Decision Making Under Uncertainty
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Lecture – 13
Utility Function

The next lecture is on a topic called utility function.

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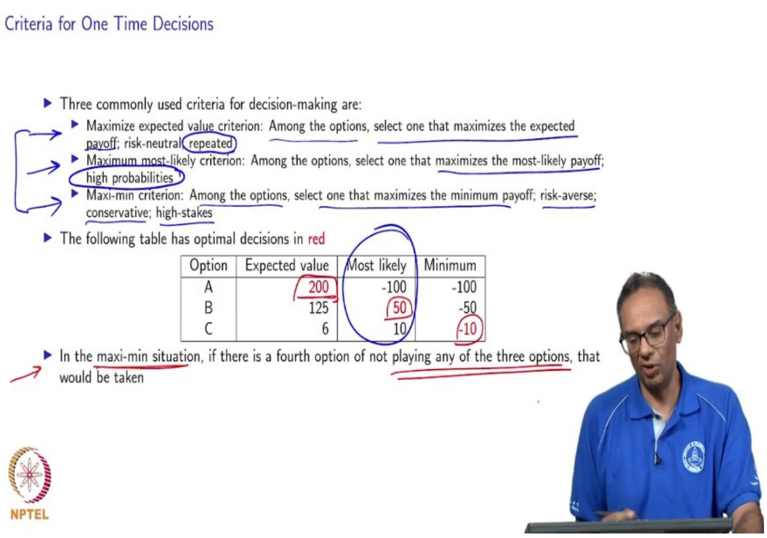

So, we saw last time that it was a little bit tricky between which objective to pick, whether should we do maximize the expected value or should we do the option of selecting the most likely or the minimum as a criterion. Which criteria should you select? Now, turns out that like I said earlier, we will not really get into this business of most likely; that is because it really is applicable only if the probabilities are extraordinarily high.

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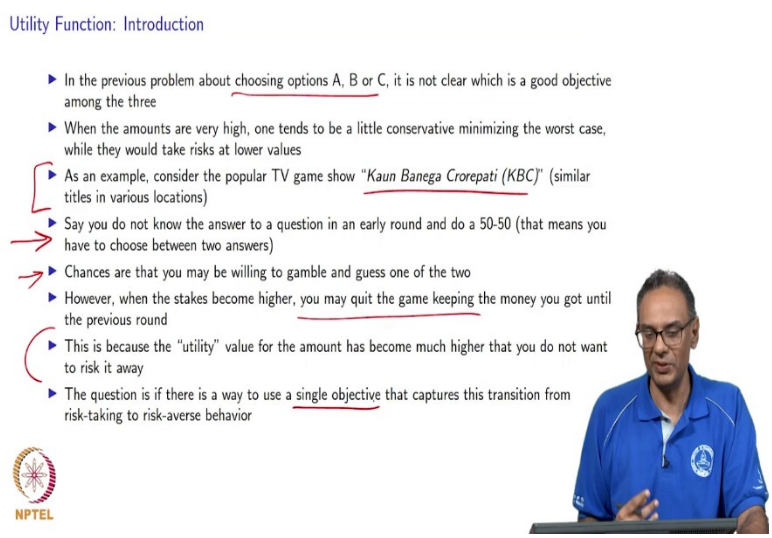

So, we will stick with these other two. Now, turns out that there is a nice way of actually merging those two using what is called utility function and I am going to give you a brief introduction about this. So, if you think about it, in the previous example, we had just three options A, B and C. It was not very clear which is a good objective like I said a little while ago, and we probably will not worry too much about the one in the middle.

Now, turns out that if you think about what you would do, many of us might prefer to play option C as opposed to A, because when the amounts are pretty low, you do not mind taking a bet. However, if the amounts are very high, you say maybe not. Now, if you have seen this TV show called "Kaun Banega Crorepati"; it turns out that this is still going on at this point when we are recording. If you have never seen the show, I would recommend pausing this video, going to YouTube and actually watching at least one episode of Kaun Banega Crorepati. There will be at least 3 or 4 people who would come and do what I am going to be talking about. I seriously recommend pausing the video and going and watching that because this example will really ring nicely after that; otherwise, this would just look like an exercise that is purely academic.

So, turns out if you have watched that show and you have come back to see this or if you watched it before, people do take some risks in the early rounds. When the number of rupees or the amount of money that is in stake is small, even if you do not know the answer, you take a guess, because going back home with a small amount of money is not terribly exciting.

So, here is the deal. As the numbers become higher, think of this situation where you are out there. You are the participant and you have amassed quite a decent amount of wealth and then you come to a question for which you have no idea what is the answer. Then, you would say or the person who is compering the show is going to say “Do you want to go 50-50?” and then you would say “Oh! Okay. We will do that.” and then two options go away; you are left with two. Now, you have a choice of either selecting one or the other by maybe flipping a coin or you are saying, “Well! I will just take whatever money I have and I will leave”; so, you have that option.

So, turns out that when the stakes are a bit low, you might be picking that option; but once the rupee amount goes higher, you might go ahead and pick that. I do want to say that we want to reflect this in what is called a utility function. Larger amounts of money are more useful to us than smaller amounts of money. Therefore, you do not want to risk away large amount.

So, if the amounts are larger, we tend to be a little bit conservative and if the amounts are smaller, we are willing to take risks. So, in some sense, in the previous case, we are saying, “Oh! We are changing our objective from maximizing, say being risk neutral, maximizing the expected value towards maximizing something or rather maximizing the worst-case situation”.

So, you are kind of changing that objective. Is there a way to capture that? I do want to say one thing. I forgot to say that the show Kaun Banega Crorepati has come in several languages. The one that is in English is called “Who wants to be a millionaire”. There are versions in other languages. I know for sure there is one in Tamil. If you are not familiar with the Hindi language, you can go ahead and listen to those; it would be the same thing.

So, “Who wants to be a millionaire” is what is the show in English. The idea is pretty much the same thing. Now, turns out that like I said before, if the amount that is up there is small you might be willing to gamble and you pick one of the two options when you had a 50-50. But, if the amount is large, usually I just take the money and go. So typically, you will quit when the stakes are high and you might risk when it is low; this is what is called utility.

So, the utility value is not linear in some sense. So, we want to find a way to see if there is a function that we could use that captures this behavior. So, we would be able to use a single objective to figure out which bet is most ideal for us; that way, we will have just one objective. I do want to say, repeat one more time that there is most likely stuff, we are going to put it away because that is not something that is terribly useful. It's not really an objective that you would use unless the probabilities are really high.

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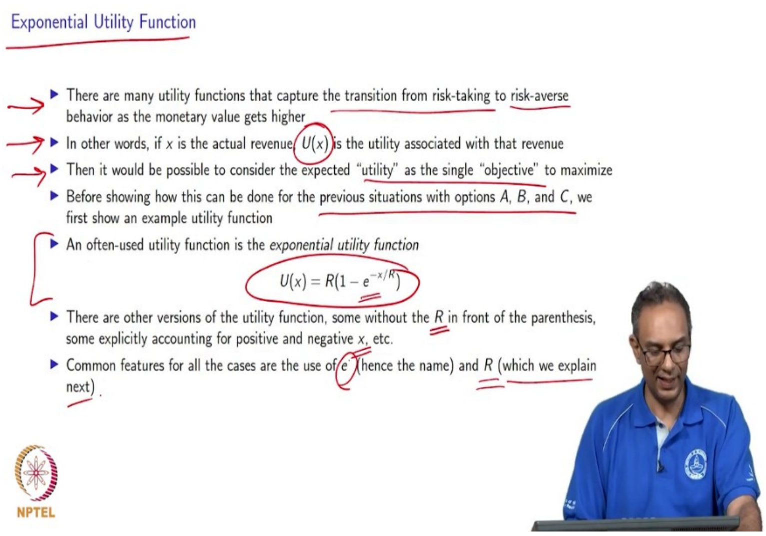

So, one utility function is what is called the exponential utility function. For the purposes of this course, this is the only utility function that we will talk about. But if you go and read up, there are many other utility functions and I would recommend that you go ahead and look at that. There are also a lot of people that have criticized the use of utility function and again, I am not a proponent of this. But I do want to give you the option of finding a way to get a common metric if you will.

So, there are many such utility functions. Essentially what you want to do is, as you transition from risk taking or being risk averse, you want to capture that behavior. You have to capture that behavior where you are going from being risk averse to being, you know, I am saying, by saying risk taking, what I really mean is, you are being risk neutral.

So, we want to use a function U(x) and we don't worry too much about the function itself. As x increases, you want to capture your utility. Now, you want to then take this utility function as a single objective that you want to maximize and compute the maxima of that function. Turns out that we could do that exact same thing and pick one objective and optimize among the three options which is what we will see here. One utility function that is used a lot is what is called the exponential utility and we write down the exponential utility function like, so U ¿

. Now, there are other utility functions as well. But we are picking the exponential just for the sake of picking one and presenting it here. Now, there are many different versions of the utility functions; some of them use the R in the front like I am; some of them remove that; some people explicitly take care of positive and negative values. I am not doing that as well. So, we are just picking something fairly simplistic. The idea is to show you how this works and we will do a little demo towards the end so that it will give you a better idea.

Now, the reason we call this exponential is because you have this $e^{\frac{-x}{R}}$. And therefore, that is the reason it is called the exponential utility function. What is this R? That is an important thing we need to talk about and that is what is going to tell you how you are going to switch from being risk neutral to being risk averse. So, the R parameter is fairly important in this analysis which is what we will see next.

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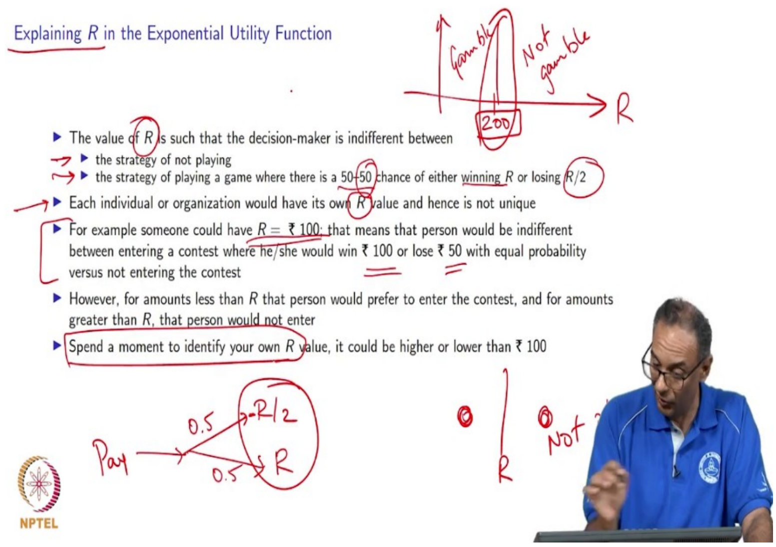

So, we are going to explain this R guy and in the context of exponential utility function. So, this R is explained in the following way. So, let us say we go on increasing the value of R. Let us say you have smaller values of R.

So, I am going to plot a graph and I am writing R here. So, let us say when R is 200 rupees and you are saying that I willing to risk if it is less than 200 rupees; when its above 200 rupees, I am saying that I would not gamble. That means, the amount is so high I don't want to lose out 200 rupees.

Now, we want to be a little bit more precise here. We are saying the value of R is such that you as a decision maker are going to be indifferent between these two options. The first option is not playing; so, essentially this is my R; however, my game is a little bit different. It is not like a gamble or not game. I am going to tell you what the game is. The game is the following: there is a 50 percent chance of getting R rupees or there is a 50 percent chance of losing R divided by 2.

So, that is the gamble. So essentially, what we are talking about is there is a 50 percent chance that I would get -R/2 at the end of it and there is a 50 percent chance that I will have R rupees. So, that is your eventual outcome.

So, let us look at this now. Each person will have his or her own R value. Some of us who are very conservative will have a small R value; for us, maybe it will be 10 or 20. Some of us are either not just conservative; if you also have a lot of money, for that person, 2000 rupees may not be a whole lot of money; maybe 20000 is probably reasonable for that person.

So, each individual has a different value for this R. Each company has different value of the R. I mean they talk about company culture; this is what we are talking about; we are talking about the culture of being risk averse or risk taking.

So, let’s look at a specific example of R is 100 rupees. So, what does that mean? So that means, you would be equally happy not playing the game where you will get 100 rupees or lose 50 rupees with same probability or play the game. So, that is the value. If you are less than 100, you will surely play the game. So, let me just put it this way. If I was smaller than R, I would gamble. If I am bigger than R, I would not play.

So, that is the same example we saw here. The probability of winning and not winning is 0.5 and 0.5. And, the amount of money you would win is R or lose R/2 with equal probability; so, that is that value. So, let us think about what would be such value of R that you would like to pick for yourself? It is important to think about that. Now, why you do that? So, you need to think about what is your own R value; so, is it higher or lower than 100? It could be higher or I would be not be surprised if it is lower than 100. So, what is an amount of R that you would feel comfortable enough to gamble.

So, basically what is the largest number that you would be willing to consider to gamble away, either you are losing R/2 or you are winning R rupees.

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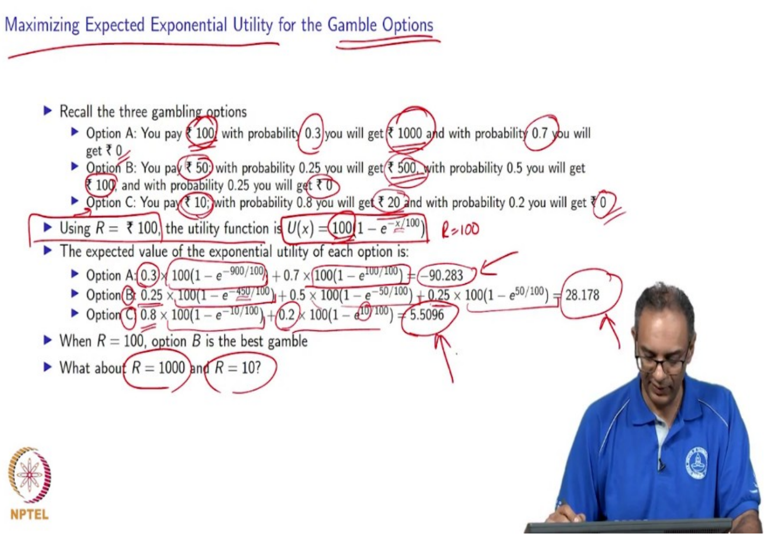

So, let us move along. Now, what we do is, we try to maximize the expected exponential utility; that's what we are trying to do for these three gamble options. So, what we will do is we will look at the three options. So, remember that you pay 100 rupees in the first option and then with a 30 percent chance you will get 1000 rupees and with a 70 percent chance you will get 0. That means, you will either win 900 rupees net with probability 0.3 or you will lose 100 rupees with probability 0.7. Notice that this is different from the R and the R over 2 we talked about earlier. This is because you want to first select your R right here like I said. Spend some time thinking about your R before you move to the next slide. Once you have done that, you are looking at the three options and then the second option is, you pay 50 rupees and with it, there is 25 percent chance that you will get back 500 rupees. That means, your net gain is 450 rupees with probability 0.25 and your net gain is 50 rupees with probability 0.5 and you lose 50 rupees with probability 0.25. The third option is one in which you pay 10 rupees and then there is 80 percent chance that you will get 20 rupees back and there is a 20 percent chance that you will lose that 10 rupees.

Now let us take a person whose R is 100. That means, in the previous case; that means, if the amount was less than 100 rupees, you would be betting and if the amount was 100 rupees, you will not enter the bet. Then, the utility function for that person is given by this, $U(x)=100\left(1-e^{\frac{-x}{100}}\right)$. So, this is the utility function that is important for us. So, if you notice here, the utility function is U ¿.

Now, let us compute the expected utility. For the first option, we compute the expected utility in the following way. Remember the expected value of a function of a random variable. So, $U(option\,A)=0.3*100\left(1-e^{\frac{-900}{100}}\right)+0.7*100(1-e^{\frac{-100}{100}})$. So, if you did your calculation, your expected utility is -90.283 units; it is not rupees, it is units. So, utility function is not really in rupees although because we multiplied by 100 here in this case, it is still rupees; however, it does if you do not have the R in there, then it is just another dimensionless quantity.

Now, turns out that the second option we do the same thing as probability times utility value. So, it is $U(option\,B)=0.25*100\left(1-e^{\frac{-450}{100}}\right)+0.5*100\left(1-e^{\frac{-50}{100}}\right)+0.25*100(1-e^{\frac{50}{100}})$ and the expected utility is 28.178. And the third option, option C tells me that $U(option\,C)=0.8*100\left(1-e^{\frac{-10}{100}}\right)+0.2*100(1-e^{\frac{10}{100}})$ and the utility is 5.5096. I mean what if it was 1000 or 10? We ask two questions. What if you might have a different utility and not 100?

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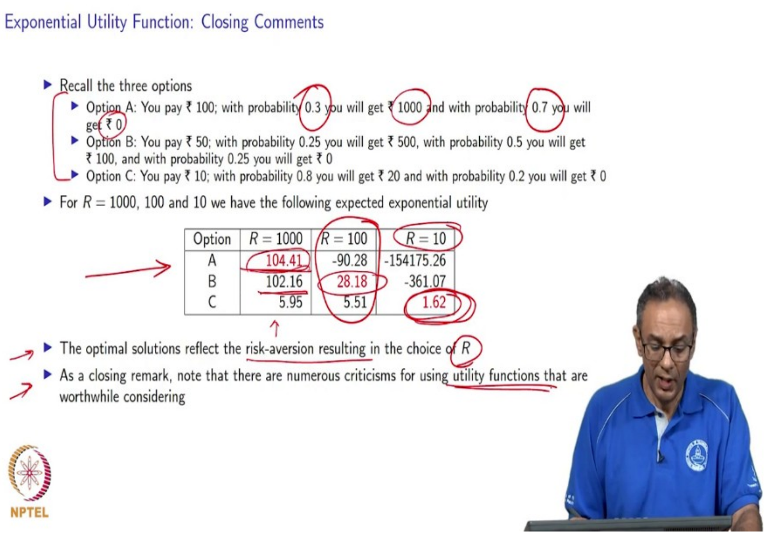

So, let’s go to the next slide which is as follows. If I plugged in a different set of numbers one more time, remember these all still remain exactly the same. The first option is to use is to gain 900 rupees with probability 0.3 and lose 100 rupees of probability 0.7 and so on.

Now, I have created a little table here. When your utility function is 100, we saw in the previous slide, -90.28, 28.18 and 5.51 which we saw here right; 90 here, 28 here and 5.5 here. That is exactly the same numbers here except I have rounded off to the second decimal to a paisa. And also, if you look at the 1000 one, notice here that because utilities are as high as 1000, this person is probably happy enough to make this bet because for this person, winning 1000 rupees or losing 500 rupees is something that they would be willing to consider as a bet. So, for this person, this is a reasonable gamble to make. But also notice how much just a little bit bigger than the second option, okay. And, that is still because the probabilities are not 0.5 and 0.5. So, you look at this and if the R value is really small, your expected utility is really tiny.

So, depending on your R value, that is what becomes a big deal in terms of which option you would use. So, you would be extremely conservative if your R value is small, and you will pick the third option; if your R value is high such as 1000, you would pick the option A; if your R value somewhere in between say 100, then you would pick option B.

So now, your best option depends on how conservative you are. A smaller value of R means you are more conservative, a larger value of R means you are a little bit more aggressive than the person who put 10. Now, you could say – Well, my R could be 10000 or something like that. Then obviously, you are not as conservative

Now, it is also a matter of what does that amount of money mean to you? For someone, 10000 rupees is a huge amount of money and they would not be willing to risk that. For some other people who are much well off financially, for them, that might not be the case. Before I move on to a demo where we will try the 10000 example, let us quickly make some closing remarks.

So, the first thing is that the optimal solutions essentially tell you a little bit about how risk aversion a person you are. So, the more risk averse person would play the less risky option. Basically, smaller the R, the more averse to risk you are. As a closing remark, I do want to say that a lot of people criticize this utility function thing. I am not again a proponent of something like this; it is just to let you know that this is something that people use quite a bit in companies; especially, if it is not your money, it is a different situation.

One of my teachers and school used to tell us this when we had our first computer back in the 80s in our school. The teachers would say - Think of it as your own computer. What my teacher essentially was trying to say there was, take good care of it like it is your own. Well, while we conveniently misunderstood that as think of it as your own, do not bother, play with it as much as you like. Well, in reality what he was trying to say was, think of that as your own thing, your own money. A lot of companies also do the same thing. When you are at a high position like a CEO, you are pretending like it is your own money when you are gambling; so that means, in taking risky ventures, alright.

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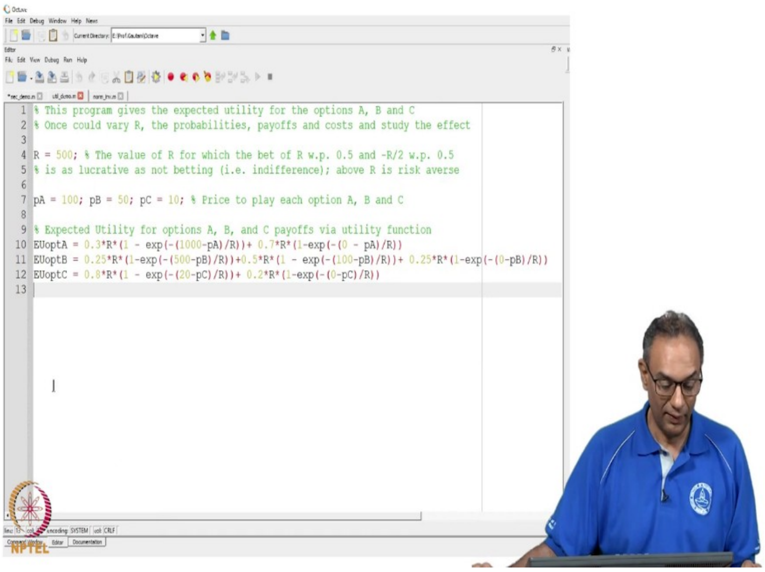

So now, I am going to move to the demo. So, what I am going to do now is do a little demo of what we just saw. So, this is again on Octave. Actually, all my demos from this course are going to be Octave. So, turns out that what we want is we are given these three options A B and C and we want to vary the values of R and you could also change the probabilities.

In fact, I am going to give you this file for you to use. You are welcome to go ahead and play around with these probabilities a little bit; play around with the payoffs, you can play with all those numbers, you can also play around with the cost and see what happens.

So now, I am going to pick somebody whose R is 500. Now, we will do the 10000 a little bit later. So, let us say I have somebody whose R value is 500. This person is willing to bet anything less than or equal to 500. This person is willing to go into a bet which says I will get 500 rupees or lower with probability 0.5 and I will lose 250 rupees or lower with probability 0.5. So, that is this 500 if you remember, alright. So now, this is the three numbers right; for playing the option A, you pay 100 rupees; for playing option B, you give 50 rupees; for playing option C, you give 10 rupees. If you look at the expected utility, I am essentially writing in the following way. In option A, you would get $1000-pA$ with probability 0.3 where pA is 100 rupees or you would get $0-pA$ with probability 0.7 and I have converted into utility values. Similar computations are performed for options B and C as explained earlier.

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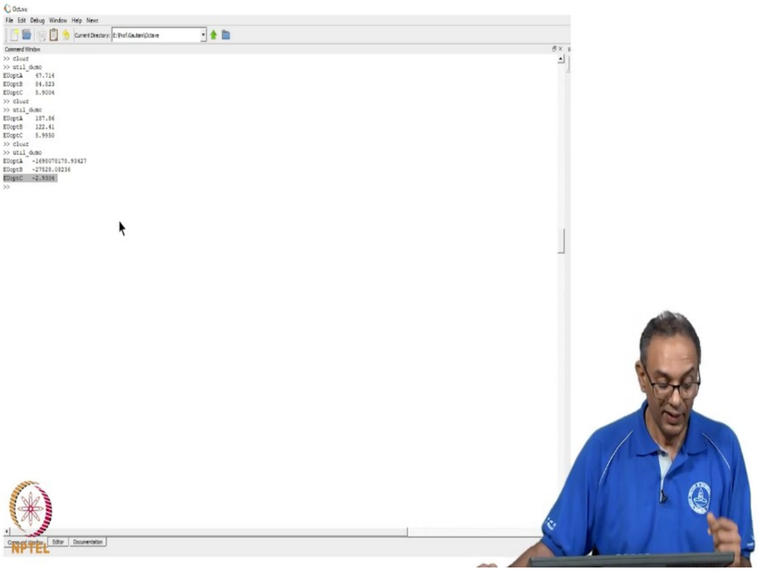

So, let us see what happens when I use R=500. So, let me go here. Let me just clear all the previous example right here and then let me hit clear anyway. I do think there is a clear there in here and right in the beginning like we do always, I guess I did not put it clear. So, I am glad I hit the clear. So, I will do util_demo. So now, if you look at the values, it says that the utility of option A is 47.714, option B is 84.823, options C is 5.9004. So, these are my utility values.

So now, what happens for me is, if my R equals 500, my best option is B - the second option which is 84 which gives me an expected utility of 84.823. So, this is a gamble that I would be willing to play.

Now, if my R value is way higher; so, it says 10000. I am going to save this file using Ctrl+S. Now, let us clear one more time and then let us do util_demo. I presume it to depict my 500; yes, I mean my 10000. Now, if I use 10000 as my value of R, then my expected utility is much higher, 187.86. Clearly for somebody whose R values as high as 10000, for them to play the first game certainly makes sense; it is the best of the three games that they should be playing.

Now, if I had a much smaller value. Let us say mine is only 5 rupees. Let us see what happens at an extremely small value. I save that. And then, I do clear and run it one more time. If you cannot see this, I would recommend downloading the program util_demo and then running it. Notice here that all the numbers are negative; even the last one is negative; that is because even 10 rupees is very large for somebody whose R value is 5 and therefore, would not be playing any of these options as the net utility is all negative.

So, I just wanted to say that depending on his value of R, different objectives become bigger. So, if R is 10000, your best bet is option A; if the R value is 500, your best bet is option B; if your R value is something as small as 5, you never even bet here. In fact, if your R value is 10, then at least there is a chance for you to pick option C; however, if your R value is only 5, then even that is not a lucrative option; you should not even be betting in that situation. So, I will stop here. However, I encourage you to play around with these numbers when you get a chance.

Thank you.