# Decision Making Under Uncertainty

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## Lecture - 15
## Decision Trees

The next topic is decision trees. Now, this is a wonderful tool to perform complex analysis like we saw before. Decision trees are used not just in the context that we are going to be seeing, but a whole lot of other applications in other settings as well.

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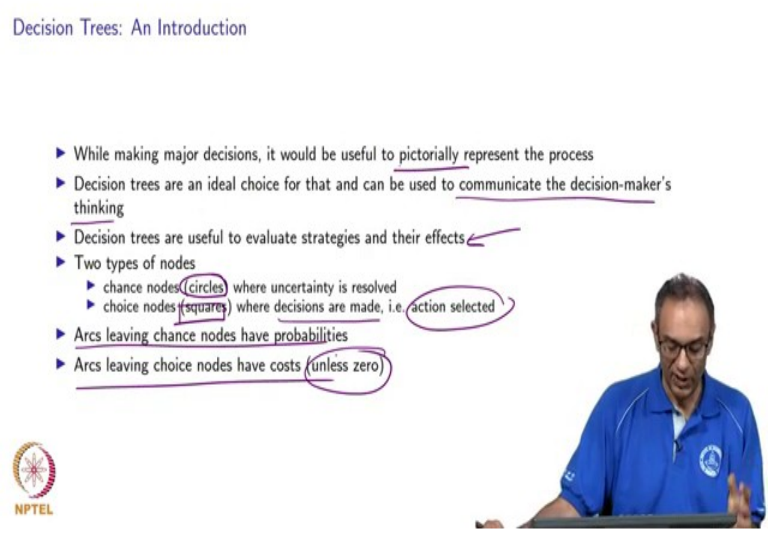


Let us look through the exact same problem using decision trees, but first let me tell you what decision trees are about. So, it is a way to pictorially represent what is going on. So, it is a way to put down the whole decisions and actions and states and uncertainties all that in a pictorial fashion in a figure and these are great, especially when you want to communicate your decision to other people.

So, for example, in the previous case which we will see next, if you want to explain the spending to some type of a stakeholder, you would want to put this together in a picture. I am sure a lot of you are probably still not certain about how we went about doing those calculations. We will do it one more time in this example.

Now, turns out that it is nice to also evaluate strategies and look at their effects. In fact, you put something down in a decision tree. You can even write a little program to do this automatically. Now, in a decision tree, unlike several other trees, there are two types of nodes. Most trees or most networks has only one type of nodes; here, there are two. Some nodes will be represented as a circle, some nodes will be represented as a square. The circular nodes are what is called chance nodes; that means, there is uncertainty involved that needs to be resolved. The second type of nodes are what is called choice nodes, this is where decisions are made; choices are made and you pick an action. So, there are two types of nodes.

So, what happens basically is you take an action and then you go to a particular state and then some random event occurs, then go to the next state. So, it is kind of like a tree structure which we will see in the next page. Now, there will be some things on the arcs. So, those of you that have never taken a class on networks, these words probably do not mean much. So, essentially the circular things and square here are called nodes, the lines are called arcs and the arcs have probabilities if they are leaving a chance node because there is a chance that something happens, one or the other, and arcs leaving a choice node where we make a decision always have a cost because there is a cost for making decision. The only exception to that is when the cost is zero. So, in which case, you do not have any anything written on the arcs; we will see that as an example next.

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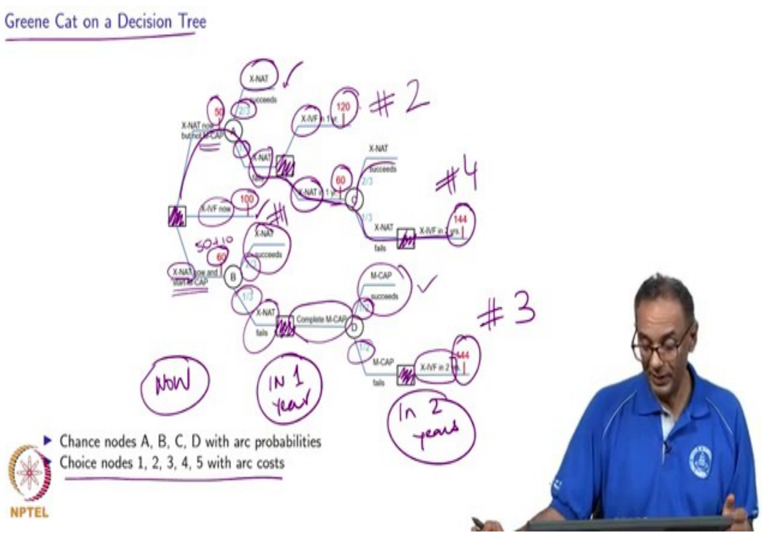


So, remember the Greene cat example. If you do not, I would recommend just going back and quickly flipping through that example. So, this number 1 is your decision that you have to make right now. So, this is now, this is in 1 year and this is in 2 years. So, this is how the time line goes; this is now, this is in 1 year, this is in 2 years.

So, here are your decision nodes; this is 1, this is 2, this is 3, this is 4, this is 5. So, these are just the numbers - 1, 2, 3, 4, 5 do not matter; these are a decision nodes or choice nodes; they are called choice nodes. And, chance nodes are your A, B, C and D; these are where there are probability.

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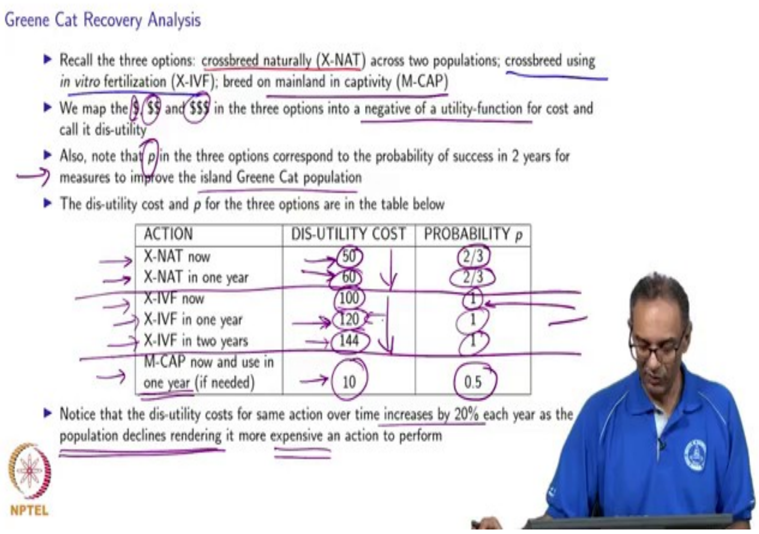


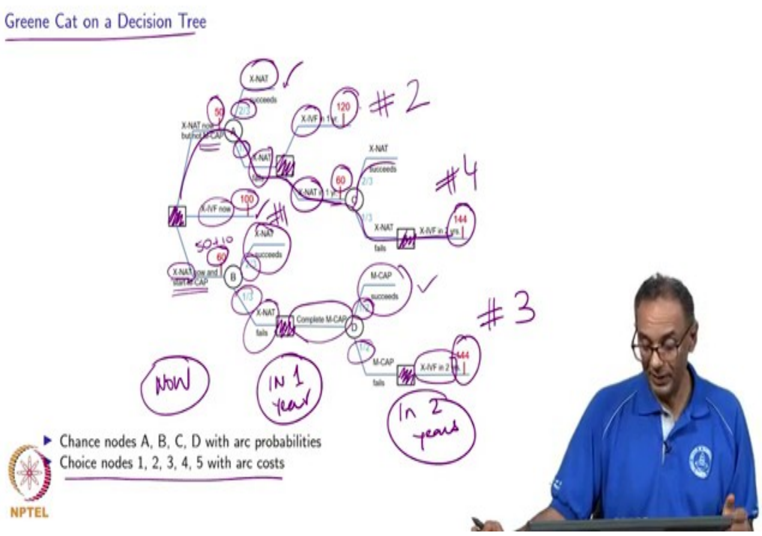


So, let me just go over this just to explain the numbers; we have seen the numbers before. I want you to refer to, if you are able to, refer to this in another page; that would be even better;

that way, I do not have to go back and forth between the slides. So, if you are able to see that page on a piece of paper or another screen, that would be ideal. So, think about this.

So, right now, we have three things that we could do. We could either do the very first strategy, we do IVF which is the in vitro fertilization right now and pay hundred units and be done. Now, there are two other options. Remember you could naturally do a cross breeding, but we do not do; this is option 4, but we do not do the mainland captivating, breeding in captivity. Now, the other option is to do the natural right now and start the mainland captivity. So, this one costs 50 units because you are not doing any mainland captivity. It is just the 50 units that you spend on doing the natural crossbreed; however, here it is 60 because that is 50+10; 50 for crossbreed + 10 for the mainland captivity breeding Now, let me just go branch by branch of the tree. This tree is done. Once it's here, you are done. Now, this tree here is if the natural crossbreed is successful, we are done; that happens with probability 2/3. If the natural crossbreed fails that happens with probability 1/3; now, we have two options. We could either do an IVF 1 year from now that is and spend 120 units. So, if you look at the IVF. I should explain this 120; this is 120 that you have here. So, we spent that 120 after 1 year, and that would be the cost that you would incur on this arc. Alternatively, you could do one more round of natural crossbreed; this time that itself will cost 60. If you remember the numbers in the table, that itself will cost 60. So, this strategy is going in this direction and this or this, this is your strategy number 4. This is your strategy number 1, this is your strategy number 2 and this is your strategy number 3.

Now, strategy number 3, I do want to explain. So, if your natural crossbreed is successful, we are done; that happens with probability two-thirds like we saw before. However, if it fails; that happens with probability one-thirds; then we will complete this thing, the operations where we bring the cats from the mainland into the island and go ahead and try to do that. Now, there is a 50 percent chance that it would succeed, there is a 50 percent chance it would fail; If it succeeds, we are done. If it fails, then we go ahead and do IVF in 2 years. So, IVF in 2 years cost 144; we have seen that before. So, these are their 4, strategies and next we will see how they map to in terms of the cost.

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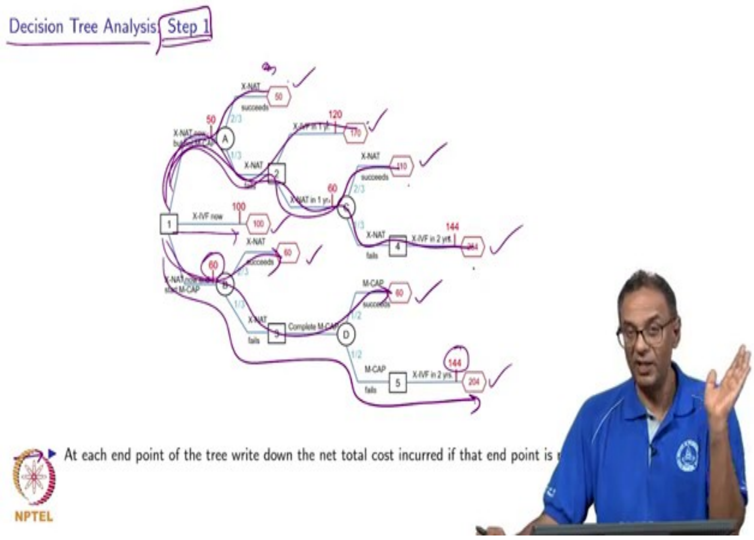


Now, let us do the analysis. So, this is called decision tree analysis. There are two steps. Step 1 of the analysis works as follows. So, the step was written in the bottom; go see the end of a tree. So, I mean you would see that in the previous picture, they were all here and so on; there is nothing there. In this picture, we put down a bunch of red colored nodes if you will; those nodes tell you what are the total cost.

So, now this is not that difficult to compute. Now, if you look at it, if you take this path then how much have you spent? Well, 60 + 144 = 204. If you go along this path, then you would spend only 60 bucks; if you went in this path, again it is only 60 bucks; if you went in this path, it is 100 bucks which is also good. If you went in this path, it is 50 because that is the only cost you incur. If you went like this, it is 50+20 and 120, which is 170. If you went in this path, it is 50+60 which is 110, whereas if you went down this path, it is 50+60 +144 which is 254. So, here are your various end costs. So, these are the various options that could happen and that will be your end cost. So, that is the first step that you need to do; you need to compute those costs and put them down in the end node.

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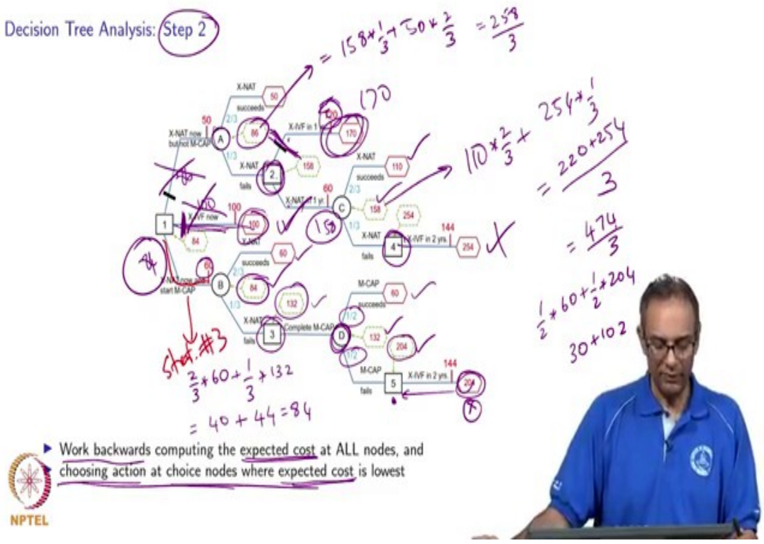


Now, let us go to step 2. In step 2 of the process, this is the last step. There were two steps here. We work backwards and compute the expected cost. So, this is again where we use the expected value computations at every node; every chance node and every choice node, we write down the expected cost. It is written in green color. You can look through that and you need to pick the appropriate action at the choice node.

The second part of the step two is you need to pick the appropriate action and cut off the action that you do not want to take and you want to pick the one that has the lowest cost. I am going to go through this very carefully. Let us do from the bottom to the top.

So, let us start here at 204. So now, if you go to node 5, you do not have any cost other than this 204. So, that cost is written here. Now, if you go to node D, now what happens is there is a 50 percent chance that you will incur 60 bucks, there is a 50 percent chance that you would incur 204 bucks, it is $0.5*60+0.5*204$. So, that is 30+102 that is 132. So, that is this number. So, when you come to this node, your cost is 132. There is nothing random there. Then, this cost 84 is essentially equal to $\frac{2}{3}*60+\frac{1}{3}*132$. So, this one is equal to 40+44 which is 84. So, that is this cost. So, that is your 84 that is here. So, your cost is basically 84 in node B. So, we leave it at that. We do not have to make any decisions here. There are no decisions here to be made.

Now, let us look at this guy. The cost here is 100, alright. Now therefore, the cost here to take this path is 100; now, 84 beats 100. So, we will smash this off ; this is what is happening; we are choosing the action. So, we do not want this action. I have a better action; so, I kill that. Now, put an X here. Now, if you look at node 4, the cost there is 254 because there is nothing random there. Now, let us look at this 158. So, that is basically $\frac{110*2}{3}+\frac{254*1}{3}$. So, that is equal to 158; it is worth checking it. So, this would be 474/3. So, that sounds good to me. So, 158 is good. Now, let us go to this node. So, this node is important; in fact, I could have even done from the top to bottom. So, if I came here, not 120; this would cost me 170 in this path and it costs me 158 in this path. Therefore, between the two, the cheaper option is to do this one. So, I will block this path off. Therefore, we are still living with this path. Now, let us say we go to node A. So, from this side, I have 158. So, this 86 basically is equal to $\frac{158*1}{3}+\frac{50*2}{3}$. So, that is 258/3 which is equal to 86. So, that is your 86.

So, now you have 86 here. So, between 86 and 184, clearly 84 is better. So, this is gone, this is gone, giving us 84 and I am going to use a different color to tell you that therefore, your best choice is to take is to do this one which is our strategy number 3. So, that is essentially what we are trying to do here.

Thank you.