Decision Making Under Uncertainty
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Lecture - 17
Decisions in Game Shows: Monte Hall

Hi. This is another game show, a second in our series and it is called the Monte Hall problem.

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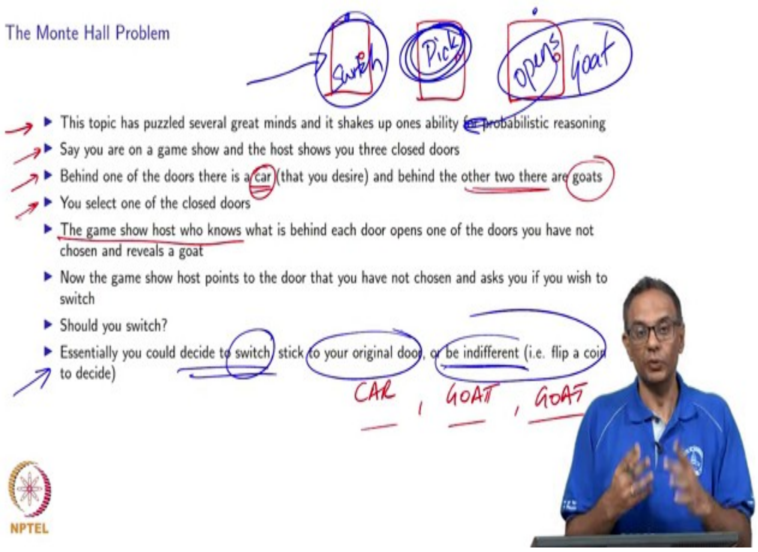

This problem is very popular in the probability courses and community in general. This is something that has really shocked, puzzled and troubled a lot of people. Very great people have really been confused if anything about this topic. So, here is the situation. I will explain first and then I will draw a picture. So, let us say you are in a game show. Again, this one is a different type of game show; it is not really crucial. So, the host shows you three doors. Now, the doors are closed. Behind one door, there is a car and behind the other two doors, there are goats. I am going to make an assumption that you are interested in the car. That is a reasonable assumption. At least today, cars are way more expensive than goats. So, you do not know what is behind each door because the doors are closed and you asked to select a door.

So, there is a car behind one; there are goats behind the other two doors - goat, goat. So, that is the situation right now. But we do not know behind which door, there is a car; behind which other two doors, there are goats. Now, the game show host knows what is behind each door. That is an important thing to know. So, like I said, behind one door, there is a car; behind the other two, there are goats and the game show host knows which is what is behind what. So, you select one of the closed doors.

So, let us say for example, you select door number. So, let us say you pick this. So, you select this; you pick the second door. This is an example; you could have picked the first or the third. Now, the game show host like I said, knows what is behind and picks one of these two and opens it and reveals a goat. So, the person knows what is behind each door and the person has a choice and decides to open one of those doors. For sure, one of the doors has a goat; he opens the door and reveals the goat.

Now, the game show host points to the door that has not been opened and that has not been selected. Remember you picked the middle door; the game show host opened the door on the right. Now, the game show host asks you the question - do you want to stick to your door original choice, the middle door, or switch to the door to the right? He wants to ask you that question - do you want to switch or do you want to stick? That’s the only choice. However, from your standpoint, you really are looking at one of three situations - you either decide to switch or you decide to stick to your door or you could be indifferent. What I mean by that is, you thinking about 50-50, flip a coin. If I get heads, I will switch; if I get tails, I will stick; something like that, but you have to make that decision.

So, this is where the decision comes. What should we decide? Should you decide to switch? Should you decide to stick with the original door or should you flip a coin? What would you do? Turn off or pause the video; think about it; give it a nice thought and say what you would do. We will give you the answer in the next slide.

So, I want you to think about the answer before we move on. So, one more time, you have three doors; behind one door, there is a goat; behind another door, there is a goat; behind the third door, there is a car. But they could be shuffled in whichever way possible. You select one door whichever it is. One of the other two doors does not have a car for sure. The game show host who knows what is behind door opens one of those doors, reveals a goat and ask you the question - do you want to switch or do you want to stick with your original choice? What would you do?

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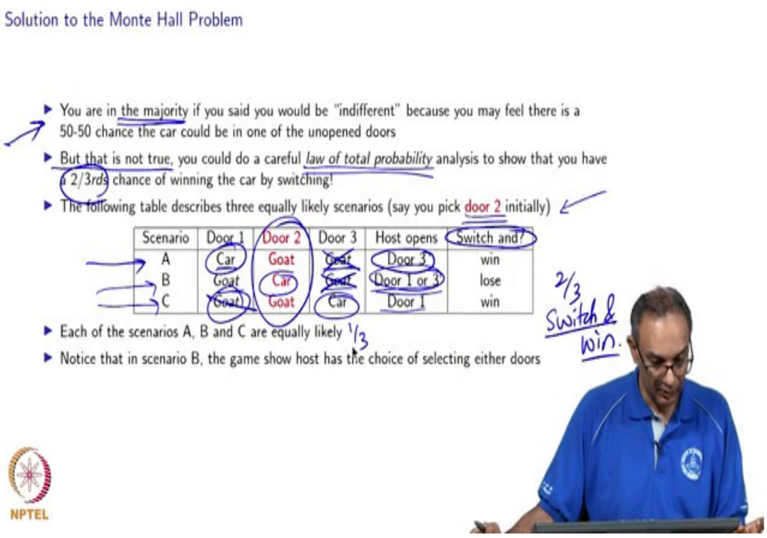

So, let us look at the solution. This like I said has really puzzled a lot of people. If you said you are going to be indifferent because you think there is a 50-50 chance that it is going to be one of the unopened doors, either the one you picked or the one you did not pick, then you are in the majority; that means, this is what most people do. When I teach a class and I ask this question, I would say about 80-90 percent of the students would say 50-50. I would say it is even larger if they have never heard this problem, but those of them that have heard of this problem would know what to answer. Now, turns out that is not true that it is 50-50. In fact, if you switch, you have a two-thirds chance of winning. Now, this would be surprising to a lot of people. One way to solve the problem is to do the law of total probability. Remember we touched upon the law of total probability. One way to solve this problem is to take the route of the law of total probability, compute the probabilities and then find out what is the probability that there will be a car behind the other door. You could actually compute your conditional probability and so on; that is one way to do.

The second way is a somewhat intuitive solution. So, like we did in the previous slide, let us say you pick door 2 initially; that is why it is in red. You could have one of three scenarios. Scenario A is where the car is in door 1, scenario B is where the car is in door 2, scenario C is where the car is in door 3. What will the host do in scenario one? The host will open door 3 because that was the one that has a goat, door 1 has a car. In the second scenario, the car is in door 2. So, the host could open either door 1 or door 3. And in the third scenario 3, for sure the game show host will open door 1; that is because that is the one that has a goat; the game show host will not open the third door. Now, if you switch, what is going to happen?

In the first case, if you switched, they will open door 3. So, this is revealed. Here, they would open either. Let us just say open door 3; here, they open this. If you look at it, you switch you win; you switch you lose here; you switch you win. So, 2 out of 3 times you switch and win therefore, the probability of winning by switching is two-thirds. Notice that in the second choice, it could either be 1 or 3, but it does not matter. The game show host arbitrarily picks one of them. Now, also notice that the three scenarios are equally likely with probability one-thirds. And therefore, you know the probability of winning by switching is two-thirds.

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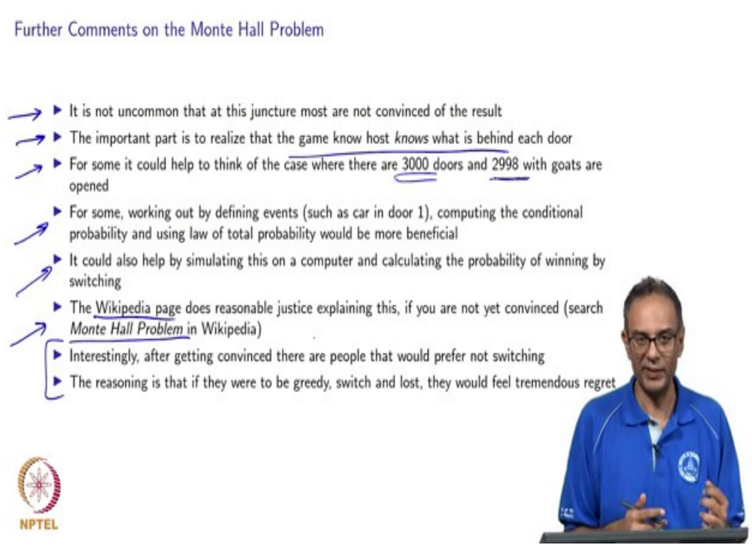

Now, I want to make a few comments. Whenever I talk about this in my classes, students are typically not convinced of the results. They usually they need another weekend or something like that to think about the problem; please do that. The first time I taught this material, I had a student who actually walked out of the classroom, went ahead and wrote a program in MATLAB or Octave and actually simulated this situation. And, his simulation; his name is Adam and his simulation was such that he could clearly show that there is a two-thirds chance of winning; he was not convinced. In fact, he walked out of the class. This is the only time something like that has happened to me. Now, the important part of this is that the game show host knows what is behind each other; the game show host does not randomly just open one of the doors. The person knows what is behind each door. Now, for some people, it might help to think of it in the following way. Think of this special case of 3000 doors, and 2999 of them have goats and one has a car. And, when you pick one door and the game show host opens 2998 of them leaving one door closed, would you pick that one door just because that is the one that the person didn't. It turns out that if you did that, there is a very good chance of you actually getting the car. So, if that is something that helps, go ahead and do that. If you are still not convinced, like I said, you can do the conditional probability argument mathematically and use law of total probability and derive the result. So, there are many ways; you can write a computer program and simulate what is going on and actually show it. This computation is a fun exercise. I would highly recommend that.

Finally, there is a nice Wikipedia page on this called the Monte Hall problem; go ahead and see that. There is a very nice justification for the result. I would encourage you to go and take a look at this. Now, turns out this is an interesting comment that I do want to make. Sometimes I do have people who know well and good that there is a two-thirds chance of switching and winning. But still, they will not switch. So, you think about this. If you stick with your original door, you only have a one-third chance of winning. But if you switch, there is two-thirds chance. Still, I know of people who tell me that I will still stick with my door. The reason is very fascinating to me; it is a psychologically, extremely amazing result. They say that, well if I were to switch, I would be perceived as inordinately greedy. Even though I have a two-thirds chance of winning, I would be perceived as someone who is extremely greedy and after that if I do not win, there is a one-third chance of losing and losing due to greed is something that I cannot take. So, I would regret for the rest of my life and therefore, instead of going with my gut feel and picking my first choice even though my probability of choosing the second one, switching, gives me a better option, I would not pick that. There are many people who think about that and that is a phenomenon called regret which has been studied really well in the psychology literature and the decision making in the psychology side of things. So, this is another aspect that we are not talking about; we are saying people are going to be purely rational decision makers and there are other issues where humans are going to behave somewhat irrationally in situations like this. Thank you.