Decision Making Under Uncertainty
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Lecture – 12
Which Option to Gamble Just Once?

In this lecture, we will look at which option to gamble just once. So, remember the title of this topic is one-time decision. So, like the secretary problem that we saw before, we were going to make just one decision on which person to hire as the secretary, again here too, we are going to make just one decision on which option to gamble. And, I am going to gamble just once; I am not going to keep gambling.

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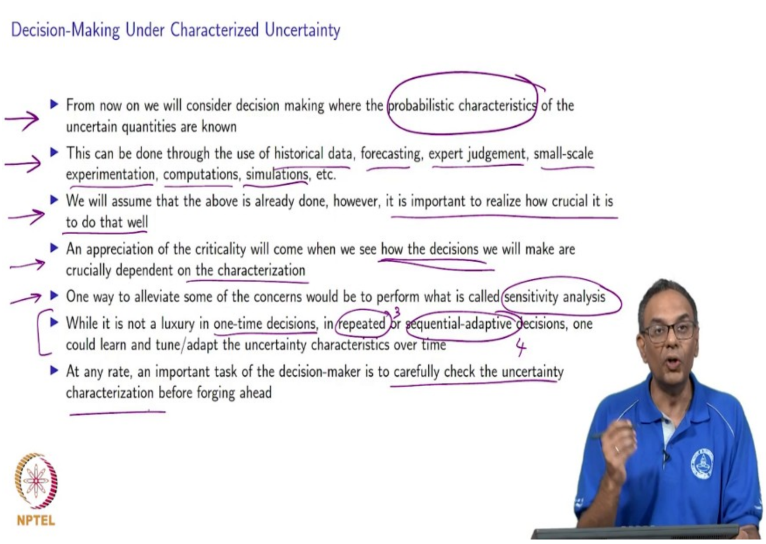

So, I do want to say a few things before we get to the gambling problem. We want to talk about decision making under uncertainty, which is the title of this course. And, we were characterizing the random variables like we saw in the first topic.

However, when we look at the secretary problem, we did not say much about what those numbers are from. Although I did generate random numbers, that was just to for me to make this program run an Octave that I needed to somehow create some random numbers. But in reality, that does not have to be known. However, for the remainder of this course, we will always assume that there are some probabilistic characteristics of the uncertain quantity that we know; we know the probabilities.

Now, this is not available for free. I mean, in this day and age where people are looking at big data, they are collecting a lot of historical data to do these types of probabilistic characteristics or to obtain these types of probability characteristics. But one could get this historical data, use some fancy forecasting models or you could look at experts and they will give you some of their judgments. We could also do some small-scale experimentation. We could try a few things, but then, you can learn from that and go from there. You could also do some significant computations and simulations. And, these are all different ways by which we could get some probabilistic characteristics.

Now, for the rest of this course, we will always assume that someone has done that hard work and given us some type of probabilistic characteristics. Now, this is an important thing for us to take away. One has to do a good job of how that is to be done. So, it is important that we do the characterization extremely well.

Now, you will see later why it is so important because a lot of our results are going to be in terms of those probabilities. They will critically or crucially depend on how these probabilities are characterized. So, our decisions will depend heavily on the characterization of the probability. So, we could take a little bit by doing some type of sensitivity analysis; that means, I perturb my numbers a little bit and see if I make different decisions; one could do that. I think that would be something that I would highly recommend.

Turns out that these types of problems that we are talking about in this topic are what are called one-time decisions. I only make decision once. I do not repeat. When we do repeat it (topic 3) or when you do sequential adaptive decision (topic 4), we will see other things we could learn and tune and adapt and things like that. However, in this, you don't have any such luxury; you only get one chance you do it and you do it right. Otherwise, you are out. Now, turns out that it is important to check the characterization before forging ahead.

So, why is that the case? Well, especially when you make a one-time decision, you want to be sure you get the right type of characterization. Now, I do want to pause for a moment and say that many times there are several companies that tell you, please give us the forecast and we will make the decisions, but then you should be sure that you have the right kind of forecasts.

For example, if you just get the averages which is what most forecasting will give you, just give you one number, this is the mean; that may not be very useful if there is a lot of uncertainty in what you are looking for. And therefore, you want to be sure that you have the right characterization before you go about making decisions.

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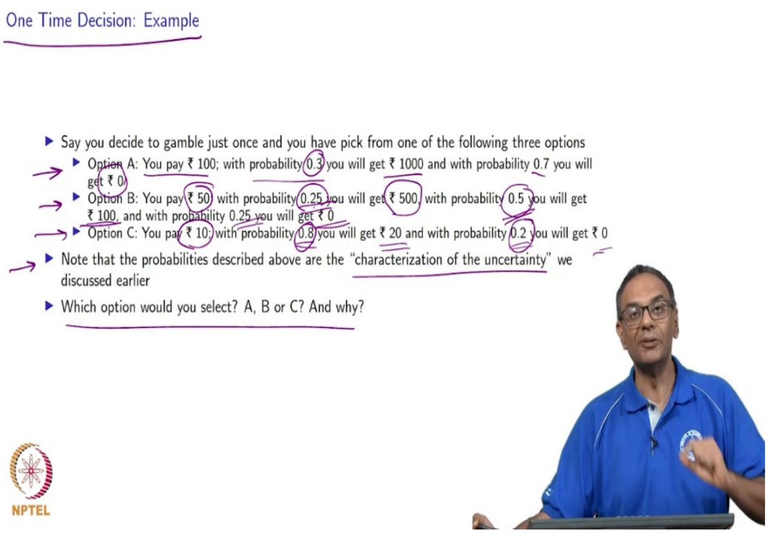

Now, we will look at an example of a one-time decision. Let's say for example, you have decided to gamble. I do want to say one quick thing; gambling is not a good idea. This is just for the sake of illustrating this topic that am talking about gambling. And also, no gambler will give you these kinds of lucrative options. But, let us go ahead and play this for the purposes of this course. So, there are three options. In the first option, you pay 100 rupees to pay this game. With probability 0.3, you will get 1000 rupees and with probability 0.7, you will get 0 rupees. That means, you will lose 100 rupees with probability 0.7 and you will gain 900 rupees with probability 0.3; that's option A. Option B is as follows. You pay 50 rupees only. Your upfront cost is smaller than before. However, there is a quarter chance that you will get 500 rupees; there is a 50 percent chance that you will get 100 rupees, and there is a 25 percent chance that you will get 0. In other words, you could either gain 450 rupees with probability 0.25 or gain 50 rupees with probability 0.5 or lose 50 rupees with probability 0.25. So, the last option is lose; the first two options are gain. The third option is the following. You pay 10 rupees much lower than 100 or 50. But there is an 80 percent chance that you will get 20 rupees back. And, there is a 20 percent chance you will get nothing. In other words, you will gain 10 rupees with probability 0.8 and you will lose 10 rupees with probability 0.2. The question is - which option would you select?

So, you can only gamble once and you can only pick one of the three options. The question is - which one would you select? Now, I do want to say one other thing. Notice here that we have characterized the uncertainty. We have said something about what the random numbers are going to be. When we say, you are going to either get 1000 or 0 with probability 0.3 or 0.7; so, you are already characterizing the uncertainties. So, it is not like I have no idea what I am going to get, which is true in many gambling situations. So, if one goes to a casino, then you would not know what is the probability that you are going to get a payoff. Chances are that you more likely than not, you are not going to get money. So, that's why am saying gambling is not a good idea. Nonetheless, let us say this artificial gambling situation where you can get 1000 rupees and you know the number is 0.3 with that probability. You could also get 500 rupees after paying 50 rupees with probability 0.25 and so on. So, which option would you pick? I want you to think through this a little bit before we move on to the next slide. So, if you wish to pause your video, go ahead and do that and think about which one you would pick? It is important that you play this. If not, this would be a purely academic exercise. We want you to be involved and think about what you would decide.

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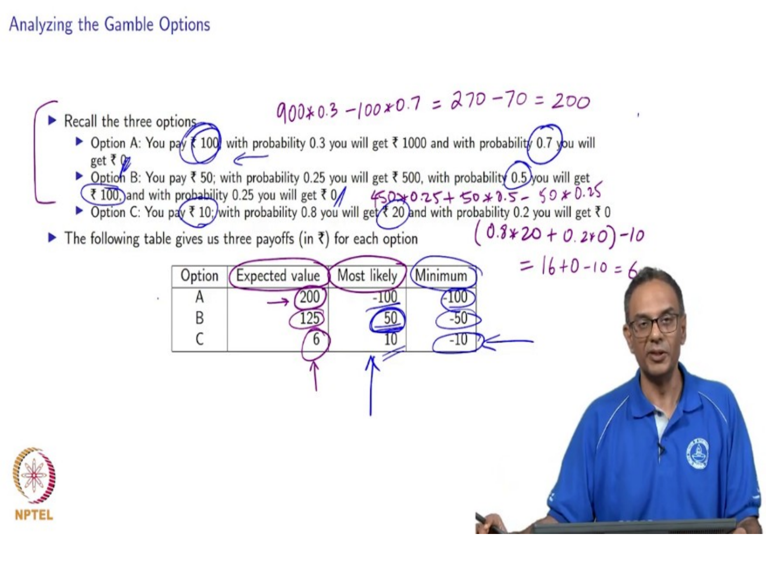

Now, let us analyze the options. So, remember the three options are written here. I will come back to them in a second, but I am going to come up with three different ways of selecting. So, most people will either do one of these three. Either, they would look at the expected value; remember this is the expected value calculation that we did before in the first topic. So, how do you get this number? So, let us compute the expected value. So, in option A, what is the expected value? Well, you will get 900 rupees, that is because its 1000-100. So, you get 900 rupees with probability 0.3 or you would lose 100 with 0.7. So, that is 270 minus 70 which is 200. So, that is how you get this number 200, which is $900*0.3-100*0.7=270-70=200$. So, option A's expected value is 200. So, what does that mean? If I played this game over and over and over again, on average, I would be better off by 200 rupees. So, this would be a phenomenal thing to bet, if you were playing this gamble over and over again, but you do not have that luxury; you get to play just once. Now, let us look at the second option B where you will pay 50 rupees. So, you will get 450 rupees with probability 0.25 or you would get 50 rupees with probability 0.5 or you would lose 50 rupees with probability 0.25. And if you did the calculations, you would get $450*0.25+50*0.5-50*0.25$ which is 125. So, you will be better off by 125 rupees on average. So, that is the expected value gain. We are not doing this over and over to be thinking about averages. This is a one-time decision. I do want to make that very clear. I am just telling you what is the expected value. Lastly, how did you get the 6 in option C? Let's say you played option C. There is an 80 percent chance that you will get 20 rupees and there is a 20 percent chance that you will get 0, but you also paid ten rupees. So, it is $(0.8*20+0.2*0)-10=6$. So, these are my expected value calculation. If my objective is to maximize the expected value, then I would go with option A because that is what maximizes; however, I am not playing this game over and over and over again. So, it is not clear that the expected value is what we should be maximizing.

One thing that some people like to maximize what is called most likely. So, now let's see what is the most likely option. I am going to change my pen color just for a second so that my second option is clear in this color. Now, in the first situation A, most likely is probability of 0.7. So, most likely, I will get 0; so, $0-100=-100$. So, that is what will happen most likely. In option B, most likely option is this 0.5 and gaining 100 rupees, and you pay 50 rupees upfront. So, your payoff is 50 rupees. And, in option C, the most likely possibility is this. So, you pay 10 rupees upfront and you get 20 rupees. So, the most likely option is 10 rupees. So, if you look at it, if you want to maximize the most likely which is another objective, then you would select option B. So, this is what is the most likely; however, we will talk next when we go to the next slide about when would you the condition like most likely. In situations like this, this is probably not the best thing to do.

The third one which is quite popular among a lot of people in making one-time decisions is the conservative option of what is called the minimum. So, what is the worst that could happen to you in the first case A? The worst thing that could happen to you is you would be losing 100 rupees, because everything that you paid in the beginning, you are going to lose it all. In option B, you paid 50 rupees and get back 0. So, you would be down by 50 rupees. The worst thing that could happen in option C is that you will lose the 10 rupees that you paid initially. So, the best option of the lot is option C where you would be losing 10 rupees; so, the most conservative option would be option C. So, the best expected value option is option A, the best most likely option is option B, the best minimum option is option C, which we will see in the next slide.

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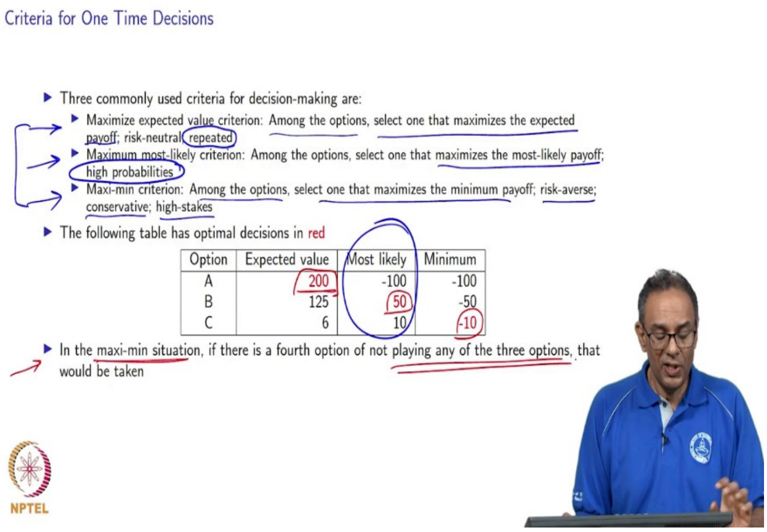

So, that's exactly what I am saying here. But I do want to say a little bit more about this. So, when would you maximize the expected value? So, that is the question. The answer is if you are extremely risk neutral, you would maximize the expected value. So, this option is what is called risk neutral option; you are not taking too much risk, you are also not being too conservative. But, this is most ideal when you have to make repeated decisions. So, which one would you pick? Well, among the options, pick the one that maximizes the expected payoff. So that is what is the maximize expected value criterion. So, that is the criterion that one could use.

Another criterion that one could use is what is called maximum most likely. Among the options, which maximizes the most likely payoff? We saw that earlier that is option B.

And finally, what we call maximin; this is a conservative option. This among the options picks the one that maximizes the minimum payoff; maximizes the worst case. This is what is called the risk averse option. You do not take any risk and you would want to pick this; this is also called a conservative option. You typically do pick this when the bets have very high stakes. You want to be a little bit careful while using something like this.

Now, the first and the third are ones that are often used. The most likely criterion is not often used. You use the most likely when the probabilities typically are really high. So, I want to say that briefly. For example, you are standing and you want to cross the road; you are standing in one of the busy roads and you want to cross the road. If you were going to cross the road when it is absolutely safe and there is no vehicle on the road, you might have to wait for several hours. What most of us do is we start crossing when we are reasonably safe. So, that is when you are saying, "Well, most likely I will be okay; so, I am going to go ahead and cross the road". That is what we all do. In situations like that, it makes sense to take the most likely option as opposed to the conservative option where you wait forever. In fact, the expected value is also to wait forever; just think about this. So, if you say for example, there is a 99.99 percent chance that I will safely cross the road. Then, there is a 0.01 percent chance that I will not safely cross the road; then the expected value, if you think of being alive and being dead, clearly you would want to be alive and therefore, you would not cross the road. In fact, in the expected value case, the best option is to stay in and cross only when it is absolutely safe; same with the minimum or the most conservative. But the most likely criterion says that with 99.99 percent chance, I am probably going to be safe; so, I will go ahead and cross it, which is what we all do all the time. So, under those extraordinarily high probability situations, I would go with a decision like most likely.

So, to summarize, what we saw was, we would take the expected value of decision and pick option A if that is my criteria; again, I would do that either I was risk neutral or I was making repeated decisions. I would pick option B if I want to maximize the most likely outcome. And, I would pick the third option if I were to be extremely conservative. I do want to say that in the extremely conservative maximin situation, the right thing to do is to not even play any of these three options and say forget it. I will take back my zero amount of money that I invested. So, that brings us to the end of this particular lecture.

Thank you.