Decision Making Under Uncertainty
Prof. Natarajan Gautam
Department of Industrial and Systems Engineering
Texas A&M University, USA

Lecture - 28
Two Stage Stochastic Optimization

So, this problem formulation is going to be modeled as a two Stage Stochastic Optimization. There is a formal way of doing this in the literature and you will find several books and probably with tutorials online. However, we keep this fairly straightforward and just give you an idea of what exactly happens in these two stage stochastic optimization.

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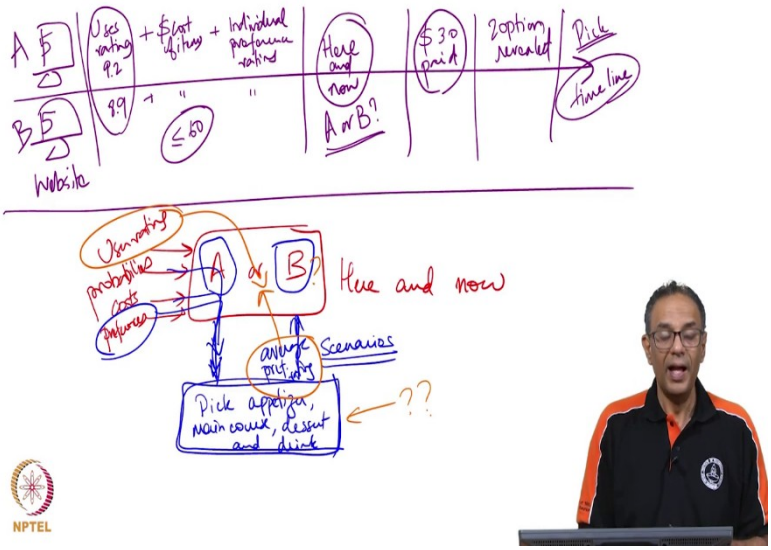

Now, let me just quickly recap the scenario in terms of a timeline. We begin by displaying; this is the first time I am going to draw a timeline. So, we begin by giving the 5 options here. So, you know what are the 5 various options on the website. So, this is the website.

So, the app first goes to the website, it downloads restaurant A and restaurant B's 5 choices and the costs ok: dollar costs. So, it uses ratings of 9.2 here and 8.9 and it also uses the dollar cost of items. In addition, it also uses the individual preference rating for each item, it uses for both. Based on that information and the fact that it knows that the restaurant is going to select: 2 out of the 5 with equal probability. So that means, it could pick 1 and 2, 1 and 3, 1 and 4, 1 and 5 or it could pick 2 and 5, 2 and 4, 2 and 3 or 3 and 4, 3 and 5 or 4 and 5. All

these 10 options with equal probability; that is an information that it assumes and that is what the website also tells you that it is going to select any 2 of those 5 with equal probability.

So, once you go to the website and say: ok. I will like you to go and search restaurant A and restaurant B, it uses the ratings from other users and it also looks at the cost because remember you have a constraint that your total cost must be less than 60 dollars and you also want to look at individual ratings. So, your first decision is "here and now" and decide what you are going to pick: restaurant A or restaurant B. So, that is your first decision. Once you pick a restaurant then you need to decide which menu options. So, this is where you pay the dollar 30. Once you make the decision and then the two options are revealed.

Then you pick the dessert, the appetizer, the main course and the drink and place an order. So, that is what you need to do. So, that is what the timeline is. So, that is the timeline of activities. So, first the website gets revealed and you pick the user rating from a popular site, you incorporate the cost of items, you have a constraint of 60 dollars and then you would look at your individual preference ratings for all the 5 items. And you will come up with some type of a number between 0 and 5 for a preference rating. Then, you use that information to make "here and now" decisions. You also use the information that you are going to select 2 of the 5 with equal probability and you need to make a decision" should I go with A or should I go with B?

Then, you pay the 30 dollar fee. Once you pay the 30 dollars fee, the 2 options are revealed and then you need to select which appetizer, which main course, which dessert and which drink. Now, this is often formulated as a 2 stage stochastic program; I will change the color to red. So, I am going to do the first stage decision, which is called "here and now" and I am going to decide whether it is going to be restaurant A or B. The inputs to that are like I said: the user rating, the probabilities- for us they are all equally likely, but in general, you need to know the characteristics of the random quantities; and then the costs, and then the preferences.

So, all these go into the "here and now" decision. Not just that, there is another decision that goes in here and that is if you picked A, then I would say: ok, now we make the second set of decision is to pick the appetizer and you select the main course, dessert and drink among the ones that get revealed. So, this one is a random event. Now, there is nothing random in to

pick A or B. But, then once you pick an appetizer, now that information is also returned to B. So, there are multiple scenarios.

So, once you are given whether it is A or whether it is B, you create the various scenarios and in each scenario, you are going to pick an appetizer, main course, a dessert and drink and it kind of gives back the average preference rating. So, it uses all the preferences, pushes it out here and it also uses the probabilities and the cost, it pushes all that information down to this level, it does not push the user rating because it does not care about user rating because you already made that "here and now" decision of selecting A or B. So, once you have selected that, you push down the probabilities or all the possible scenarios and also their probabilities.

In our case they are all equally likely, you push down the costs and you push down the preferences to the second stage. In the second stage, given all these, we create all the possible scenarios and then it computes the average preference rating and sends it back here. Now, here you go back to the "here and now" decision, you look at the user rating and the average preference rating.

So, I am going to use a slightly different color and say: well, I am going to use the user rating and the average preference rating in combination to make that "here and now" decision. Once we make the decision depending on which one gets revealed, we can go back to see what was done here, what the choices was here and that is going to be how we are going to proceed.

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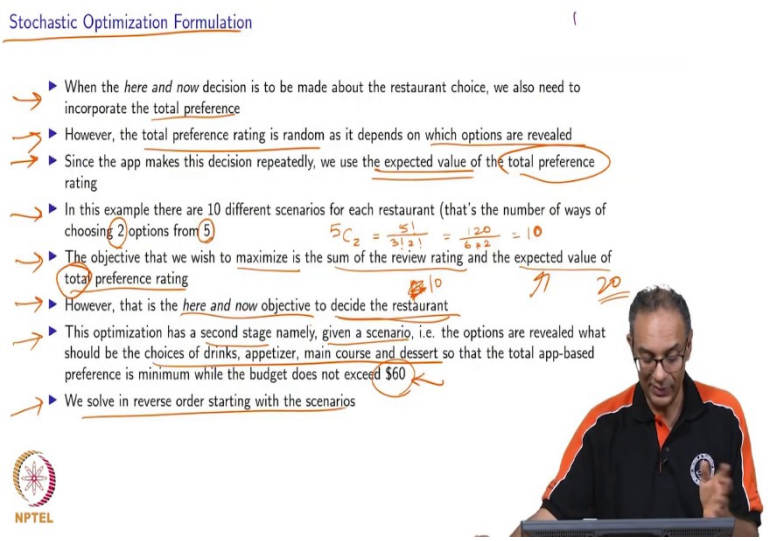

So, now let us see, how to formulate this problem- stochastic optimization formulation. So, we will incorporate the total preference into making this decision because we cannot just decide based on only the user rating. Remember, last time you saw: if you only took the user rating, where we are probably going to pick restaurant A, we also want to get an idea of the preference ratings. But, the preference rating is uncertain because we do not know, which option is going to get revealed or which 2 options are going to get revealed.

So, that is what I am saying here- the preference rating is random and depends on which options are revealed. Now, pretend like you have this app and you do use this everywhere you go. So, in some sense your preference ratings expected value seems to be a reasonable choice. So, you are outsourcing your decision to your app to figure out what to do. So, let us say our little stochastic program there will crunch some numbers and tell you: hey! here is the expected value, this is what I would expect as my total preference and it keeps doing this over and over again.

Now, I recall that there are 10 scenarios, why do I know there are 10 scenarios? Well, I have 5 options from which I choose 2. So, 5 choose 2 or 5_{C_2} and that is $\frac{5!}{3!X2!}=\frac{120}{6X2}=10$.

So, that is the reason we have 10 choices. So, let me just tell you what the 10 choices are: Options 1 and 2, 1 and 3, 1 and 4, 1 and 5, 2 and 3, 2 and 4, 2 and 5, 3 and 4, 3 and 5 or 4 and 5, these are your 10 choices. So, these are a 10 different scenarios that could occur. Now, what you want to do is you want to maximize the sum of the review ratings and the expected value of the total preference rating. Now, why should you pick the sum of the review ratings and the expected value of the total preference, by a total preference I mean the sum of the ratings, the expected value of the appetizer rating, the main course rating, the drink rating and the dessert rating.

So, you are going to get a maximum of 20 points there, this would be out of 20, this would be out of 10. So, you get a total of out of 30. Now, you could say: why should I maximize the sum of those two, I could do something different. I could possibly do 2 times this number and this 20. So, that they are both equal. I could do 4 times this and that.

So, that depends on now your personal preference and I am going to pick just the sum of those two, but from a personal standpoint you could think of something like this. If you are someone who says: well, I really care about what others think of the restaurant and I am

going to give a lot more weight to that, sure by all means. On the other hand, you might think: well, I know what I like and I do not really care what the other people thinks, I might give a lot more weight to this guy. So, that is completely up to you. Now, turns out that, in the "here and now" decision that we talked about, we have to decide the restaurant, but for that we are going to incorporate the expected value of the preference rating.

Now, there is also a second stage like I was telling you earlier given a scenario. So, like I said let us say, I do say 2 and 5 options are revealed for restaurant A, then you need to decide which of these choices to make. So, there are two stages, first stage you decide which restaurant, second stage you decide which of the choices you want to select. Now, there is also a constraint, you have to be sure that, the budget does not exceed 60 dollars. Remember that the 30 dollars you pay is counted towards the 60. So, you really do not have to worry about the 30. The 30 is there just to tell you that you have to make this "here and now" decision.

If that was not there, let us say it was 0, what we would do then is: we would try both restaurants and see what is available for today and then pick the one that is better. So, then it does not come in the same framework of a 2 stage stochastic optimization. There you are revealing, then after that it is not really much of a deal, it is a much simpler problem, if everything gets revealed. So, the 30 dollars is there just to illustrate that: well, there is an uncertainty there and how to make a decision and how do I go about doing that.

Now, when you solve the problem, you will actually go in reverse order because we do need the scenarios and we do need the average cost, in order to be able to calculate. So, we will actually go in the reverse starting the scenario.

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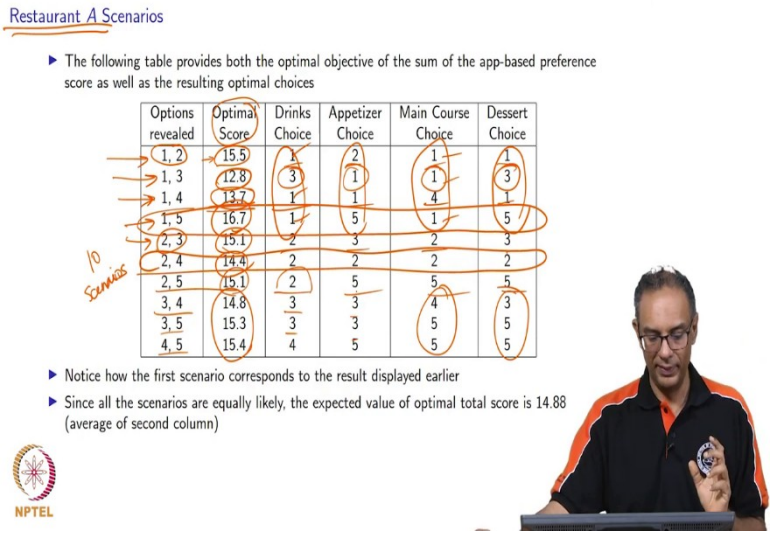

Now, let us see what all the 10 scenarios. We enumerated this earlier. So, we are now talking about restaurant A alone and not restaurant B and if you look at the optimal objective function, now what is that? This is the optimal score, this value is the 4 numbers together. So, for example, if options 1 and 2 are revealed, now this is what we had before, remember we also had this 15.5, I am going to go back and show you that.

So, this is restaurant A, if you look at restaurant A and let us say options 1 and 2 are revealed, right here 1 and 2 and we did the calculation. We found that this one, first time this happened, we had exceeded our budget. So, we went back resolved the problem. Then, we said 15.5 and my budget is within the budget and we are going to go ahead with this costing us 15.5. So, if this scenario is what gets revealed, then we are going to be adding 15.5 points for our preferences. So, we go back to where we were, we go back here and say, if options 1 and 2 are revealed then it will cost us 15.5 and then our choices is to pick the first drink, second appetizer, first main course, first dessert. So, only the appetizer is second, if you go back here, that is basically what we do right. First drink, second appetizer, first main course, first dessert, the only thing that we do second is for the appetizer.

So, others all from option 1. So, that is what we are doing right here. Similar fashion, what if scenarios 1 and 3 are revealed, we could do a similar calculation and what happens is: the total cost is then going to be equal to 12.8. Now, similarly if options 1 and 4 are revealed, you can do a quick calculation. So, notice that you in our case, what I ended up doing is wrote a

little octave programs, the octave program will actually look at the various options and pick the one that is minimum among all the options.

Now, if you had larger than this, you could write a mathematical program to actually do this in a careful manner. So, here if you look between 1 and 3: from option 3, I will pick the drinks and from option 1, I will pick the appetizer and the main course. And from option 3, I will pick the dessert. Likewise, if I had 1 and 4 options revealed, instead of 1 and 2 or 1 and 3 then, my optimal score is this. So, if you look at it there you will pick options 1, 1, 4 and 1. So, somewhat like in the first case except now this time we are picking the main courses choice.

So, notice how it is a little bit different, although 1 is common in all of them and not in all cases, you are actually picking 1 even in the main course choice, in appetizer or in the drinks, you do not always pick 1 even though it gets revealed. That is because, you are trying to do 2 things, you are trying to maximize your optimal score and you are also trying to be within your 60 dollar budget.

So, if you are doing both, it is likely that the same choice does not show up all the time. So, if you look at it, it is not only that, let me just go back a little bit if you look at this, it is not only that option 1 has the best- the costs of going all over the place and the ratings are also all over the place. So, for example, in the main course, you should have picked it all the time because it is the best among the 4. You should have also almost always pick the drink because it is the best. Now, if you look at it, the drinks and main courses more often than not- if you see 3 out of the 4, you do pick it as the main one. Only in 2 other cases you do not, which kind of makes sense. Now, if you look at the options without a: 1. So, let us say: 2 and 3 get revealed. Similarly, you get a number 15.1 and then you would use these 4 as your 4 choices.

Now, interestingly in the next one, look at this if you get options 2 and 4, your total score is 14.4 and everything you pick is option 2. So, let us go back and see what is going on between 2 and 4. So, if you are back here between 2 and 4. So, this is 4 and this is 2. So back here 2 and 4, you are always picking 2 more than 4. Now, notice that in case of the appetizer: 2 is better than 4, in case the dessert 2 is better than 4, but when they are better, they are significantly better. So, you are clearly picking those.

However, if you look at the main course: 4 is better than 2 and the drink as well 4 is better than 2, but then if you notice the betterment is only slight compared to what it is in the

dessert, in the appetizer and you know cost wise obviously if you went that way, you would probably incur a higher cost because this guy is more expensive and did I pick the right ones? Yeah, so I am sorry- so, here there is a huge reduction, the huge reduction in cost. Not so much here, but those things kind of add up. So, therefore, you are really picking option 2 although you would have picked 2, 4, 2, 4 if budget was not a constraint.

So, this is a constrained type of situation and you have to be a little bit careful, when you make a choices. Now in 2, 5 you notice it is 15.1 and then your choices are 2, 5, 5 and 5. So now, the 2 which is so lucrative before is now all of a sudden picked only once, the 5 seems to be doing way better. So, really depends on what options you get, 3 and 4, 3 and 5, 4 and 5. Likewise, I have these as your costs. So, essentially your best bet is if 1 and 5 show up- if 1 and 5 show up, your benefit is as high as 16.7, that is probably the best one so far.

However, if you had the other choices, it is kind of going like this and if you look at these numbers again you do not really see a common pattern. However, these are mainly because of two reasons like I said: one is you do have a constraint on your cost and that really pushes your decisions one way or the other and secondly you also have to make sure that your preferences are appropriate.

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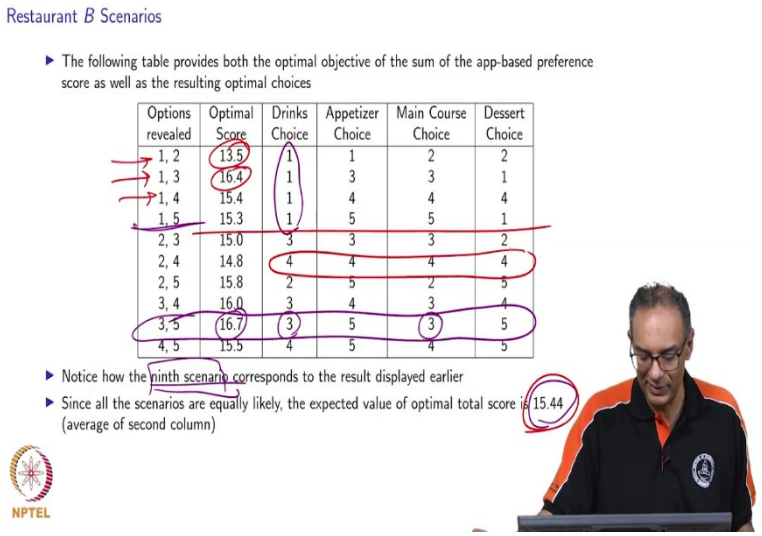

Now, let us look at the same situation with restaurant B. We are going to look at what happens with restaurant B. Now, let us look at the ninth scenario and I do want to say one other thing that I forgot in restaurant A. So, if I took the average- now all these are equally

likely. So, you are equally likely to get any of these optimal scores and then I take their average- their mean.

Well, why do we take the average that is because they are all equally likely- each of them has a probability of one-tenths, you could have formally done: $15.5\,X\,\frac{1}{10}+12.8\,X\,\frac{1}{10}+\ldots$, like we normally do to compute the expected value. But, notice that that is exactly the same as taking the averages. So, you do $\frac{15.5+12.5+\ldots.+15.4}{10}$; you are doing exactly the same thing. I just wanted to remind, I forgot to mention that earlier, but you could actually obtain the average score of 14.88.

Likewise, in the restaurant B situation, the average score is 15.44 and the ninth scenario- so that is this guy: 3 and 5 is what we saw earlier and it has an optimal score of 16.7 coincidently; actually these numbers are completely made up- it is the exact same 16.7 that we had here as our best score. So, the best score actually is no different between the 2 cases which is somewhat interesting. Now, let us say we had options 3 and 5 revealed then we saw earlier that we will pick 3 as our drink's choice and main course and dessert and appetizer of 5. So, let us just go and confirm that that is the case. So, drinks and main course from option 3, dessert and appetizer from option 5. In fact, this would even give us 7 dollars to spare, but it would make our ratings extremely high.

So, you would make these choices. Here, if you notice again, we do not see any major patterns like for example if you look at it I do want to point that out: you would always pick in any case drinks from choice 1. Yeah! as long as there is a 1, none of the other choices has a 1. So, let us go and investigate how the first choice of drinks and restaurant B actually looks like.

If you see here, turns out that the first drink choice is actually the highest of the lot and it is also reasonably inexpensive. So, it kind of makes sense that you are picking option 1. However, it is not that obvious because the other drink choices are not terribly cheaper- I mean for example, but terribly weaker.

So, for example, this drink is only slightly worse- this 1 is only slightly worse, this guy is also the same 8 dollar, so the fact that you know then it is low enough and if you look at this guy. So, it is not obvious, it is not intuitive or obvious that you should have always picked choice

1 because if you look at it there are some cheaper choices with almost the same rating, as our original choice. So, it is not clear why it actually selected option 1 every single time for drink.

So, these are interesting things that do not get revealed unless you do a thorough optimization analysis. That is why many times we prefer not to eyeball and let us go for this type of a deal. It is very important to run through an optimization routine in order for you to select what is the best choice.

So, similarly here as well, we can compute these assuming that these options are revealed, all of them are equally likely with probability one tenths. And therefore, the total average score is 15.44. I went a little fast explaining this since it is exactly the same as restaurant A, I did not really take a whole lot of time trying to explain each and every line, but I think I would highly recommend that you go ahead and look at this situation; you always for all the choices between 2 and 4, 4 seems to dominate 2. So, let us take another look at that, I mean it is useful to look for these patterns and see what is going on between 2 and 4, let us take a look between 2 and 4. So, mark 2 is here and 4. Now, notice that, 2 is better than 4 in all except the drink.

In the drink alone, 4 is better. However, 4 is so much cheaper than the drink 2 that it is possible that we had a problem with the dollar cost for the drink and therefore I mean reaching the 60 dollar limit and it is possible that it picked this guy, this guy, this guy and this guy. So, that is probably what happened.

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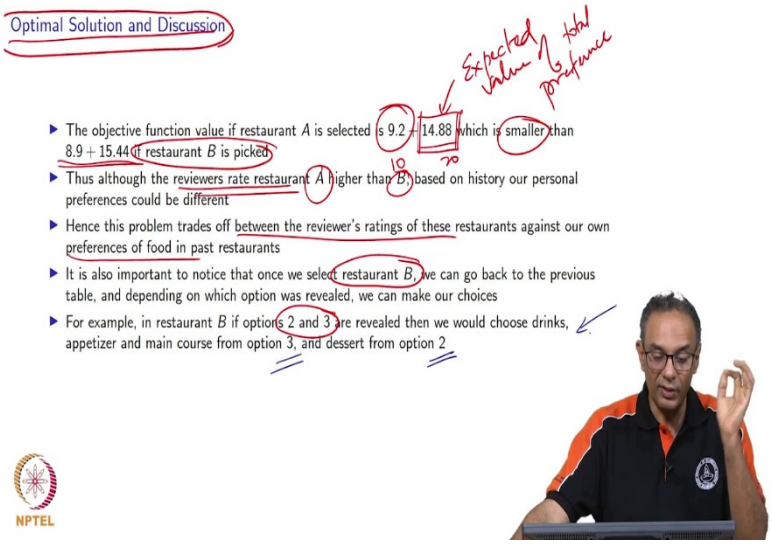

So now, we move on and see now how are we going to solve this problem. So, we have 2 choices, this is the expected value of total preference and the maximum number this could be is 20. So, out of 20 on average, I am going to get 14.88 and this is the restaurant choice. So, this maximum is 10, this maximum is 20 and this number 9.2 plus 14.88, which is the average. So, this is where we are actually solving. So, this is where, we go in this situation, this information gets input. This information gets input and we are saying: ok, which is a better choice. This is where we are making the "here and now" decision. Only now we are actually solving the problem.

So, now we are looking at this and saying between 8.9 plus 15.44, versus 9.2 plus 14. 88: which is better? Turns out that A is smaller and you should actually select restaurant B. So, that is what this tells you. If you select restaurant B, on average you are going to be doing better. Now, it turns out that although the reviewers rating is better for restaurant A than restaurant B. Based on your preferences, you could be better off with restaurant B.

So, now, it is a question of- we are really trading off between your preferences versus the restaurant. So, your preference means- what you like among item, like for example if you like the ice cream, your preference in general versus the restaurants preference, you are kind of taking a combination of the two. So, it really trades off between the reviewers ratings against our own preferences of food based on past restaurant behavior.

Now, once we select restaurant B, this is important. We go back to this table, so we actually have, we know what to do. So, we have already made this calculation. So, let us say we are in restaurant B and I believe I have that example. Let us say, options 2 and 3 get revealed then, then what will we do? So now, let us say: you actually picked restaurant B and options 2 and 3 got revealed. So, that we are here.

So, we let us say options 2 and 3 get revealed then what this one says is: you will have to pick this 3, you will have to pick except for the dessert, where you pick from option 2, all the others you pick from option 3. So, except for dessert that you pick from option 2, all the others you pick from option 3 and that is the optimal thing to do. So, you do not have to recalculate it, you already have made this calculation, which is wonderful.

So, in the second stage, you already have the solution because you have already created that scenario and many times in stochastic programming this is a luxury that we have. So, once we get to what gets revealed- once we get to the situation, we may have to make a decision in

the second stage, we have already done that calculation. So, you just go take a look and make this decision. So, one more time just summarizing by saying: In a 2 stage optimization what happens is- in the first stage you make “here and now” decisions. For that, you get inputs from the costs in terms of what you have to make and “here and now” decisions as soon as the average cost; you could do something fancier than the average by the way.

From the second stage and say: here is the average now, which decision I make? Once you make that “here and now” decision, now when the uncertainty is revealed, you look at- what is the best choice under those scenarios and you make that decision in the second stage. So, this brings us to the end. I do have some concluding remarks that I will make about this and in the next lecture.

Thank you.