Decision Making Under Uncertainty
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Lecture – 14
Nested One - Time Decisions

The next lecture is about nested one-time decisions. So, in some sense, this is a case where we are not making just one decision, but this one decision results in a few other subsequent decisions. But if you think about it, this is just about one thing. So, it still is one time, but there are some nesting issues that we will talk about.

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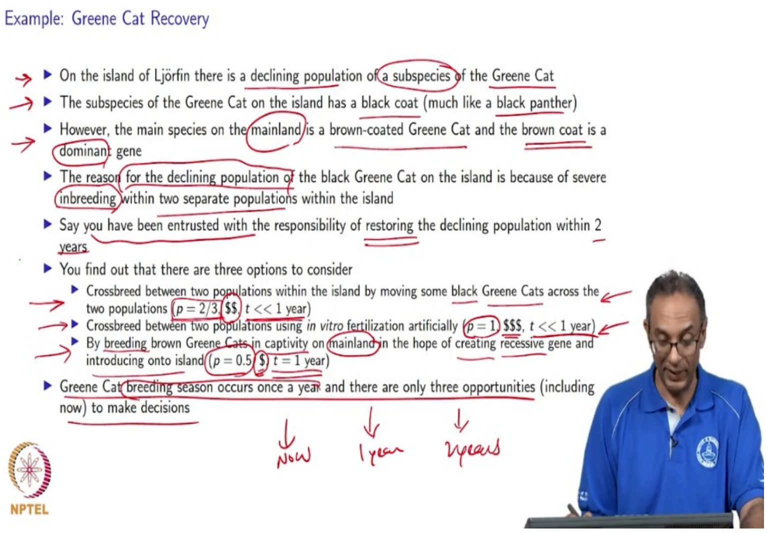

Now, this is an extremely contrived example, completely made up; however, it is from Denardo's book where he talks about a mosquito situation. We have adapted that for a different application here and Denardo book is one of the books that we are using in our references. So, this problem is called the Greene Cat recovery. My son is the one who gave me all this context which is pretty interesting. I will draw a picture in the next slide to explain it one more time, but let us go over this carefully line by line. The first point is there is this island of Ljörfin and there is a special type of cat called Greene Cat. Greene is the name of a person and it is a subspecies of that.

So, inside of this type of Greene Cat, and its population is declining. So, that is what is the situation. Now, turns out that this Greene Cat is very special because its coat is black in color much like a black panther. So, if you think about a black panther, it is basically a regular panther except that its coat is black if you go really close you see all those rosettes and other things.

Now, the main species which is the more popular species it is what is called the brown coated Greene Cat and that is only available in the mainland. So, there is an island which has this Greene Cat which is a black colored one and a brown coated Greene Cat in the mainland. Turns out that the brown coat itself is what is called the dominant gene and the black coat is a recessive gene. So, it turns out many years ago, there were bunch of these black coated green cats that were in the island. And, if you go to the island, you will only see the black coated subspecies.

And, the reason the population is declining, is because there is tremendous amount of inbreeding and I come to the population’s later. Inside this island, these cats are not breeding with other cats and there is some genetic defect that get spread. And therefore, the population is declining because there are some genetic diseases associated with that and turns out there are two separate populations of this Greene Cat.

So, this is an island and the island has two separate forest areas and in each of these forest areas, there are two separately booming black coated Greene Cat population. And, they are not mixing with each other either because this in this day and age, there are so many things that are built that these two forests in the island are far away that there is no mixing.

Now, turns out that if they stop this inbreeding, it is told that we could restore the declining population within 2 years. So, that is what we need to do. So now, what are the options to stop the inbreeding? So, you have been trusted with doing or entrusted with restoring the declining population in a given 2 years of time. Now, here are the three options that one could consider and I will go ahead and re-explain all this once again using a picture in the next slide.

So now, turns out that you have three options. The first option is, remember I told you there are two populations in two different forests within the island, you could crossbreed by taking some of these black green cats from one and put in the other population. So, you do a switch; it is called crossbreeding, you could do that.

But because of the way these people are out of their comfort zone, there is a chance that this will not be successful. So, the probability of success is only two-thirds; so, there is a two-third chance that that we could restore the population. And in fact, it would be restored well before the 2-year mark; in fact, much smaller than 1 year is what is expected pretty much right away. Now, I am putting a dollar sign. This is frequently used to denote qualitatively how much it would cost. So, for example, you see some of these are three-dollar signs or just one-dollar sign. So, this is midway; it is not terribly expensive, it is not terribly cheap here. The first option is to crossbreed between the islands

The second option is to do the same thing; however, instead of physically moving the cats, the black colored Greene Cat physically moving them, we do what is called IVF, in vitro fertilization. So, that can also be done. It is a lot more expensive because you would have to manually take a cat, put them to sleep, move genetic material and so on. So, it gets a little bit more expensive; however, it is guaranteed to work. So, if we crossbreed using in vitro fertilization; now, in practice, it is probably not guaranteed, but to keep this problem more tractable, we are going to assume that its success probability is one. And, it is also right away; it does not take much time if we decide to go with this; it just costs a lot of money.

Third option which is the cheapest of the lot is by breeding brown green cats. Remember that the brown coat is the dominant coat or a dominant gene and the black coat is the recessive gene. So, if you go ahead and do some type of breeding in captivity in the mainland and not in the island, but in the mainland, we could create this recessive gene by chance. However, that will take a lot of trying, but it is very cheap comparatively; so, that will take 1 year. Because then the cat will have to grow and then they would introduce it to the island, there is a lot of uncertainty involved. So therefore, we are saying there is a 50 percent chance that this would be successful. So, there is a time aspect and there is a low probability; however, it is the lowest cost. So, you have three options; one option is extremely expensive, the second option, it will take a really small amount of time, but it is guaranteed. The first option is not so expensive, but also not guaranteed and will take a very small amount of time. Third option takes a large amount of time, very cheap, but has a low probability. Now, we are also allowing to do combinations or all of the above or something like that and let us see what all we could do. There is another important thing. You only have three decision opportunities. Now, after 1 year, and then after 2 years. So, you only have 3 times you could make this decision because the breeding season occurs only once a year. So, the breeding season happens only one time in 1 year and so you either catch it now which is the breeding season or 1 year from now or 2 years from now. That is another important aspect that we need to consider, alright. This is not the most straightforward commonly used example. So, I am going to try to recreate this example in the next slide using a picture.

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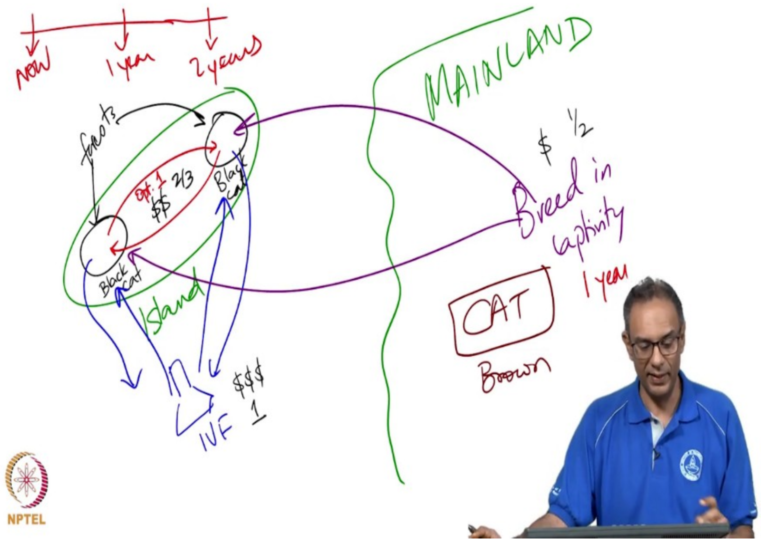

So, let us say we have this, is our Island of Ljörfin and this is the mainland and then you have a brown coated cat here and assuming this is brown in color. So, this is not really brown, but let's pretend that this is cat this is brown. So then, here is black coated wildcat; I can probably just click here, but they are in two separate populations.

So, this is one forest where there is a black colored Greene Cat and there is another black colored Greene Cat. So now, this is an island and these are two forests and these two populations are separated. So, these populations are the two black cat populations in the island are separated. Now, there is also brown cat family sitting at the mainland.

So, your three options. The first option is to move one or few black cats from one to the other and move another one from here to here in the island. So, they actually physically move the cats. The second option is to do the same thing except that instead of physically moving the cat, you will do it in a lab; we will do an in vitro fertilization in the lab. So, this is the IVF option and this is the first one, the physical option. And, the third option is to breed in captivity. In captivity and once that happens, move them here. Those are your three options and they cost different things.

Now this, the first option for example, this one is two dollar signs and has a probability of two-thirds of being successful. The second option has three-dollar signs, very expensive; the probability of success being one. The third option is the cheapest and the probability of success is only half. And now, let me see if there is anything else that I missed out, the time as well. So, the third one takes 1 year. I do want to emphasize that quite clearly so that when we come back, we could do the breed in captivity and after 1 year you bring them there so that they are mature and they can be moved. So, that is that is what we need to do. So, these are our options and we need a timeline. What do you do now? What do you do a year from now and what do you do 2 years from now? We can see the results and also make a decision. So, you are allowed to do that too. You are not just making a decision for everything right now. You are making what you want to make right. Now, you will see what happens and then make your decision next year and the year after. So, let us see what our various options are.

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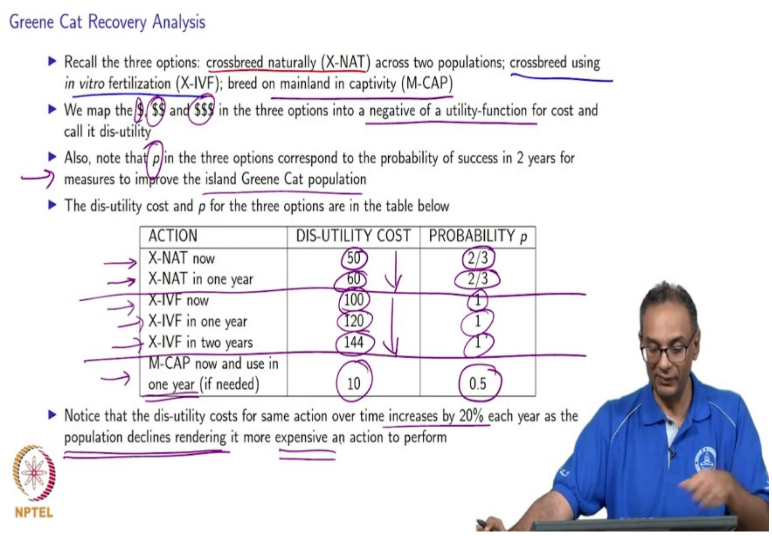

So, let's do a quick analysis. So, remember I am going to use some alphabets here. It might look like an alphabet, but I will try to elaborate whenever I can. So, we have three options. The first option is crossbreed naturally called X-NAT. So, naturally move them; so, this is our red color, one here in this picture. The second one is crossbreed using in vitro fertilization; so, that is a blue option crossbreed using IVF. So, X is for crossbreed. And finally, the third option is how to breed in the mainland in captivity. So, it is called M CAP, M for mainland, CAP for captivity. So, that is your third option, right.

Now, remember that we have this dollar amount. By saying dollar, it is really not going to be a physical money; it is the amount that just tells you one is cheap, one is not. So, one is cheap and one is not quite expensive. So, we were going to use what is called a negative of a utility function. We saw in the previous lecture what a utility function was. Well certainly, the utility function always was a benefit; it was something that was positive. So, it could have a negative benefit; that is one way of thinking about it or we will just call it disutility and we want to minimize the disutility function on average. It is the same idea that we saw before; as in some utility, I am calling it disutility because it is a cost.

Now, p is the probability that in each attempt each action is successful. Also, we must be successful in 2 years and until success, we must do something in each time period. We have a little table here. I want to go over this table very quickly. So, if we do a natural crossbreed right now, remember it has a probability of success of two-thirds; then it would cost us 50 or 50 units. Again, this is a utility cost; it is not real rupees or dollar cost. It is utility cost and it will cost 50.

Now, if you were to wait for 1 year and do what we call as natural crossbreed, this is where you move one from one forest to the other forest and crossbreed them, then that would be more expensive a year from now. Why is that the case? Well, there is an increase in 20 percent. Because what happens is as time goes by, the population declines further and thereby it becomes more expensive. You need to find a cat, move them and then the population is very low there is very less chance of actually crossbreeding. So therefore, you know the cost that way is kind of going to be higher; the probability of success still remains at two-thirds. Now, that is the first option; our second option is to do IVF. In the IVF option, this is the in vitro fertilization and that you could do now itself costing 100 units and with probability 1, you are all done. So, that is a wonderful option.

Now, if you were to wait for one year and do it, you would spend 120 and with probability 1 will be done. If you wait for two years, it would be more expensive, but you would be done. The reason you may want to wait for longer and not do it now is because it is so much more expensive. Maybe if some of the cheaper options work out, I do not even have to go for the more expensive option. So therefore, I might be inclined towards using a disutility cost that is higher in the future as a worst-case situation. As a last option, we could do captivate sorry in we could do a mainline captivity right now, but we only get to use it one year from now and there is a cost of 10 units.

So, remember the only time we could do that is once right now and if we do not do it now, there is no point in doing it anytime later because it takes a long time for you to see the effects; turns out the probability is also 0.5. So, this information is stated very clearly here.

So, notice that all the costs, this cost as well as disutility cost, both of them increased by 20 percent each year; that is mainly because the population is declining and so, it becomes more expensive to perform the same action in the future.

So, this is our setup. We want to come up with a strategy that minimizes the expected disutility. Once again, we are using expected disutility in the same vein as we use the expected utility function in the previous case. That is because we already are taking care of this business of going from being risk averse to making a common objective by using a disutility function.

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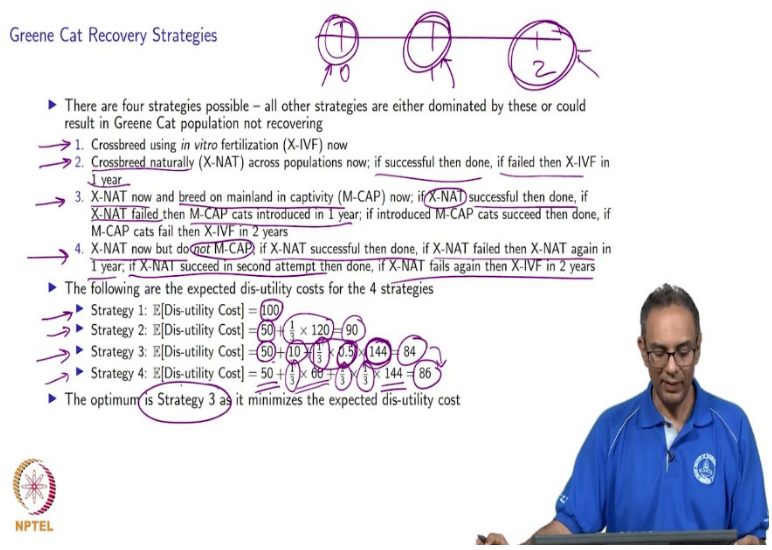

Now, here are our strategies. We have four strategies that are possible. Strategy is different from what are all options. So, the first strategy as you would have guessed, is to do IVF now and be done with it. Remember, you do this option and there is a 100 percent chance of it being successful; you spend 100 units and you are done. So, it will cost you 100 units and you will be done.

So, we will also do the calculation as we go. If we look at the first strategy 1, the expected disutility cost is 100, because you spent that 100. There is no uncertainty. You are going to get the 100 back and that is all it is going to cost you.

Now, let us see what happens in the second strategy. In the second strategy, we crossbreed naturally. So, this is the one where this is the red option and we crossbreed naturally. And then, if it is successful, we are done; if we fail, then year 1. So, remember we have year 0, year 1 and year 2. 0 is now. So, right here, if we try to crossbreed and if you are successful, we are done. If you are not successful in year 1, we will do an IVF and then we will be done. So, we want to be done by the way at the end of year 2. So, let us see what is that one's cost if you look at the cost, the expected disutility cost. So, you will spend 50 units just for the initial crossbreeding; that is the number here, the very first number here. So, you will always spend; then after that, there is a two-third chance that it will be successful and there is a one-third chance it would fail. So, there is a one-third chance that you will have to now spend 120 bucks, or 120 disutility cost to take care of the problem. So, that is why you had $\frac{1}{3}*120$. So, the expected disutility is now $50+\frac{1}{3}*120=90$; so, the expected disutility has become better. So, this strategy is better than the first strategy like we guessed earlier that we are giving this a shot. It is because there is a two-thirds chance that I will be successful and I could do that with a lot less money.

Now, let's look at a third option; the third option is a little bit tricky because we do all of them. The third option we do the following. We do the natural crossbreeding right now. So, this is the red option we had in this picture; we do the natural right now and we also at the same time, breed in captivity in the mainland. So, at time 0, we start the first one which is to crossbreed naturally as well as breed in captivity, do both in year 0. So, this is right here; do both. Then what happens is, if the natural crossbreed is successful, we are done. Yes, we did spend some money doing the captivity stuff, but that is okay. Now, if it failed, then what we do is we introduce these cats in the island after 1 year. So, at year 1, if it fails, we introduce those cats and if that is successful, we are done. Otherwise, we do IVF in year 2. Now, there is a reason we do not do IVF also in year one because after all we have gone ahead and done this captivity breeding. So, it does not make any sense. You can actually do the analysis and see that those strategies need not even be considered. But if you think about it, yes in theory, you can write down all the various strategies. But those are not going to be there for us. The cost of the strategy is the 50 bucks that you have to pay to do the crossbreed naturally in your year 0 that is the time right now. Then, there is a ten charge to also do the captivity in mainland; so, that is the 10 bucks. You also pay that. So initially, you do both, 50+10. Then there is a one-third chance that you would fail and there is a 50 percent chance that the breeding in captivity will fail and you must then do the IVF in year 2; so therefore, that would cost 144. So, if you think about it, we are looking at this probability multiplied by this, $\frac{1}{3} * 0.5 * 144$. So, the only time you are going to incur this cost is if both failed; both the zeroth year X NAT and the first years M CAP. If both fail, then you will have to incur this cost and if you did the calculation, strategy 3 costs 84. So, by far is the cheapest. In fact, you will see that that is the optimal thing to do, but let us look at the last strategy.

So, last strategy says we will do a natural cross breeding right now, but we will not do the mainland captivity. So, the natural one is successful; then we are done; if the natural one fails, you try natural again in 1 year; if it is successful, then we are done; if not, we will just do IVF at the last year. Remember that you have to be successful at the end of the at the end of 2 years. So, at the end, if nothing works, you will do IVF here. So, he tries once using the natural crossbreeding; if it does not work, try that again after 1 year; if it does not work, then go ahead and do IVF in year 3. Let us compute the cost 50 bucks first time anyway. If it is unsuccessful the first time, with probability one-thirds, you multiply that by 60 which is a cost to doing it the second year. Now, if that is successful, you are done. If it is unsuccessful again; so, both years unsuccessful, then you will spend $\frac{\frac{1}{3} * 1}{3} * 144$ units and do an IVF. The resulting cost of strategy 4 is 86, slightly more than strategy 3.

So, the optimal strategy is strategy 3, which is to do the natural crossbreed right now and at the same time, start planning the in the mainland to do the breeding to see if we can get a recessive gene. And then, introduce them a year from now and then, if that also fails, finally do the IVF; so, that is the optimal strategy. So, we will stop here and move on to decision trees next.

Thank you.