Decision Making Under Uncertainty
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Lecture – 16
Decisions in Game Shows: Final Jeopardy

This topic is decisions in game shows. This one and the next topic will talk about a game show called Final Jeopardy. This is a popular quiz show in the US. A lot of times this is available in YouTube. I would highly recommend that you watch the final jeopardy once at least in YouTube before going through this entire exercise. I'll say that one more time.

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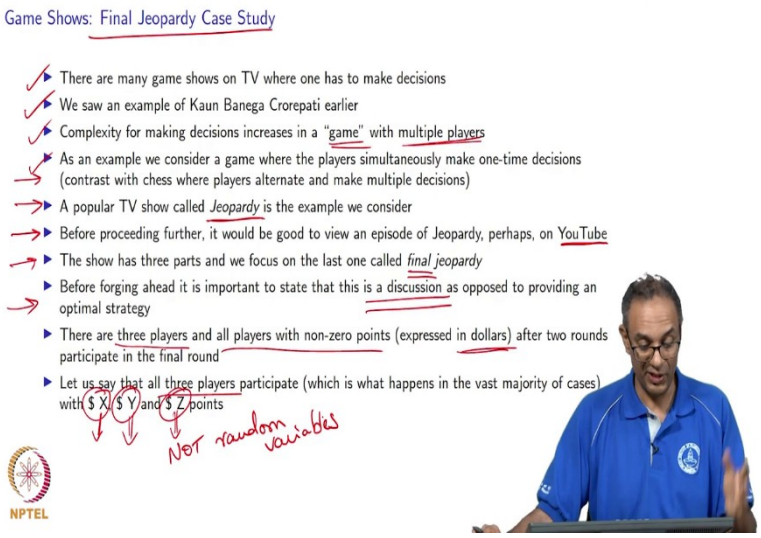

So, we will do a little case study of this game called jeopardy. Now, there are many TV shows like jeopardy where we have to make decisions and we saw the Kaun Banega Crorepati example earlier where you would say - do I want to continue or do I want to stop? There are many such game shows on TV that we see and this is just to list a couple of them here, but there are many more. Now, there are some complications here. When there is a game, there are multiple players. So, it is unlike the Kaun Banega Crorepati where you are the only player. Therefore, the situation is a bit different here as there are multiple players. So, it adds a little bit to the complexity. We will take a look at it. I will explain the game reasonably well. Now, there are three parts of the game. We going to look at the last part. So now, essentially chess is another game. Obviously, there are many other games. But in chess,

the players alternate and make decisions; the white goes first, makes a decision and then black goes first. By decision, what we mean is the player makes a move. Turns out that they do this one after the other. You can see what the other person has done. Here, what happens is we will see where all the players make a decision in one shot and one decision and they do not see what each other is doing; so, at the same time. So, in some sense, this is like an auction. There are many times you would be familiar with what is called an auction. This is something similar to that. And this jeopardy is what we are going to be looking at and this is a very popular TV show that has been running for a long time, several decades. I would highly recommend that you go to YouTube and watch one episode of jeopardy if not more. I would recommend watching a few. Even if you do not have that kind of time, at least watch the last part. So, there are three parts in the jeopardy game. The first part is the basic jeopardy, then there is something called double jeopardy, then there is a final jeopardy. The final jeopardy is all you need to see for today. This is what we are going to be talking about in this course.

But I will tell you a little bit more about what happens in final jeopardy in case you do not get a chance to see. I want to also say that we have to just do a discussion. We are not going to come up with an optimal strategy like we saw in the Greene Cat before or the secretary problem; all those had an optimal strategy. Here, we are not going to have an optimal strategy. I do want to warn you; so, do not get too excited.

We will just analyze and I just want to give you an appreciation for decision making under uncertainty. Now, turns out and I will give you a picture when I get a chance, but there are three players in the game and everybody who has nonzero points. And, these points are expressed in dollars. Actually, the word dollar there does not make any sense mainly because the players do not really get that dollar amount, unlike a Kaun Banega Crorepati or Who Wants to be a Millionaire where that is the actual amount that they go home with; here, the amount is just notional. So, we can just think of them as points.

Now, after two rounds, they have certain number of points. So, I am going to say the first person has X number of points, second person has Y points and the third person has Z points. So, more often than not, there are three players left; if somebody has 0 points, they are basically eliminated from the final round, but that happens very rarely. So, we will just take the more generic case of having three players.

So, first player has X amount, second player has Y, third player has Z. Now, X, Y and Z are assumed to be actually known; not just assume, it is actually known to everybody. Everybody can see how much points each person has. Unfortunately, I do not like using upper case X, upper case Y and upper-case Z mainly because these are like random variables, but here these are not at all random variables. I just want to warn you that these are not random variables like you seen before. I do want to warn you; maybe I should have used a different letter.

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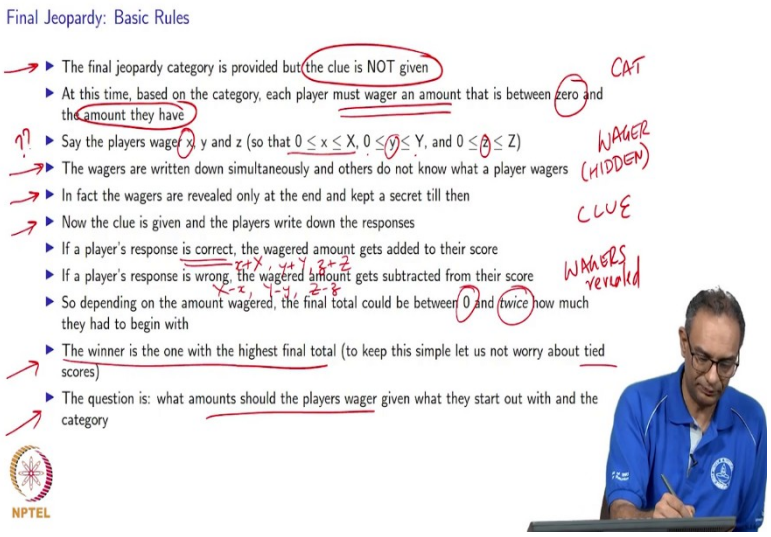

Here are the basic rules. I am going to tell you the rules once. It might be difficult and I'll draw a picture and explain it one more time next. In a blank slide here, I will do that. So, essentially this is what happens. So, when you come down to final jeopardy and each of the player, say first player has X amount of money, second player has Y amount of money, the third player has Z amount of money; at that point, the category is provided. So, the category for the last final jeopardy is provided, but the clue is not given. So, you do not know what question is going to come, but you just know the category. Now, once the category is revealed, you as a player need to wager an amount, need to say, "I will wager this amount of money" and that number has to be between 0 and how much money they have. So, for example, player 1 can wager between 0 and X. The little x is the amount that the player wagers, that is, puts down. Then, the little x must be between 0 and big X.

Likewise, if player 2 wagers y which should be a number between 0 and upper-case Y; and the third player wagers z. Now, the wagers are such that if you win and get the answer

correct, then that x, y or z gets added to your score. If the wager is wrong, it gets subtracted from your score. So, now, everybody writes down their wagers at the same time. Nobody knows what other people have wagered.

So, in fact, you see a big screen on your left and your right and when you see it on TV. You will not see what the other persons are writing down. In fact, you will never know how much they wagered until the very end. So, here are the steps. Step one is the category. Step two is the wager is written, but it is hidden; wager is written down, but it is hidden. The next step is the clue is given or the question is given. The quiz question is given and the players write down the answers on their screen much like the screen that I am writing down right now; they write down the answer.

Then, the quiz master, most likely Alex Trebek will actually show what each person has answered. So, he goes the first player and looks at his or her answer and if the response is correct, then the person would now get x added to their scores. This score will be little x plus big X. Now, if y also gets it right, there was going to be y+Y and finally, if the third player gets it right, it is z+Z.

Now, if the question is answered incorrectly, then the x amount or y amount or z amount gets subtracted. So, this is when the wagers are revealed; wagers are revealed and you know how much they all are better. So, you could as a player either have 0 or twice how much you started with. So, if you wager all your money, then basically if you got the answer correct, you would double and if you got the answer wrong, you would have 0. So, you would be anywhere between 0 and 2x if you are player 1, 0 and 2y if you are player 2, and 0 and 2z if you are player 3. Now, the winner of the jeopardy is the one with the highest final total; this is after adding or subtracting. We will not worry about ties; we will for the purposes of this course, ignore that. So, the question is what should the players wager? How much should each player wager? So, what should x be? What should y be and what should z be? So, this is the question that is what we are trying to decide. This the decision you need to make under uncertainty. What is uncertain? While making the wager, what is uncertain is what question is going to come. However, what is known a little bit about it is, some information on what category it is. So, if it is an easy category, you might think, maybe I will get the answer correct. If it is a hard category, maybe not something like that. So, that is what is going to happen.

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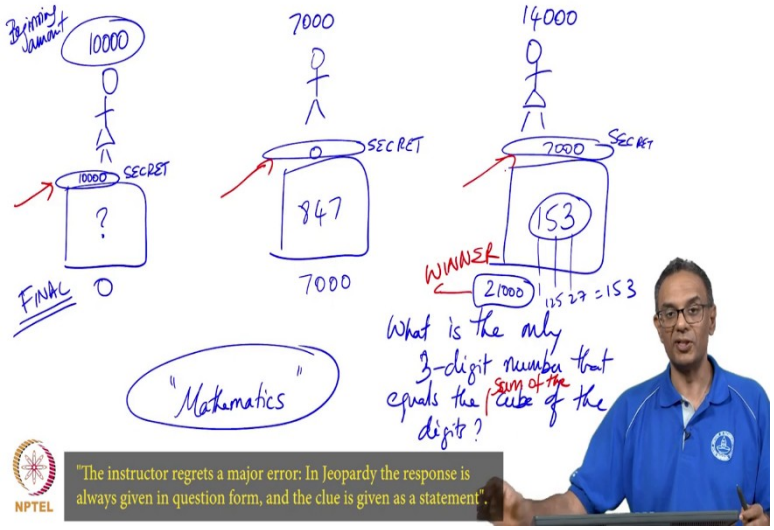

So, let me give you an example. So, let me give you an example just to illustrate this whole process. So, when you look at the screen, you will see player 1 and then they will have a screen here; this is player 2 and will have a screen here; and player 3 will have a screen here. And, when they give out the category, let us say the category is mathematics. So, they tell you the category is mathematics, alright. So, that is what Alex Trebek will come out and say - the category is mathematics.

Now, this person has initial amount like the beginning amount. Let us say the first person has 10000 and say the second person has 7000, and the third person has 14000, alright. Now, the category is mathematics. What happens is the first person needs to decide mathematics is something that I am strong at. So, maybe I will wager all my money. Now, the others cannot see this is secret; nobody can see this other than the player; this is kept secret. Now, this person said - Oh! I do not know; I am not very strong as mathematics; I am going to be wager 0. This is also a secret. And, the third person says Mathematics, I am with it, maybe I will wager 7000; and this is also a secret.

So, you do not know how much they wager. So, right now, the question hasn't yet come. So, they put down this amount of money; they cannot change this till the end of the game. Now, the question is revealed. So, now, these people have made their decisions based on how strong they feel about the subject of mathematics. The first person feels a bit strong; they put down all their money. The second person is not very strong; so, puts down zero thinking

whatever they ask I am not going to be able to answer. So, I will just put down zero. The third person is kind of ambivalent and said I will put down a half way, 7000. We will talk about what is a good number to put down. These are not the best numbers. This is just to illustrate what could happen, but you need to be thinking about how each person should be wagering at this point of time.

Now, let us say the question comes up saying for example, what is the smallest number or actually the only number, 3-digit number that equals the sum of the cube of the digits. So, that is a question, alright. So, the first person thinks about it and is not sure and puts a question mark. The second person who remarkably things about it and then just writes down 847, and the third person thinks about it and says I know this is 153 which by the way is the right answer. Because 1^3 is 1, 5^3 is 125, 3^3 is 27; so, 1+125+27=153. So, that is the only number that can be written in that form.

So, what happens is when they reveal the answer, the first person had does not write anything; so, they would go home with 0. Second person goes at 7000 and the third person gets 21000 and the winner. So, the third person is the winner of that day. This is the winner because they have the maximum amount of money. So, now, the question is - what is a good number? So, what should this number be? And, what should this number be will be our study next.

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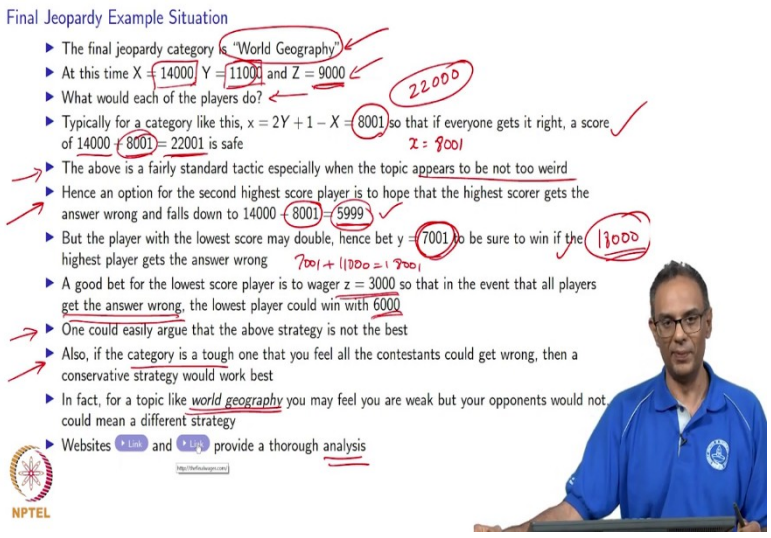

So, let us look at this. Let us say the category is world geography. So, now, this is another different category. Now, I am giving you different numbers for what X, Y and Z are. So, let us say the first person has 14000, the second person has 11000, and the third person has 9000. The question is what should each person do? What should each of the players do?

Now, when a category something like world geography, you will see that most of these guys are pretty good in world geography. So, they would feel somewhat confident. It is unlikely that anybody is going to put down a 0 for a topic like world geography. So, especially you would expect a question that they could potentially answer.

So, I do want to say that the topic is somewhat important although some of the websites do not say too much about it. And so, what should each player do? So, now, let us check player by player. In fact, let us take the first player. So, if the first player has the maximum number of points, 14000. What is the worst thing that could happen is the second player wagers all their money and gets the answer right and thereby going to 22000. So, Y's amount could be 22000. So, X must wager just enough so that if they get it right, which is 14000+8001 to get 22001 and thereby win. We do not want a tie for now. Let us pretend that is the case. So, what we will do is we will say let us go ahead and wager 8001; so, little x is 8001; this is a reasonably good strategy. Now, this is a very standard tactic almost all the top players generally do especially when the topic is not terribly weird. Let us say for example, the topic is Shakespeare and you are not strong in that. Yes, it does not make sense for you to wager somewhat lesser. But otherwise, it is a pretty good choice. Now, the second player's best hope is they do know that this being a very popular strategy this is what the first player is going to do. So, the second player is the second highest scorer, is going to say, well, I can only hope that they get the answer wrong and if they get the answer wrong, they would be down to 5999. However, what could happen is the third player who is at 9000 could double and go to 18000 and we do not want that. So, we want to top that one because this guy goes to 5999, then you are still at 11000; so, that is not a problem. However, if the person gets it wrong in this situation, that will be an issue. So, given that this topic is something that they feel confident about, they would wager 7001. So, that 7001+11000 gives me 18001. So, 7001+11000 if they get the answer correct is 18001 and they can win provided the first person gets it wrong. Now, the lowest player's best bet would could be several things. One good bet is for them to wager 3000. So, if everybody gets the answer wrong, then the lowest player is going to win 6000 because they know that they are probably guessing that this is a

strategy that the others are going to pick. Now, we could really talk about is this the best strategy; probably, not; but this is a strategy and you also want to pick strategies that are dependent on the topic and a whole bunch of other things. The other thing is if a category is very tough, you could do something different. You could think where everybody is going to get it wrong and so, I will be conservative and put down 0. And, topics like world geography, maybe you are one person who feels a little bit weak about the topic, but your opponents may not. So, then you say let us go ahead and do that. So, that is enough. I have given you two websites I am not going to click this, but this website is called thejeopardyfan.com; go ahead and click it and look at the analysis that they provide; it is really interesting. And also, go ahead and click this one I am not going to do it. Here, it is called thefinalwager.com; click that and it give you some information about various ways that people who have played this game a lot or at least have been in the show, tell you how best and what are the good strategies to adopt. And, I would be interested in hearing about what you think should be a good strategy for just this one. So, feel free to email me your personal options.

Thank you very much.