**Decision Making Under Uncertainty**
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**Lecture - 18**
**Project Network and Analysis**

The last item in this topic is project network and analysis.

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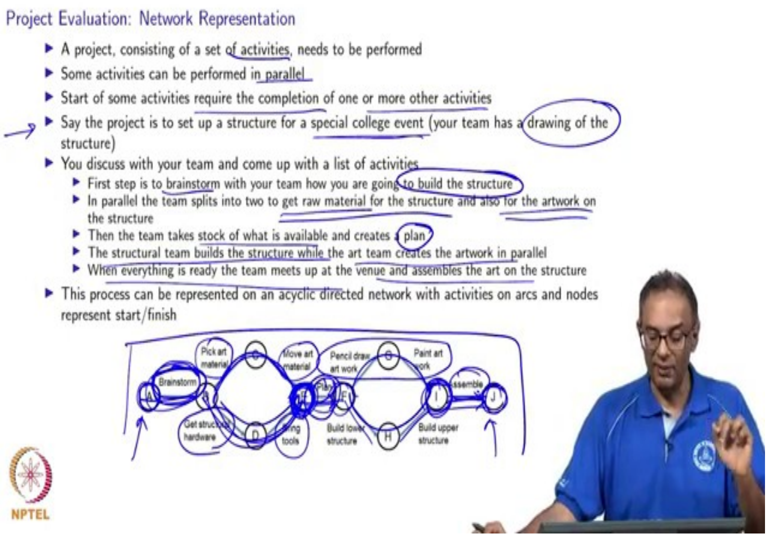


Here, we have a network representation of a project. A project typically consists of several steps; some steps or activities can be performed in series; some can be performed in parallel. So, this project itself has a bunch of activities; some of them can be performed in parallel whereas some of them need to be completed before starting other activities. I will try to explain that with an example in a short while.

So, as an example, let us say you have a college event, a cultural event for example, and you and a bunch of your friends are there who have a little drawing of the structure that you need to make for that you need to build. So, you need to build a structure for an event in your college and you and your team sit down and write down all the activities. So, what do you need to do first? Well, you first need to brainstorm with your team and figure out how am I going to build this target. How are we going to build this structure as a team?

Then what happens is the team splits into two groups. One group goes and gets the raw material for the structure and another group goes and works on the artwork. So, the structure has a physical form. So, that has to be engineered first and on top of that, you need to put some art. So, the art raw material and the structural raw material can be done in parallel and once they are all made; so, what happens is this can be done in parallel and then once both of them are ready, we can sit down and plan.

So, we take stock of what is available because many times you can plan only if we after figuring out what we have with us. So, you go and pick art material, you pick structural material and put everything together and sit down and come back for another round of brainstorming; you try and plan what to do. The structural team perhaps will build a structure again. That can happen in parallel while the art team creates art work again in parallel and then puts it on the structure.

Now, once everything is ready, we meet together, go up at the venue and assemble the art on the structure. So, create the structure, assemble it and then create the art and then put it on top of the structure; put it all together and you will be done. Now, everything that I just said can be put down in a network form. So, you start with node A; so, these nodes and letters in the nodes do not mean much. They just denote when an activity starts and when an activity ends. There are other ways of representing project activities; we will not go into that.

So, this is a network and it goes in one direction. You start at A and end at J and in that process what happens is we go through multiple arcs and each art tells you what activities there are. So, the first activity is to brainstorm which all the team members have to be there, then the art team goes and picks art material and then they would move the art material down to the place where they are going to discuss. Similarly, the structural hardware and the tools that are needed are brought together. Now, we have all the materials E. So, once we have everything with us, we have to wait when whichever happens last, we wait for that to happen; then we sit down and plan. So, while at the end of the planning process, we are saying we have two pieces, the art guys can go ahead and start to pencil draw the art, paint the art and then it will be done. Likewise, the structural folks can build the lower structure and then they build the upper structure, the two parts of the structure, and then they are done. Now, when once everything is done, they bring it to the venue and then they take time to assemble it and once everything is assembled, we are done.

So, notice that there are some tasks that can be done in parallel. There are some tasks that can are to be done in series, in other words, have to wait. For example, to start planning, you have to wait to be done with both the other parts. To start assembling you have to wait for both the art and this structure to be ready. So, these are some of the constraints.

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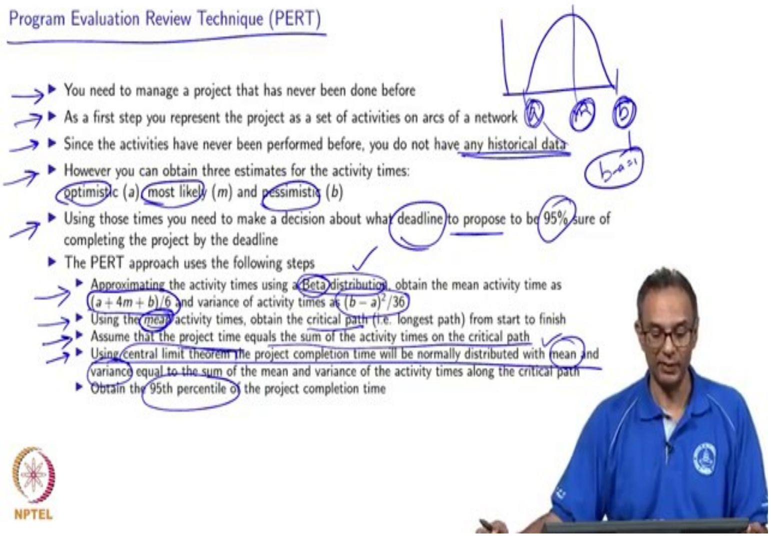


Now, this type of problem falls under what is called PERT; PERT means Program Evaluation Review Technique. So, let us say you need to manage a project that has never been done before. Now, this is a very important part of PERT. It is not like I have tons of prior data. Think about this project where each of us will probably get chance once in your entire college career to do a project like that.

So, it is very important to realize that this is not a repeated type of situation. This is one in which the project has never been done before. So, what we do is we are thinking about putting down this project in a formal fashion in terms of a network to see how we can analyse. And, the activities also have never been done before, at least not by the people who are pursuing the activity.

Therefore, we do not have any historical data. So, now, this is again back to a situation where we are saying we do not have any data, we do not know, we have never done this before; then what do you do? Well, we are not going to be completely what is called nonparametric. We are going to have some parametric estimate for example; we are going to say, well, I could at least guess what is the fastest I can do this, that’s what is called optimistic; or what is most likely, how long it is going to take me; or what is the pessimistic estimate, the worst case. So, I have three numbers a, m and b; a is the smallest, next is m; next is b. a is the optimistic, what is the fastest I can do this; b is the pessimistic, what is the worst case time that it will take, and m is somewhere in between which is the most likely time.

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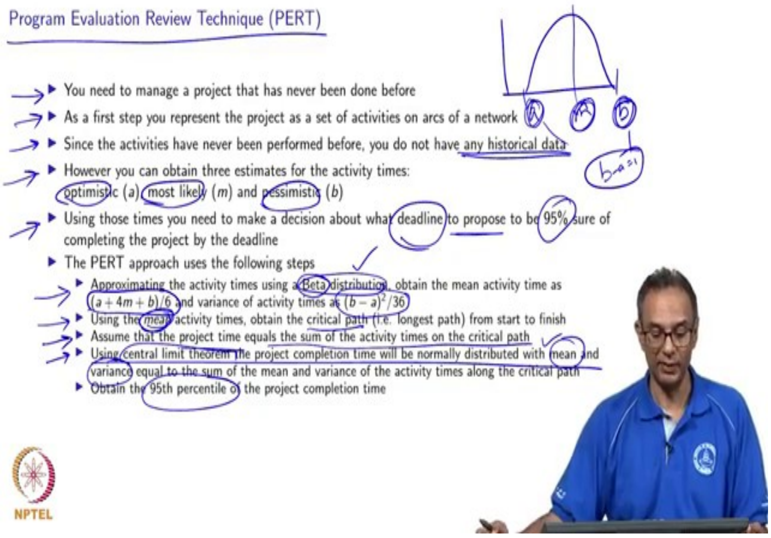


So, usually the graph usually looks kind of like this. This is a, this is m - the mode of the distribution and that is the reason it has an m. Now, we assume that this distribution is what is called beta distribution. When we used the beta distribution in the previous topic, there were only two parameters; now, we have three because this is what is called a three-parameter beta distribution with a, m and b. In the two-parameter distribution, basically the b and a were such that b-a was equal to 1 or something like that.

It is a special case and this is a more general case. I like this a little bit better than what I presented earlier. So, we need to make the following decision - we want to figure out what deadline to propose, what should I tell the person who wants the project to be completed as to when they should show up. When they show up, I want to be 95 percent sure of completing the project. So that means, there is a very high chance that I will be done by the time the person comes and says is it ready.

This is an important aspect and we want to be sure that you want to provide good service to your client if you will. Now, the PERT approximation uses the following steps. There are five steps. The first step in the PERT analysis says we are going to use the beta distribution for the activity times. Really, the beta part is not important; the reason we use a beta part is if I know this a, m and b, I can compute the mean and the variance of the activity time. So, if you look at look at it here, how long is it going to take me to brainstorm? I can compute the mean and the variance of that time; that is all I really need it for. So, I compute the mean using this, $\frac{a+4m+b}{6}$ and the variance as, $\frac{(b-a)^2}{36}$. So, that is how I compute the mean and the variance. So, once we do that, the beta part is gone; there is no need for that. Now, the next thing we do is we compute what is called the critical path. This is the longest path from the start to finish. So, this is the longest path from A to J. Why do we pick the longest path? Well, that is the time that it is going to take if you think about it. So, for example, if the art work gets done faster than the structure, you still have to wait till the longer arc gets done, the structural part for you to start activity E. So, the total time is going to be the time along the longest path. Now, the path times are also random unlike the standard shortest path problem that some of you may have seen.

Now, turns out that we are going to make a major assumption which is that the project time equals the sum of the activity times of the critical path. So, it is only the critical path. And, the critical path is computed using the mean values, the average values. So, it could so be that the one that has a smaller average could take longer sometimes. You remember we looked a lot of those kind of things. We will see that again in the next topic.

However, as a decent approximation, we say that among the average times, take the one that takes longer. Then, we use the central limit theorem that we saw last time in topic one and say that the project completion time is going to be normally distributed with means equal to the sum of the means along the critical path and variance is equal to sum of the variance along the critical path.

So, remember the critical path is assumed to be the time and it is the sum of a bunch of other times. And, remember when you add up a bunch of independent random variables, a resulting random variable is normally distributed if you had a lot of them and then we say therefore, the distribution is according to normal using central limit theorem, and then we will say what is a 95th percentile of the project completion time.

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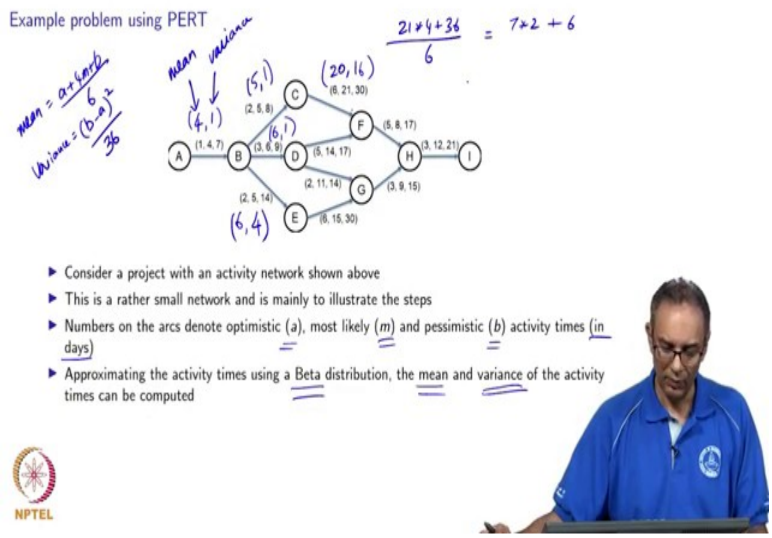


Let us do a little example using PERT. So, we are going to take a network that is shown here and this is somewhat of a small network. The reason it is small is so that I can show you all the steps and tell you how these things are done. The numbers on the arcs are three values.

The first value is a - the optimistic estimate, m - the most likely activity time and b - the pessimistic activity time. So, those are the times which could be in minutes or could be in hours depending on the problem. I guess in this case it is days. So, it is the number of days it is going to take me to do this. So, that values a, m and b are the optimistic, most likely and pessimistic values.

Now, we are going to use the beta distribution and we are going to compute the mean and variance of the activity times. I am going to erase these numbers and instead, what I am going to do is I am going to write down the mean and the variance values on top of the arc. So, if you remember, the mean equals $\frac{a+4m+b}{6}$ and the variance is $\frac{(b-a)^2}{36}$.

So, now, if you look at this, the first one being arc A-B, the mean is going to be 4; that is because 1+16+7; so, 7+1 is 8; 8+16 is 24; 24/6 is 4 and then the variance is b-a=6; 6*6 is 36 and then, 36/36=1. So, the mean is 4 and the variance is 1. Likewise, I go here to arc B-C and I can compute the mean as 5; that is because 5*4 is 20; 20+10 is 30; 30/6 is 5; 8-2 is 6; 6/6 is 1; so, the variance is $1^2$=1. Likewise, in arc C-F, it is 21*4 + 36 divided by 6 and that is equal to 7*2 + 6. So, the mean is 20. And, the variance is 30-6 is 24; 24/6 is 4; 4*4 is 16.

Now, we will look at this arc B-D; this mean is 6, the variance is 1 and the standard deviation is 1 as well. Here, if you look at this arc B-E, mean is 5*4 is 20, + 16 is equal to 36; 36/6 is 6. And, the standard deviation is 4 because 14-2 is 12 and $12^2$ is 144 divided by 36 is 4. Likewise, we can go ahead and write down the other numbers as well. I am going to go to the next slide and show you what the actual numbers look like.

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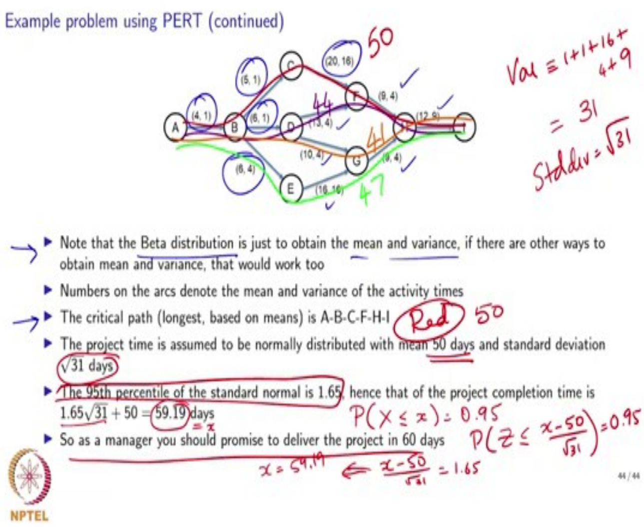


So, we saw that (4,1) at A-B, (5,1) at B-C, (20,16) at C-F, (6,1) at B-D and a few others and (6,4) at B-E and so on. These are the numbers we saw. You can compute and check all these other numbers. Now, remember that we have only used a beta distribution to compute the mean and the variance. After that, we do not need the beta business. So, we can let that aside.

Now, what happens is let us look at all the numbers there. There are two numbers on each arc; one number is the mean; the other number is the variance. So, we want to look and see which is the critical path. So, let us look at all the possible paths. I am going to illustrate all the paths. So, one path is this in red colour. I am going to pick another colour for the second path and the second path is this. And then, the third path, let me pick a colour like this; the third path is this. And then, there is a fourth path which is here and there are four possible paths and for this let us take orange and the fourth path is this.

So, in the red path, the total the expected value is 4+5=9, plus 20 which is 29, plus 9 is equal to 38, 39, 40, 50. So, that is 50. The second path which is in violet if you look at it; it is 6+4=10, plus 13=23, 23+9 is 32; 44. Then, for the third path in orange, the things add to

4+6=10 and 10, 29, 31, 41. And finally, the green path is 6+4=10 and 16; so, 10 and 16 is 26 and 9 is 35, 47. So, the longest path is A-B-C-F-H-I. So, the red path is indeed the longest. This is the red one and the length is 50 which is written here and the standard deviation is $\sqrt{31}$. How did we get that? Well, we have on the red path, the variance equals 1+1+16 and remember the variance add up; so, plus 4+9 I believe. So, that is equal to 16, 20, 22 and 31. So, the variance is 31. So, standard deviation is s$\sqrt{31}$ days.

Now, I am going to do a little bit of the software situation and I am going to show you in two ways that the 95th percentile is going to be 59.19 days. There are two ways of doing this. If you recall, one way is to say that the probability that the random variable X is less than or equal to little x, we want that to be 0.95, $P(X \leq x) = 0.95$; in other words, we want the probability that Z the standard normal which is $\frac{x-50}{\sqrt{31}}$; we want that to be 0.95 and we know that the 95th of the standard normal is 1.65.

So, $\frac{x-50}{\sqrt{31}}$ is 1.65. If we use this and solve for x, we will get x equals 59.19 days and that is because this 1.65 gets multiplied by $\sqrt{31}$ and you add a 50, that is my value of little x. So, you should promise 59.19 days. So, typically as a manager you would say, I will deliver the project within 60 days to be 95 percent sure. So, that is what PERT essentially does for you, tells you the probability of finishing in 60 days as this. Now, I am going to use a little demo. It is a very straightforward demo in Octave before we wrap up.

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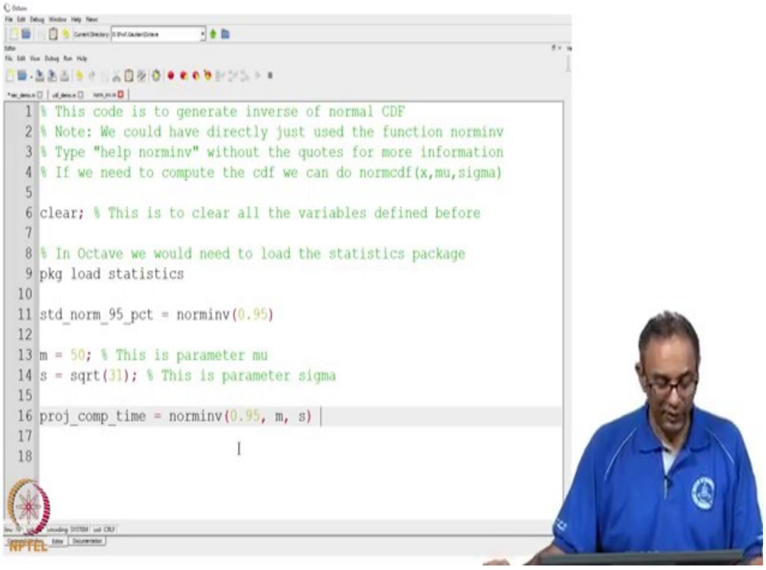


So, this code is to generate the inverse normal CDF. Remember we want to compute the inverse of the CDF because we do not want the probabilities, but we want its inverse. You can type the command norminv to compute the inverse of the normal. We could have just directly used the function norm inverse but we are going to take a couple of steps. If you just write norminv(0.95), what you would get is the 95th percentile of the standard normal. Instead, which we saw earlier in this slide was 1.65, it will probably be a little bit less than that. 1.65 is generally considered as a nice rounded up number. Instead, you could have not even gone the standard normal route; you could have just said, well, I will just go ahead and take the inverse normal of a distribution with mean 50 days and standard deviation $\sqrt{31}$ days. I could have just done that and written down the project completion time.

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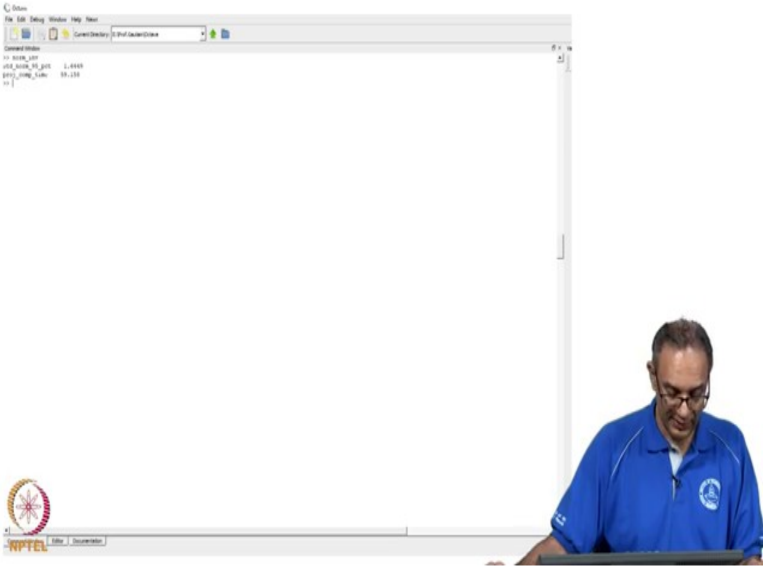


So, let us go ahead and do that and so, this file is called norm_inv.m. So, I do norminv.m. So, that is the name of the file. So, if you look at this, it says that a standard normal 95th percentile is 1.6449. We used 1.65 in our calculation and the project completion time is 59.16; that is the project completion time. So, basically what I am doing is I am directly computing this without even worrying about the standard normal. But if we do the standard normal route which is very common in most textbooks because years ago, they did not have this kind of software and so, everybody use the standard normal table and computed this.

If you did that, which is what is presented here, then it would take 59.19. But if you directly use a software, you would get that same result without going through this whole business of standard normal. I just wanted to mention that, but please feel free to play with this, because this day and age, it is a good idea to just use some of the software to directly give you these values without worrying too much about the notion of standard normal. I will stop here and thank you very much. Next class, we will see the next topic.

Thank you.