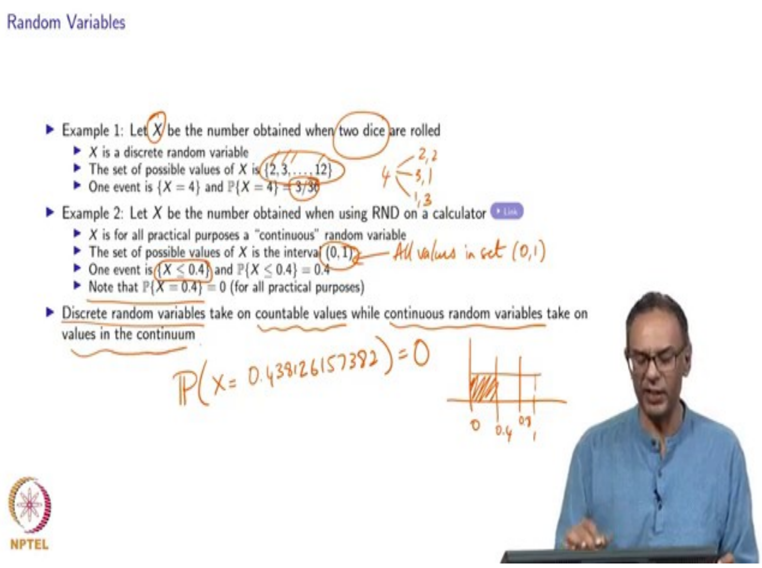

Okay, the next topic is about random variables. So, we start with the general notion of what a random variable is and then, we will go specifically into a random variable called discrete and then, random variables called continuous, okay.

So, what is a discrete random variable? Let's look at an example. Remember the two dice example; we rolled two dice and then, we looked at the number that we got; so, these numbers could be a 2 or a 3 or a 4 and so on till 12. So, it's any one of these numbers; they are not equally probable. We saw earlier the probability of rolling of 4 is 3/36 because there are 3 ways of getting a 4; you can roll a 4 as either a 2 and a 2, a 3 and a 1 or a 1 and a 3. So, there are 3 out of 36 options that will give you a 4.

Now, X is a discrete random variable. Because it takes on specific discrete values, it will either take the values 2, 3, 4, 5, 6 and so on until 12. So, these are the values that it takes. Because it takes specific values, it is called a discrete random variable. The other type of random variable is what is called a continuous random variable. I am going to do a quick demo on a website.

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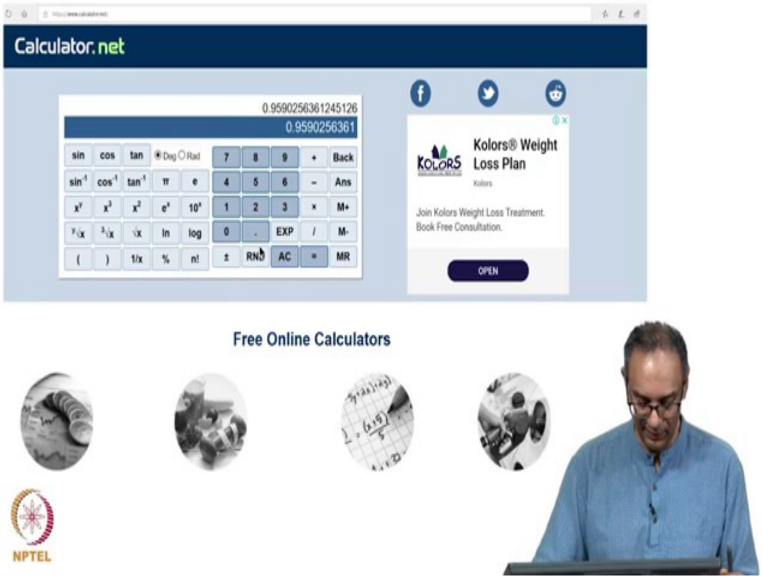

So, if you click on this thing that says link, it will take you to a website and the website is a free calculator. Essentially, you click this function called RND; RND stands for Random. You click RND; it gives you a long random number. These random numbers are all numbers between 0 and 1. Let me click RND one more time, you get another number; you click it a third time, you get another number. So, with several significant digits, you get a bunch of random numbers - 0.39180 and so on. I click it again and you get another random number. Click it again; I get another random number. Notice that all the random numbers are between 0 and 1. So essentially, what this does is randomly generate a number between 0 and 1. Now, one thing to notice is that this random number takes on values in the continuous spectrum between 0 and 1; it does not take a specific value like 0.1, 0.2, 0.3 and so on. As you saw, there are several decimals that get displayed. So, for all practical purposes, we think of that number we saw as continuous because it takes values in the continuum anywhere from 0 to 1.

In the discrete case, the set of possible values were 2, 3, 4 and so on till 12. The set of possible values here is the entire range, that is, all the values in the set 0 to 1; it could take any value in that set. So, that value it is called continuous is because the value that it takes is in the continuum between 0 and 1.

So, an event is something like this. This is how you write an event - the event that the random variable is less than or equal to 0.4. So, if you ask the question - What is the probability that I will get a number that is less than 0.4? Well, turns out that all these values are uniformly distributed between 0 and 1. So, they are all uniformly distributed between 0 and 1.

Therefore, a number less than 0.4 will have a probability of 0.4; a number less than point 0.8 will have a probability of 0.8. So, it is pretty straightforward and that's the reason I gave that example. We will look at many other examples later, but this one is for our purpose. An important feature of a continuous random variable is that the probability that it takes on a single value is 0. So, for example, the probability that X will be exactly equal to 0.438126157382 is 0; the chance it will get exactly that number is practically 0. In fact, it would be exactly 0 if you allow it to take infinite number of decimal places.

In summary, a discrete random variable is one that takes countable values; you can count them, it does not have to be a number; it could be a red, blue, green if you want to, but usually, it is a numerical value because most of our quantitative analysis tends to have a numerical flavor to it.

And, the continuous random variable takes on values in the continuum, usually from 0 to infinite, sometimes negative infinite to infinite, sometimes finite range from 0 to 1. So, that is what we're going to see in terms of random variables.

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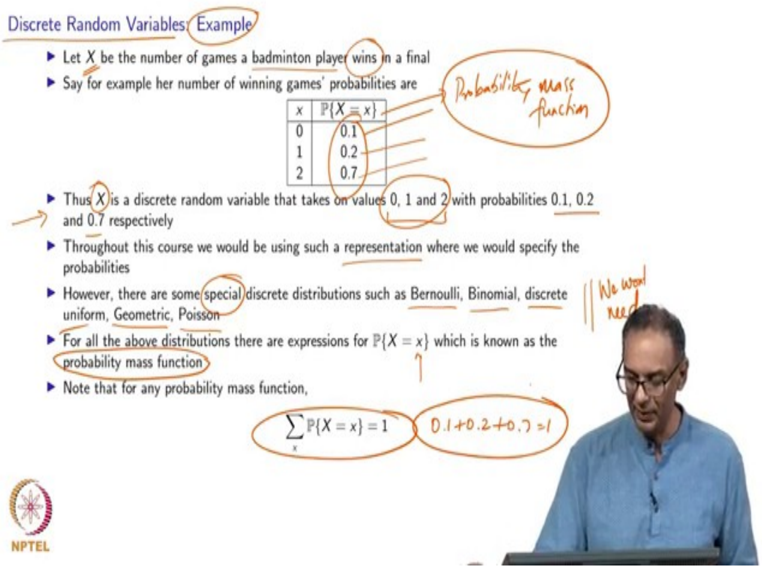

The first topic is going to be discrete random variables. Till the end of this lecture, we will basically be talking about a discrete random variable. I am taking an example. So, I am going to let X be a random variable which denotes the number of games a particular woman badminton player wins in a best of 3 finals. Let's say she has these probabilities: with probability 0.1, she will win exactly 0 games; with probability 0.2, she will win 1 game; with probability 0.7, she will win two games. Remember that it is a best of 3. So, if she wins 2 games, she has won the final. Therefore, she would never have to win anything more than two games; however, she could have played three games. So, that's what this is - the number of games that she wins in a final, alright.

So, X is a discrete random variable. We call X as a discrete random variable that takes on three values. Last time, we had a discrete random variable of rolling two dice and the number that we got took on values 2, 3, 4 all the way till 12. This time, it only takes three values - 0, 1 or 2. And, the probabilities were 0.1. 0.2 and 0.7. So, that is how we characterize in this random variable.

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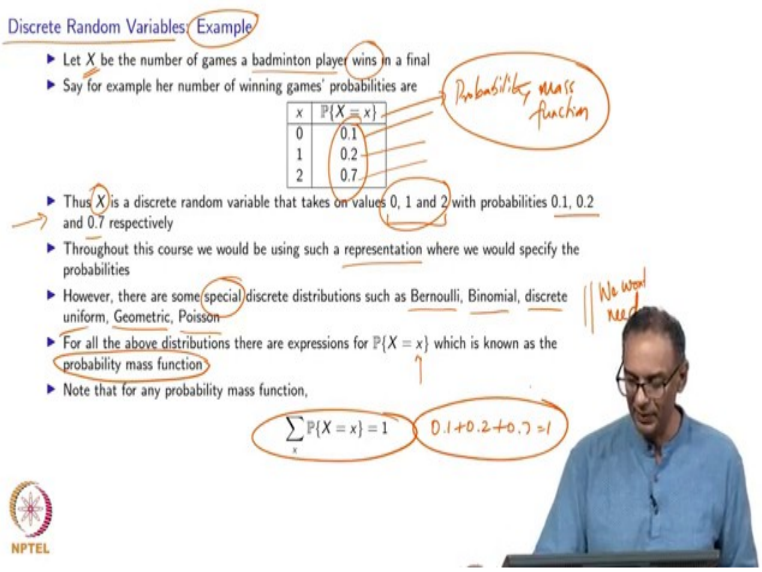

We will call $P(X=x)$ as the probability mass function. However, we won't use that notation; we will just represent it as a probability. This is so that we do not have too many notations in the course. So, I am just keeping it simple. I will just write down the probabilities whenever we need.

However, there are some very special discrete distributions; you may have heard some of these names - Bernoulli, Binomial, discrete uniform. So, discrete uniform is like rolling one die and then, you are getting any one of the 6 values - 1, 2, 3, 4, 5, 6 with all equal probability of 1/6. What we saw before while we hit that RND function in a calculator is what is called continuous uniform distribution. This is called discrete uniform, but it takes discrete values with equal probability.

Then, there is a distribution called geometric; there is a distribution called Poisson. Now, we don't have to hit any of those distributions in this course. We will not be talking about any of this; we will not need this for this course. However, for other courses, I am sure you have probably seen some of this. Like I said a little while ago, this probability is called the probability mass function that I said here.

So, the probability mass function is the probability that the random variable X takes on a value "little x". And, for all probability mass functions, the probability of all the values of X, the probability mass function adds to 1. So, if you notice here, $0.1+0.2$is 0.3; $0.3+0.7$is 1. So, notice that $0.1+0.2+0.7$ equals 1. Likewise, in the previous example, if you added up the probabilities of these guys, you will again get the value to be equal to 1. So, that's a very important rule.

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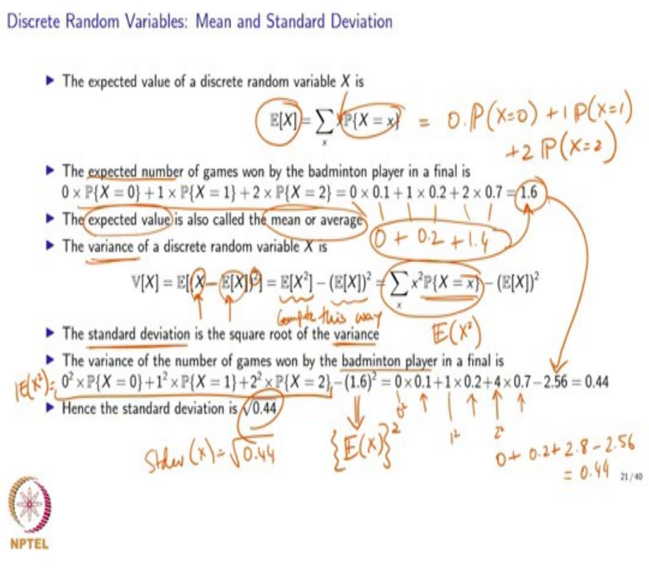

Now, we want to talk about two other notations or definitions. So, $E(X)$, the expected value of X for a discrete random variable is the sum of the product of the values that X takes, multiplied by the probability mass function; so, that sum is called the expected value. So, expected value is also known as the mean or the average.

So, there are three names for it. The word "expected value" is actually not a nice word because it is not a value that you ever expect to get. In fact, there is a good chance that you will never get the expected value. We will see that in the example. So, we use a different word called average or mean. If you think about this, we are asking the question - On average, how many games does the badminton player that we saw earlier, win in each final?

So, that is like asking the expected value or the average. So, let's compute that. From the formula here that we have the previous line, it is X times the probability, which is the same as 0 times the probability that X equals 0, plus 1 times the probability that X equals 1, plus 2 times the probability that X equals 2, that is, $0*P(X=0)+1*P(X=1)+2*P(X=2)$.

So, that's what exactly what is written here. So, if you plug in the number, $0*P(X=0)+1*P(X=1)+2*P(X=2)$; if we do the calculations, you get $0+0.2+1.4$; if you add those up, you get 1.6. So, on average, she wins 1.6 games per final.

So, that's an average. Now, 1.6 is not a number that you would ever see, because she either when 0 games or 1 game or 2 games. So, the expected number of games that she wins is on average, how many games does she win or what's the mean? Okay, let's say she plays a lot of games; we took the average number of wins in a final divided by the total number of games, you will get the number 1.6.

This is a concept that is important. At this point, if you did not follow the computation of the mean, I highly recommend that you go online and read up a little about this. One of the good sites for that is perhaps, Khan Academy or something like that. Go ahead and read about it because it is an important concept. I know I kind of quickly ran through it.

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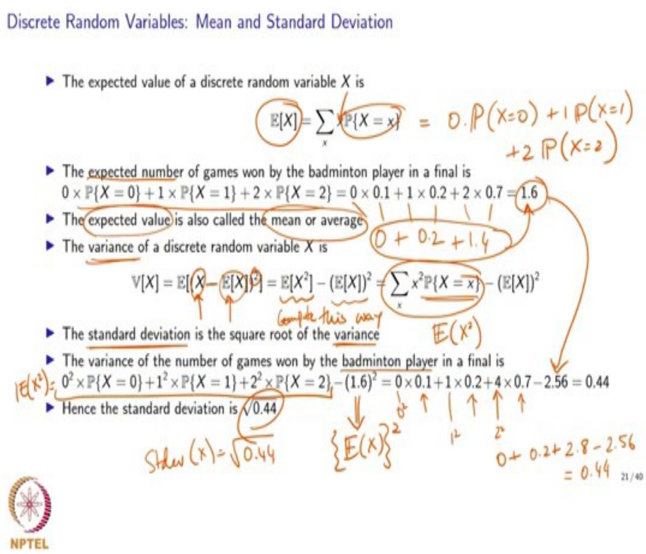

The next concept is the concept of variance. So, variance tells you how much does a random variable stay away from the mean. So, what we do is, we take X, subtract it from the mean and square it. Why do we square it? Well, if we did not square it and just took the expected value, on average, you would be as much to the less than the mean as you would be over, so that value will be 0.

So, either we could take the absolute value or we could square, which the people way before us decided to go ahead to do the square; so, we typically square. To compute, we normally do it like this. We take the expected value of X squared and subtract off the mean squared, $E(X^2) - (E(X))^2$. So, this needs to be computed.

So, how do you do that? Well, you would sum over all values of X, the quantity X squared times the probability that the X equals “little x” - so, this is the probability mass function, and subtract off the expected value squared, which we already computed before. We compute $\sum_x x^2 P(X=x) - (E(X))^2$.

So, the standard deviation is a term that is frequently used. One of the reasons is it has the same units as X; so, we like to take the square root of the variance. So, when we do that, we get the standard deviation. Let’s do a little example. So, if you look at the badminton player that we had in mind a little while ago, it is $0^2 P(X=0) + 1^2 P(X=1) + 2^2 P(X=2)$. Here,

$P(X=0)=0.1, P(X=1)=0.2, P(X=2)=0.7$. So, if you did the calculations, you would get $0+0.2+2.8-2.56$, because $1.6^2=2.56$. Then, you get $3-2.56=0.44$.

So, if you want to get the standard deviation, you take the square root. So, standard deviation of X is this square root of 0.44; the square root of the variance. So, this is another topic that is often misunderstood and I would highly recommend going to a place such as Khan Academy to learn a little bit more about this topic. I would again touch upon the mean and standard deviation in the next topic of continuous random variable. So, if this is not clear, we will see that again. So, I think that's how much I have for this topic and we will stop here.

Thank you.