Decision Making Under Uncertainty
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Lecture – 24
Route Planning

The next lecture is route planning. I often call this Route Planning, please bear with me.

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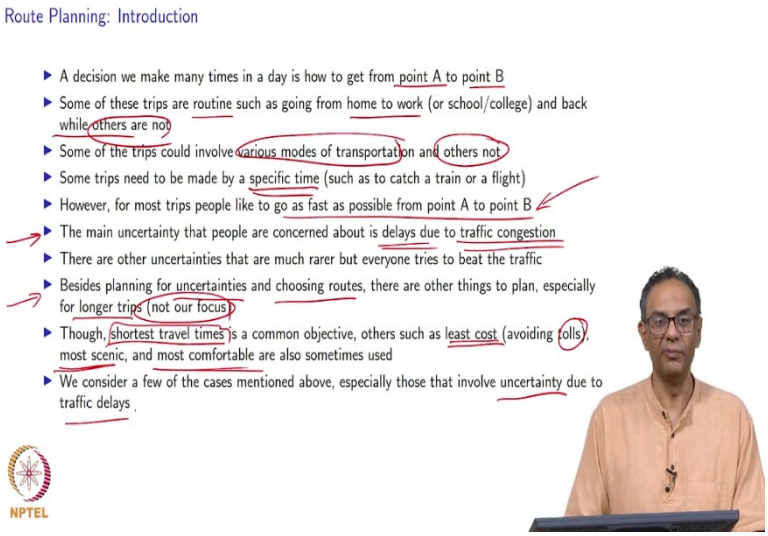

So, route planning- let us do a quick introduction first. Many times in day, we think about how to go from point A to point B? What is the best way to go, what is the fastest way to go, what is a way to go that is exciting? Sometimes these are very routine trips like going from home to either your school or college or work and coming back in the similar.

However, there are other times you are going to places that you have not been before. Sometimes, you go in various modes of transportation, sometimes you do not. Lot of you know travel times and routes that we are thinking about. Sometimes, you need to make a trip before a specified time. Like if you have to catch a plane or go in a train, there is a time by which you have to reach, this is another important aspect to consider. For the most part unless there are some strange exceptions, most people like to go as fast as possible from point A to point B. I think that this is in general a very common objective.

But, the main uncertainty that we are facing or we will face is delays because of traffic congestion. There are other types of uncertainties, but they are much rarer. So, we will just talk about beating the traffic for the most part, there are some other uncertainties in travel.

Now, there are other items that we should think about especially for longer trips. Let us say: you have to figure out what is the best way to go from point A to point B? Should I take a train or should take a plane and so on? So, there are other things besides uncertainties and choosing routes, but this is not going to be our focus in this course.

Now, the objective is going to be shortest travel time. Now, there are other objectives such as least cost. So, there are many routes with perhaps some type of toll. So, you want to avoid those tolls. So, that could be there. You might want to maximize the scenic: this case you want to go into a route that is scenic. You also want to go sometimes in routes that are most comfortable- there are fewer potholes and things like that.

So, in general however, the shortest travel times is a common objective; you could attain the others by all of them as trading all of them as some type of a cost. It is not that difficult, we will stick with the shortest travel time as the objective. Now, we will look at a few of these cases and we will go over in some amount of detail especially when there is uncertainty due to traffic delays and that is going to be the focus of this lecture.

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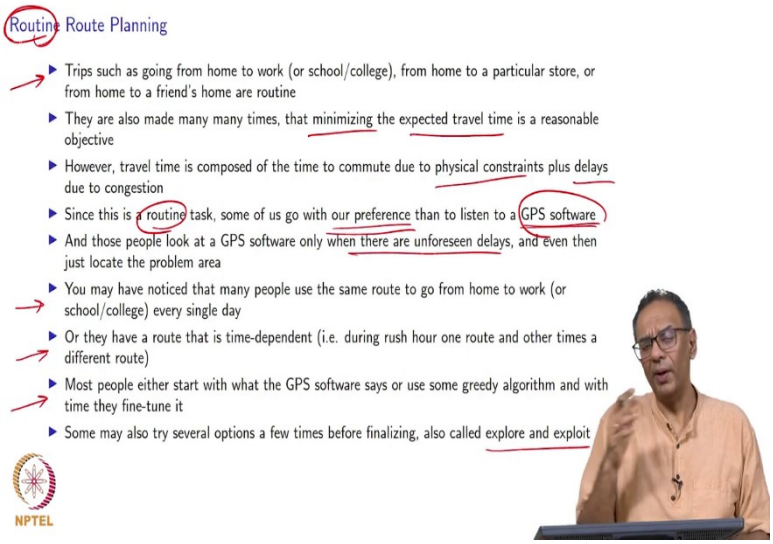

Now, one thing that we should remember is routine routes ok- that means, things that you go often, one point to the other point and back like home to work or home to a particular store that you buy stuff usually or from your house to your friend's house. These things are what is called routine.

How do you decide what is the path you want to take to go from your house to let us say your school or college? Well, what you normally do is since you make this over and over and over again, you try to find a path that minimizes the expected travel time and that is a very reasonable objective that is because you do this in a routine fashion over and over and over again. So, on average you need to be doing well and selecting the path that minimizes the average travel time is a good objective.

However, there could be delays and there could be other physical constraints that require you to consider other objectives and we will look at that some time towards the latter part of this lecture. Now because this is a routine task, many of us go with our own preferences than listen to a GPS software. I do not know what you guys, I for sure do that. I never look at my GPS, when I have to go from home to work and back. Unless of course, when there is some unforeseen delays, there is suddenly a traffic congestion, I pull up my Google maps and say: "oh there is a congestion, I will go through another route". Other than that, we almost always use our own preferences, we typically do not use a GPS software.

However, we do use GPS software sometimes such as when they are unforeseen delays like I was telling you. Even then we just look at the problem area and then we solve it for ourselves. Now, you may feel that there are many people who use the same route every single day, they do not really change. Now, that is a fairly common situation that is you know I have the same route and that is quite called routine- I just keep going there over and over in the same way.

Sometimes we have two plans: plan A, during certain times, a rush hour if I do not go during rush hour, I might go in a different route. You could have something like that. So, you could have different strategies. I use one strategy when it is rush hour, one strategy not, but other than that basically we try and go by the same route.

Now, another common thing to the lot of people is when you are new to a job or new to the town, you first start with what the GPS software tells you or you use some type of greedy algorithm. A greedy algorithm would be- basically go in the direction from home to work

whatever road leads me there. And then you start fine tuning it a little bit, tweaking it here tweaking it there; somebody said hey you have never went this way, that is a wonderful road you should try that, you try that, you try this that and finally you decide.

Now, this is the second time you are using the word "explore" and "exploit". So, you explore some options especially when you start a new; when you are looking at a routine route planning, you first explore several options. You either look at a map or look at GPS in the beginning or you try out different things and then you say: "ok, now I do like this". So, I will talk about GPS next a little bit later, but the focus of this particular slide is in routine planning, what we all normally do is this notion of we explore first and then we exploit. That means, we try all different things and then we will say: "I like this and I will keep doing this exact same route over and over again". Now, we will talk about exploration and exploitation in the next lecture.

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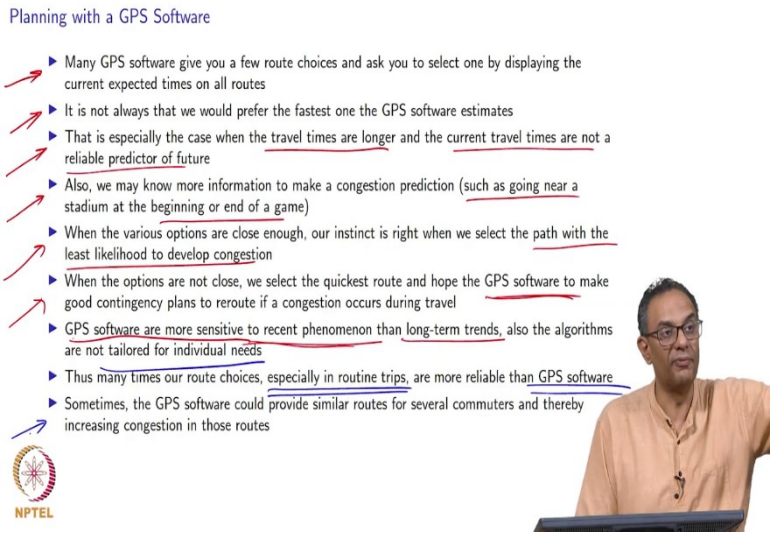

However, let us continue on talking about what if you have a GPS software and how should you go about planning? So, many GPS software, Google maps in particular will give you a few route choices and tell you pick one. So, it will probably give you 3 route choices and then it will tell you: "go ahead and select one of them". Typically its first choice will be the one that is shortest time.

Right now if you were to travel, this is the shortest time. But, you may not necessarily prefer that and that is why they give you your second and the third choice. And there are reasons

and I am going to go over that in a little bit because sometimes you do not necessarily prefer the GPS software. That is especially when the travel time are somewhat long and the current predictions are not rest necessary reliable. So, let me give you an example.

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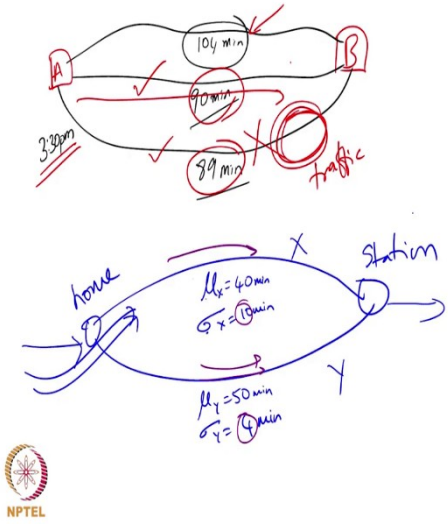

So, let us say you are here: this is point A, this is point B. Then Google maps gives you 3 routes: one route like this, one like this and one like this. And then let us say it is right now 3:30 PM and you need to reach here and you do know if you went through this route, it will tell you this will be let us say 90 minutes, this route will let us say be 104 minutes and this route may be 89 minutes.

Now, you might think 104 is long enough, I do not want to take that. But, between 89 and 90 there is no big difference. Although Google map picks this as the choice, you may think: "well after about 1 hour I expect huge traffic here". For example, let us say it is 3:30, by about 4:30, rush hour would start and I expect a big traffic jam here and I'm saying I do not want this route, I would rather go with this with a minute slower. But, I know by the time I show up somewhere here, I would be in much better shape. And that is kind of what we mean here when we say when the travel times are longer and the current travel times are not a reliable predictor of the future, we end up not necessarily using the fastest GPS route.

Now, you also may know something about more information. Like you know it is going to be rush hour or you might know that: well there might be a location, where there is a like a sporting event like end of a match and when match is going to end and then you do not want

to be driving there because everybody is going to come out of the stadium. That place is going to get really crowded. You have that information, it is not necessarily that every single GPS software has that.

Now, when we are in a situation where you have options like this, where things are very close, you are really better off using your intuition. Your intuition generally is right, when you pick a path which is less likely to develop congestion. So, that is a good decision from your standpoint. Now, when the options are not close like you probably ditch this idea of doing the 104, the options are not very close then you are thinking: “well, we will just pick the quickest one and then you know the GPS software will figure out, reroute and things like that if there is congestion along the way”.

Now, I do want to say a few things: GPS softwares are not terribly sensitive to long term trends because they are really based on what is the condition in real time. There is another problem and that problem is that these softwares are not tailored for individual needs. Now, you have to pay careful attention to think about this. The Google maps software has been developed to so that everybody can use the software.

Now, you know better how to go from point A to point B in your favorite route. Now, Google maps is not going to have one route for each of these 7 or 8 billion people that are on this earth. They are not going to do that, they are just going to have an algorithm that will do it. So, most of the time, you are probably better off using your own judgment especially when you have to make routine trips than using a GPS software.

However, sometimes a GPS software could provide some several options that you could use and make a good choice and the other interesting point that comes here is that sometimes the GPS software could say. And I am not saying that the Google maps software does that, I do not think it does, But, sometimes the GPS software- if it is not sensitive enough, it tells everybody to use a particular route and everybody follows that, that route becomes congested. So, let us think about this.

Now, let us say it gives you an option. There are two ways to go: this and this, and then for everybody who is coming from here and going here it says: this one is 45 minutes, this one is 60 minutes, and everybody goes this way. Soon enough, this will become 45 and that will become 60. That is another issue that you need to think about, when you are especially developing these types of software for sure, but as a user you have to be careful about that.

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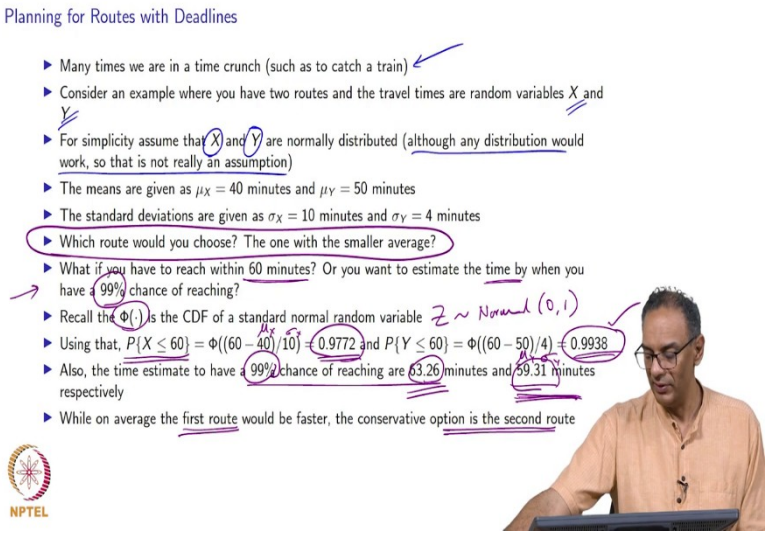

Now, I want to close this up with this minor point about planning routes with deadlines. Many times we are trying to get to a place before a certain time. Like say for example, you have a train to catch and you want to be in the station at a reasonable time. But, you have two routes and the travel times in the two routes are X and Y, these two are random variables. X is a random variable that is on one route and Y is a random variable that is in another route.

So, think of it like this. So, this is route X and this is route Y. So, you can ignore these other numbers here for now. So, let us not worry about the 60 and 45, we will just pretend like we have two routes, we call them X and Y. So, there are two ways of going from home to the station. So, that is your situation.

Now, we are going to assume for simplicity that X and Y are normally distributed. Now, there is no reason to use normal, we could actually work with any distribution. So, this is really not an assumption in the truest sense of the word. In the previous example, we needed that assumption in the safety stock example because we were adding up a few normal random variables and summation of normal are also normal, when that is not the case you do not need their assumption

So, now we are going to assume the following two routes, the mean is 40 minutes in one and 50 minutes on the other. So, the mean μ_X is 40 and μ_Y is 50 minutes. You are also given that the standard deviations are 10 minutes and 4 minutes. So σ_X is 10 minutes σ_Y is 4 minutes.

So, the question is which route would you go? Clearly, the route on the top is 10 minutes faster on average, but the route on the bottom has a smaller standard deviation. So, we want to see: which is a better route to take? So, which route would you use? The one with a smaller average which is X, which is what Google maps or anything like that will suggest or you want to pick the other one- the Y route?

Now, remember you are going to a train station. So, you would like to reach within 60 minutes. So, let us say that is your objective. You want to show up within 60 minutes or you want to pick the route so that you want to obtain the time by which you have a 99 percent chance of reaching. So, let us compute that, I am going to show that to you in this slide and then again after that show it to you in the octave software.

So, remember once more that this $\phi(X)$ is the CDF of the standard normal. This is the Z which is normal with mean = 0 and variance = 1. If we use that, we want to find the probability that X is less than or equal to 60, the usual thing is to subtract it from the mean. So, this is μ_X and this is σ_X, this is this is μ_Y and this is σ_Y. So, if you subtract it from the mean divided by the standard deviation and you compute the $\phi(X)$ value for that, it comes out to be 0.9772, likewise this guy is 0.9938.

Although the second route is actually slower on average, there is a high chance of reaching before 1 hour. So, if you want to reach before 1 hour, this is the better route to go. Now, we could also compute the time when you have a 99 percent chance of reaching and that is 63.26 and 59.31. Now, I do want to get these two numbers and show it to you on octave. So, 40, 10, 50 and 40.

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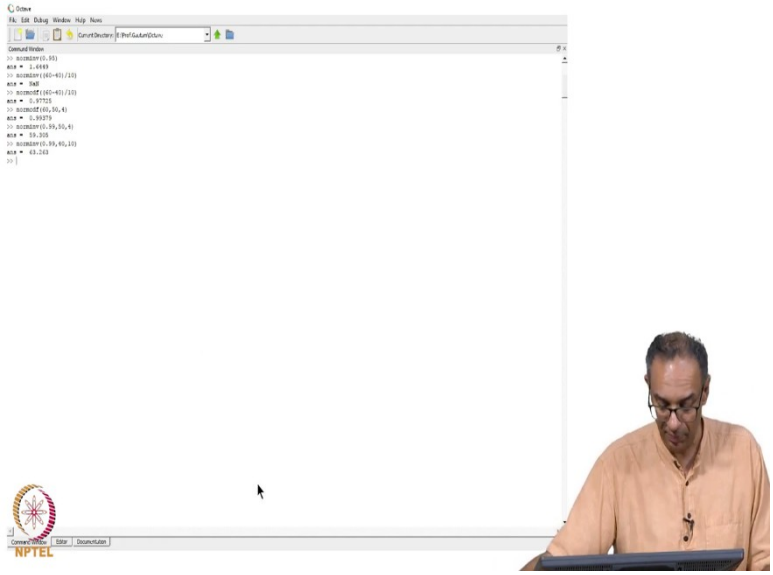

So, I go here *normcdf* , we want the probability. We do not want the inverse of the normal. I am sorry! That is why I wondered it is not a number, we want the inverse. So, its 0.97725 which is exactly what we have here 9772 and the other one is 50. Now, you do not have the *normcdf*. You can do the usual *normcdf* of 60, 50, 40. Now this is the not the standard normal, but this is the regular normal with mean 50 and standard deviation 4 and that is 0.9938.

If you look at it here it is 0.9938. Now, if you want to get the 99th percent probability that is the one that I should have used *norminv*. So, if I do *norminv,* so this is the second one of 0.99, then I would get 59. So, this is a second one, this one- the 99th percentile. So, there if you want to be 99 percent sure of reaching, the time that you reach there is 59.31 minutes from now. Whereas, the other case, let us do that in octave. The other case is 40 and 10 and the answer is 63.26, if we look at it here just 63.26.

So, this probability of reaching within 63.26 is 99 percent. In other words if you want to be 99 percent sure of reaching, then the time that you will reach is well after 60 minutes, this is before 60 minutes. So, if you really have to reach before 60 minutes, I would pick route Y. Notice that the first route is 10 minutes faster than the second, but the conservative option is by going with the second route. I will stop here and continue in a little bit.

Thank you.