Decision Making Under Uncertainty
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Lecture - 29
Concluding Remarks and Simpson's Paradox

In this lecture, we first start by making some concluding remarks; nobody starts with a concluding remarks, but this time we do and then we talk about the Simpson's Paradox.

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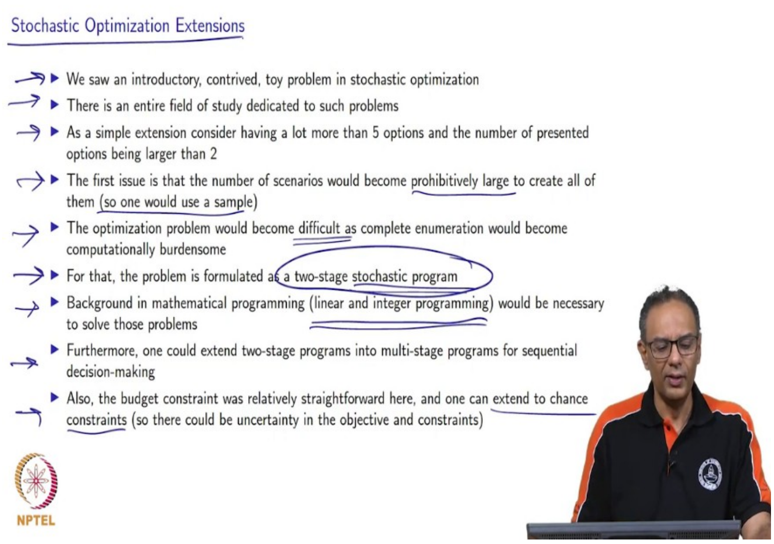

So, when we talk about stochastic optimization, the problem that we saw was a baby problem. We really looked at a very introductory level- a problem that was completely contrived; ok, nobody really has a restaurant like that and we were looking at a toy problem, where we could really explain nicely, but on the other hand you know the real problems are much harder. There is an entire field of study that looks at this two stage stochastic optimization- it is a very hot research area.

As a simple extension, let us think about this right away, I mean why you only had five options. Well, surely no restaurant is going to give you just you know five option, they are going to give tons of options and you may have to display more than two, you might have more constraints than budget. For example, they could say: it is going take this much time to make this, and you could have other constraints. So, the problem itself could be a lot more complicated; for that we need much more significant machinery. There is research that can tell you what machinery you need, but it is a little bit beyond the scope of this course.

So, what are the issues? The first problem is that the number of scenarios becomes really large very quickly. You think about it, just in these in this simple example, where you have this the five options and you show two. Now, the moment you make some of these continuous- not just discrete. Let us say: you have you know something randomly get selected from a distribution, there is looking at an infinite number of scenarios, and what you do? Well, then your best option is to use a sample.

So, what typically people do is to take nice representative sample scenarios, and then run it and compute the averages. Now, average is not the only thing that they can compute, you can look at some risk measures like some type of value at risk or conditional value of risk and things like that. Depending on what you like to do, you could pick an appropriate objective, you could sample and things like that.

And then, there is a second issue, which is that the optimization problem itself would become extremely difficult to do by hand because complete enumeration is hard, then you have to also do the optimization problem. Remember, we were looking at that and said: we did not have much intuitive results such as you know this is how many you need to, and this is when you do option 1 and option 2. Well, just looking at the numbers, it was hard even in the tiny problem. If you had a larger problem, eyeballing is not even an option, you need to do some significant computational exercise to actually go about doing this, even that could be very complicated.

So, the problem is actually solved using what is called a mathematical program or an optimization technique called two-stage stochastic program. There is a lot of literature on that, this one requires some amount of background. The background is in topics like linear and integer programming. If you are already familiar with linear and integer programming, this would be something that you could definitely read up on your own.

I am not going to assume that that is the case for all our students. However, those of you that are familiar with it, I would recommend going and looking up one of the tutorials or reading up one of the fine books written on the topic of optimization especially stochastic optimization. You will find the two-stage stochastic program dealt with in almost every single book there. Interestingly which I found is that there are very few stochastic optimization books, and in fact my presentation here may have been the only one that I know of, where they do not actually present a stochastic program by writing it as a mathematical optimization problem.

So, you know I think most of the people in that area directly assume that that is something that you already come with. So, you would typically find those books harder to read if you are not familiar. I would recommend first familiarizing yourself with linear program and integer programming before going there.

Now, you could also think about having multiple stages of decisions right. We do not have to have one stage of decision, you could think of multiple stages. In fact, we had done some research along with some students on looking at what is called the energy procurement problem. So, let us say you are a utility company- a company that buys energy especially from solar and from wind. Solar energy is getting extremely popular this day and age and this is a nice renewable source. Let us say you want to combine solar and wind energy along with your regular fossil fuel based or nuclear based energy in order to provide your day-to-day consumption.

Turns out there is a lot of uncertainty in solar and wind because we do not know how much wind is going to blow or how much light we are going to have. If you have a cloud cover, then the amount of solar power that gets generated could be reduced. And because of that you have to figure out how best to handle this. Now, the utility companies also are concerned because these other fossil fuel companies need to know how much of electricity you need to buy from them.

So, the problem that we typically solve is to say: a day in advance, it is called the day ahead prices that get revealed, and a day in advance you have to look at your expected demand. And based on that you decide how much to buy each hour and then when the actual amount gets revealed. So, this is a “here and now” decision for tomorrow: how much energy do I want to buy- each of the 24 hours of tomorrow?

And then, once the day rolls in every 2 minutes or 3 minutes or 5 minutes or even 10 minutes, you look at your consumption level and demand level and you could buy more using the real time prices. This is a wonderful stochastic optimization exercise that one can do. In fact, we have some work on that and you are welcome to take a look at that.

There also, you are looking at multistage because now what happens in "here and now" is to make tomorrow's decision, but once you go to tomorrow, every hour or every 15 minutes, you have to make a decision. And you have to do that every 15 minutes like from 1 am to 1.15 AM, 1.15 to 1.30 AM, you make this in a sequential fashion and that is really- what is the motivation behind this topicThe reason we call this as part of sequential decision making is because you have these multiple decisions to be made into the future, not just one stage that we saw in the example we had today.

Now, doing the budget constraint was quite straightforward, but to look at constraints, where there is a probability. So, it is like this- think of the actual dollar cost not being deterministic, and say it should be less than or 60 units with probability 0.95, there is 95 percent of the time you have to be less than it. Well, that also can be applicable in the cost as well.

You are thinking: ok, I want to be 95 percent sure of being less than or equal to my budget, but of course, for that you need something that is random. In our example, we do not have that situation, but you could think of situations, where there would be uncertainty in your values and therefore you have a chance constraint.

So, these are many things that could be incorporated into what is called stochastic program. So, there is a lot of interesting extensions. Now, the next topic I want to talk about a little bit because we dealt a lot with this whole idea of using ratings.

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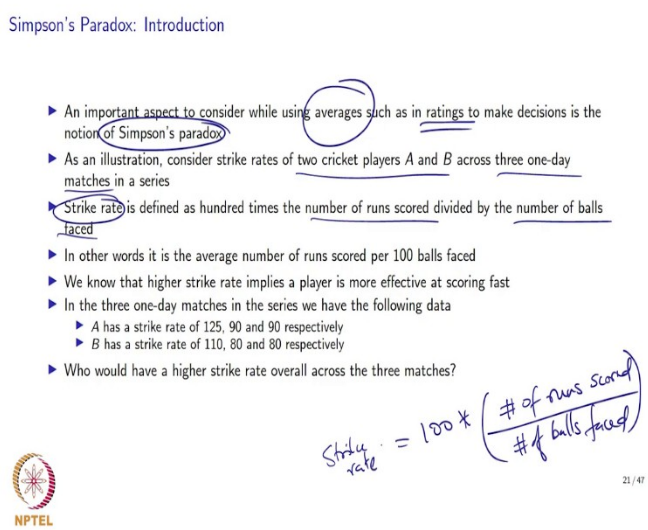

But, there is something to be very careful. We use averages to make these rating decisions. Now, it turns out that we have to be careful about something fascinating called Simpson's paradox because whenever these companies give you ratings, they may not necessarily give you the full picture.

Let us look at this example through a completely different situation such as there are two cricket players: A and B, and there are three one-day matches in a series. So, I would look at a series of one day matches and I am going to illustrate what is going to happen and what is the Simpson's paradox about.

We are going to look at the strike rate of these two players. Now, what is a strike rate? Well, strike rate is basically the number of balls faced in the denominator and the number of runs scored in the numerator. So, the number of runs scored divided by number of balls faced, that thing multiplied by 100 gives you what is called the strike rate.

So, if you had two cricket players: A and B, then you would look at their strike rates as the number of runs they scored in each of the three games divided by the number of balls faced, that number is this strike rate. Now, it is typically defined as 100 times the run score divided by number of balls. In other words, it is also called the average number of runs scored per 100 balls.

So, in some sense the strike rate; the reason I call it a rate is because it is divided by thing is there, but it is also a rating in some sense. So, the rating of a player can be thought of as a way to decide how good a person is. So, for example, the strike rate typically implies that one player is more effective at scoring fast.

So, let us look at this scenario that we have. So, let us say player A has a strike rate of 125, 90 and 90 in the three one-days respectively. The first one day is 125, second one day is 90, and the third one day is 90. Player B, on the other hand has a strike rate of 110, 80 and 80. I am not telling you anything besides the fact that- these are the two strike rates.

Player A has a strike rate of 125 versus 110 for player B; 90 and 80 in the second and the third one days for player A versus player B. Now, I am asking the question: who would have a higher strike rate overall across the three matches? Do not over think in; what is the first thing that comes to mind. Typically the first thing that comes to mind is: we take the three numbers and say A is better than B in all three cases, it must be A.

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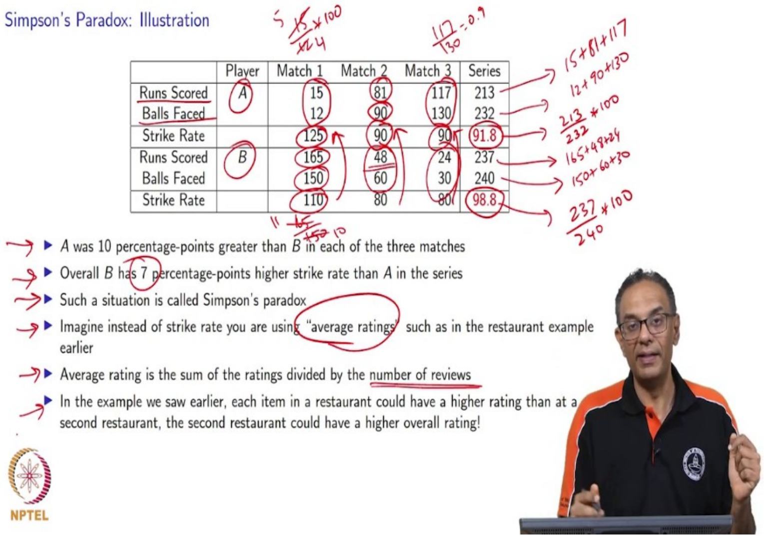

Now, let us look at what actually happened because I did not give you the numerator, I did not give you the denominator, when that gets revealed let us see what happens. Now, let us look at Simpson's paradox. Now, let us see here and look at player A. The top number is a runs scored, the bottom number is the number of balls faced.

In the first match, A scored 15 runs and faced 12 balls. So, therefore, the strike rate is $\frac{15}{12} X\,100 = 1.25\,X\,100$, the strike rate is 125. Likewise, in the second match, the person faced 90 balls and scored 81 runs. So, 81 is 90 percent of 90, so therefore the strike rate is 90. Likewise, in the third match, the person scored 117 runs, so $\frac{117}{130} X\,100 = 90$, this strike rate is 90.

Now, let us see what happened to the second player- player B. Player B scored 165 in 150 balls. So, $\frac{165}{150} X\,100 = 110$, so, the strike rate is 110. Likewise, in the second and the third matches, this is what happened- in the second match, he scored 48 or she scored 48 out of 60 balls. So, $\frac{48}{60} X\,100 = 80$, strike rate is 80. Likewise, in the third match, the player scored 24 in 30 balls, so the strike rate is 80.

Now, if you look at this, you are saying: ok, now what happens to the entire series? So, the entire series, if you look at it, you will see that the first player scored at total of 213. So, 15+81+117=213. Likewise, 12+90+130=232. So, this person's strike rate- this guy is 213/232 = 91.80.

Now, let us see what happens to the player B. Player B had a total of 165+48+24=237. Likewise, that was done in 240 balls because 150+60+90=240. So, if you look at this guy, it is 237 and 240 balls times 100, it gives me 98.8. So, clearly 98.8 is larger than 91.8. Although each of these if you compare, 80 is smaller than 90 and 110 is smaller than 125.

So, A is 10 percentage points better than B in every one of the three matches in terms of these strike rate. In fact, in the first game is more than 10, in fact, it is 15 percentage points higher. However, over the three games, player B has 7 percentage points higher at 98.8 is 7 more than 91.8. So, player B is a better and has a better strike rate throughout this series. This is bizarre because if you look at each match, player A seems to be better than B in terms of strike rate, but overall B seems to be better than player A. This is kind of a situation is what is called Simpson's paradox. So, there is a paradox here.

Now, in the restaurant example that we saw in the previous two lectures, we are using average ratings, but we did not pay attention to how many reviews were there. In fact, many of the good review sites will also tell you how many reviews were there. And you have to also take that into account. You need to take some type of a weighted average, you need to weight by the average number of ratings like here: the number of balls faced is different every single time in this cricket example. Likewise, in the average ratings if we just get the averages, you have to be sure, you know how many times you know those data points are taken.

So, for example, if you look at the average rating, which is your denominator is a number of reviews, and if you are not careful what could happen exactly like in the Simpson's paradox situation, you could be such that one particular restaurant for every single item might be higher rated than another restaurant. But, overall it could be lower, exactly the same situation. Instead match 1, match 2, match 3, it is item 1, item 2, and item 3. Item 1 might be better than restaurant 1 than restaurant 2; item 2 might be better than restaurant 1 to restaurant 2; item 3 might be the same across various customers.

But, if you look at the entire restaurant per say, so like the 9.2 that we had versus 8.9 in restaurant A versus restaurant B, 9.2 might be higher than 8.4. However, each individual item that the restaurant brings might be such that one restaurant might be higher than the other. You have to be careful about something like this because these kind of issues are not always dealt with carefully and if you did not do that, you might actually make the wrong decision. So, these are some other things that you need to take into consideration before you think about solving some of these problems especially when there are ratings involved.

You are better off by getting all the information that you can. Make taking averages by just averaging a bunch of numbers is not a good idea. In fact, a lot of people and a lot of companies just eyeball and take averages thinking that the denominator is roughly the same everywhere. Now, you should always ask for the denominators. And in fact, whenever you go to review, you look at how many reviews were taken, and what was the average rating. That will give you a better idea in order for you to mix and match among various things. I will stop here and we will continue with the next lecture next time.

Thank you.