Decision Making Under Uncertainty
Prof. Natarajan Gautam
Department of Industrial and Systems Engineering
Texas A&M University, USA

Lecture – 10
Introduction to One-Time Decisions

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We are ready to start the first major topic. This is called topic 2 for this course and the course is called decision making under uncertainty. The first topic is called one-time decisions.

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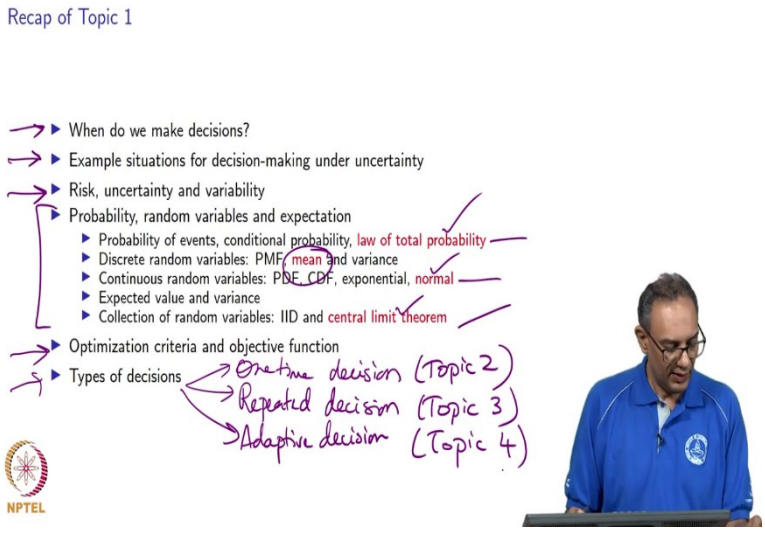

So, here is where we start making decisions; however, before doing that, I want to spend a little bit of time recapping what we saw in topic 1. We started with asking the question - when do we make decisions? We saw that we make decisions every day, all the time. We also looked at a few example situations in personal life where we were making decisions under uncertainty.

After that, we went into a somewhat controversial, it is not like people are fighting over it; but different people use different words for describing things and I just want to clarify when I say uncertainty, what I essentially mean is I do know some probabilistic information about what is going to happen in the future, based perhaps on historical data or some type of expert opinions.

Now, the mathematical part of topic 1 is essentially in the part that I just marked which is called probability, random variables and expectation. We started by talking about probability of events and then, we went into conditional probability and law of total probability. In general, I do assume that this material should basically be like a review for most people.

However, if you are not familiar with this, I would recommend going through some of these topics. For topic 2, it would be important for us to go over this topic which is the mean of discrete random variables; that's one thing we will hit a lot in topic 2. These other three things marked in red – law of total probability, normal random variable, central limit theorem; we will hit once exactly once during this topic 2.

So, I just wanted to let you know that if you are thinking about, perhaps going back and reviewing some of this, I would review the topic on mean extremely carefully; especially if you have not been familiar with the topic. And, I will also go over these topics if I were you - law of total probability, the normal distribution and its properties as well as the central limit theorem.

And, we do want to let you know that the other topics that are listed there, that are not in red color, are some things that we will do later on in the course. Actually, every one of those topics, we will touch upon during this course. So, if you are not familiar with some of them, it might be a good idea to touch base.

Now, at the end of topic 1, we looked a little bit at various criteria for optimization and what objective functions we should use and we also looked at different types of decisions. Here is

where we made the major distinction between one-time decisions and then, also talked about repeated decisions. And, we will talk about adaptive decisions. These are three things we will look at. One-time decisions is topic 2, repeated decisions is topic 3 and adaptive decisions is topic 4. So, that's the plan.

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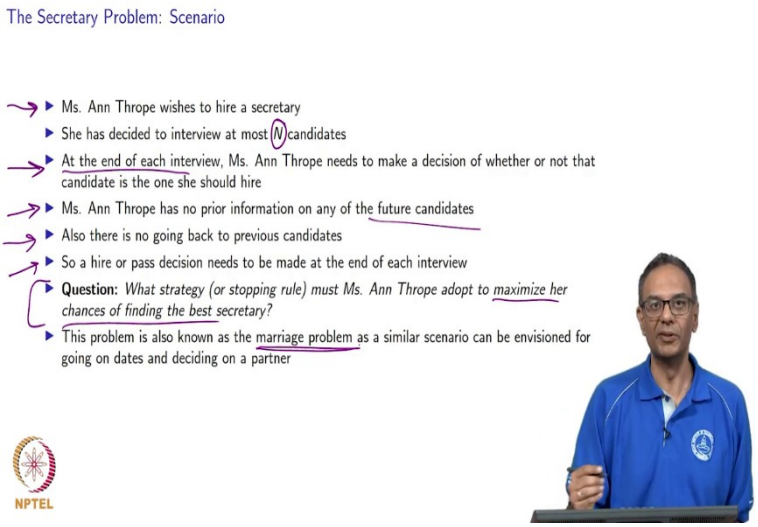

So, let's move on to topic 2. Now, I am going to give you first a problem called the secretary problem. So, turns out in topic 2, we are going to hit various problems which will be somewhat independent of each other, except two of them in the middle.

This problem is called the secretary problem. I am going to give you a scenario. So, there is this lady whose name is Ms. Ann Thrope. You will see why she is called so later. She wants to hire a secretary and this person Ms. Ann Thrope, decided to interview a maximum of N candidates. So, N is a number. At this point, let's think of this as a reasonably large number; it is not one or two; it is a decently large number. However, here is a difference. So, what we normally do is, we interview all our candidates and then pick the best one. That's not what we are doing in this problem. This problem has been specially created for a situation like what we want to do. So, we are looking at Ms. Ann Thrope trying to decide whether or not each candidate she interviews, whether or not to hire that person. So, what happens is, when an interview ends, this person decides - Do I pick this candidate or do I not? And, I cannot see who is coming later. I cannot go back to somebody before. So essentially, Ms. Ann Thrope has no prior information about who is going to come in the future. She does not have

their resumes or anything like that. Also, she cannot go back on previous candidates. Once she says no to someone, that's it; that person is out. So, what she needs to do is, she needs to interview someone and decide to hire them or to pass them. If they pass, that person is gone. We move to the next person.

So, the main question is - what strategy or what stopping rule? Stopping rule means when should she stop interviewing so that she would like to maximize their chances of finding the best secretary. So, that's the objective, to find the best secretary possible. And, we want to see what strategy should this person use.

Now, I do want to say that this problem has another name. It is also called the marriage problem. This is what the literature says. I am not recommending this procedure to find a partner. I just want to let you know that this is called the marriage problem. This is not what I did in my life. Nonetheless, this problem is called the marriage problem because if somebody would like to get married, they would like to perhaps go and date at most N number of different people. Here, it makes a lot of sense. These conditions like once you go on a date with someone, at the end of it, you have to decide whether this is a person or you want to pass and meet the next person. So, you cannot go back. Nobody is going to say, "Well, you ditched me first and then now you are coming back to me". That does not work. So essentially, that problem actually makes a lot of sense in terms of modeling it in this way. So, you meet one person and decide whether that person is to be chosen or move on to the next person; you keep doing this. So, what is an appropriate strategy? Alright.

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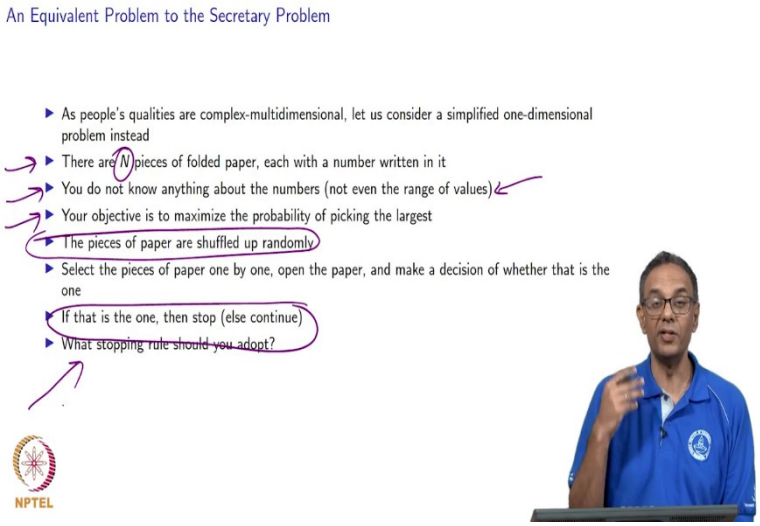

Turns out that this problem called the secretary problem, is somewhat tricky because people are very complex and they have very multi-dimensional characteristics. It is a little bit tricky. So, from a mathematical standpoint, we simplify this problem and I know it is not that exciting, but still for the purposes of tractability, let's say we have N pieces of paper and there is a number written in each paper and you fold up the paper. So basically, you cannot see the number. You just know that there are N different pieces of paper. Let's say N is equal to 30. Then, there will be 30 pieces of paper and you need to select the papers one by one. Now, you do not know anything about the numbers in the paper. This is an important thing. Now, if we are not assuming that the numbers are according to some distribution or some parameters, you have no idea what is in the numbers. You do not know the range of values either. You just do not know the distribution or the mean or the variance or the range; you know nothing about it. Your objective is to pick the largest in that set. What you need to do is you need to randomly select one paper at a time. So, either you shuffle up the paper and then select one, open it up and then, decide is this the largest. If you do not think it is, put it away, open the next one, read it and see if that is the largest; put it away and keep doing this till you think this is the largest. Now, it is important for you to make that decision. You cannot go back. You can make the decision on only on the paper that you have opened.

So, it is the same thing as the secretary problem or the marriage problem. You open it and decide whether this is selected. Yes, you have memory; you remember what you opened before. That is something that we assume. Now, you stop when you think you have hit the

largest one or you keep continuing. So, the question is what stopping rule should you use. What should be the rule that you use in order to decide when to stop? By stopping, what said is you stop and say, yes this is the largest one.

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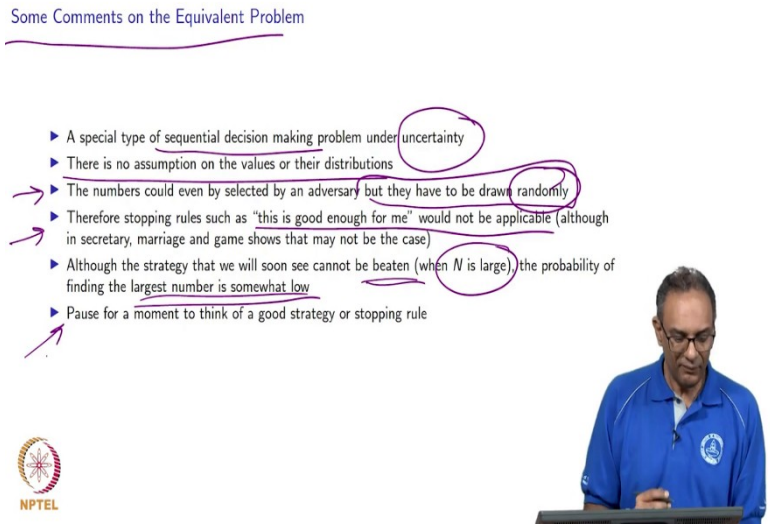

Now, I want to make some comments on this, what we call as the equivalent problem where you have a single number written on it. This is a very special type of what we call sequential decision-making problem and there is uncertainty, because you do not know what numbers are going to come in the future. So, what is in each paper? You have no idea what is inside that.

We are not making any assumptions like I said, on the distribution of those numbers. We could also allow the numbers to be selected by an adversary. What we mean by that is, let's say somebody wants you to lose in this game. They could put down any numbers they want to. I mean, you can see that there are various ways that you could pick numbers to make sure that the person selecting does not win.

However, we do want the numbers to be selected randomly; so, that part is still there. So otherwise, an adversary could just keep giving you numbers that are going on increasing and then, something like that, or some type of pattern going up and down, or give all the good numbers right away, some way, shape or form for you to lose often. So now, it has to be drawn to randomly; so, that is a requirement.

Now, there is another thing that is important. In life, what we normally do is not go through this and that's the other reason I did not pick the marriage problem or the secretary problem, is that in life you typically stop and say, "Well! This is good enough for me; "This secretary will work for me; I am happy with this". We tend to do that or you get married saying, this person is great for me and we will go ahead and get married.

So, that is what one does in life; however, here we will not allow for something like this. Your only job is to pick the largest. Now, turns out that there is a strategy is going to be optimal especially when N is large. In whatever way you pick the numbers, there is going to be an optimal strategy when N is large. However, I should say and in fact, that we will do a demo when we get to it, we will see that you would not win most of the time. Your probability of finding the large number is somewhat low.

So, I do want to say that it is not like this is guaranteed to give you, but this is a strategy that would maximize your chances of finding the largest value. I want you to seriously consider the difference between the two statements. Finding the maximum has a low probability; it is not like you are guaranteed to find it, but there is no strategy that would be better than what we are going to talk about.

Now, before we stop here, I want you to pause the slides and think about what is a good strategy, what is a good stopping rule that you will adopt. I want you to spend the time thinking about it. I would not recommend going forward and looking at the next set of slides until you have figured out a good strategy for you to use. Thank you. We will stop here and we will continue with the next lecture.

Thank you.