Introduction to Operations Research
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Module - 02
Graphical and Algebraic Methods
Lecture - 07
Graphical Method (Minimization)

In the last class, we saw the Graphical Method to solve a linear programming problem.

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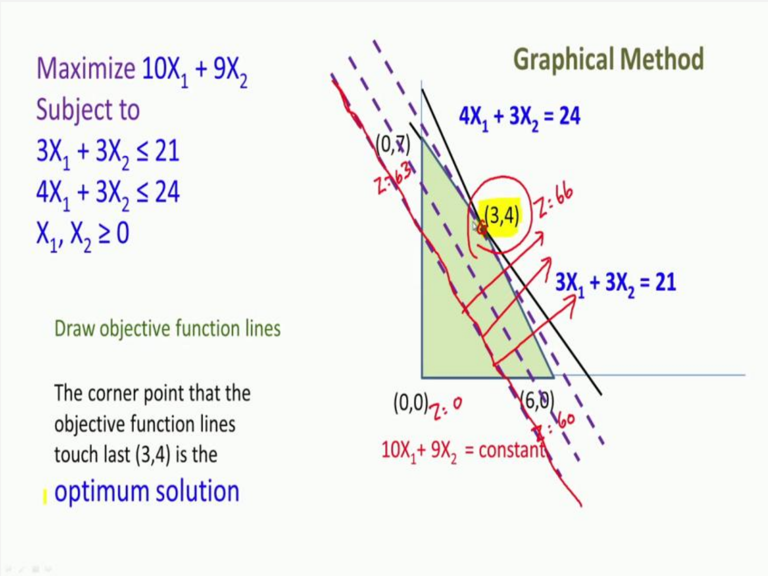

We considered the example that is shown which is a maximization problem that maximizes 10 X 1 plus 9 X 2 subject to 3 X 1 plus 3 X 2 less than or equal to 21, 4 X 1 plus 3 X 2 less than or equal to 24, X 1, X 2 greater than or equal to 0. We do the graph corresponding to the constraints, we identified the feasible region which is shown in a different color and we also computed the corner points and then, we observed that it is enough to evaluate at the corner point. And we also found out that this corner point the corner point 3 comma 4 the optimal solution lies in this.

When we evaluated the objective function value at the 4 corner points 0 comma 0 gave us z equal to 0, 6 comma 0 gave us z equal to 60, 0 comma 7 gave us z equal to 63 and 3 comma 4 gave us z equal to 66. Since, we are maximizing this function, we said this is the corner point. So, this is the corner point 3 comma 4 that has the best value of the objective function and hence is the optimal solution. Now, let us look at the same thing from a slightly different way.

Now, what we do now is, we draw objective function lines I have shown a few objective function lines. So, what we have essentially done is the objective function is 10 X 1 plus 9 X 2 and we keep 10 X 1 plus 9 X 2 equal to a constant. For example, we could keep 10 X 1 plus 9 X 2 equal to 50. So, this line could represent 10 X 1 plus 9 X 2 equal to 50 and this straight line is drawn and then we draw objective function lines which are parallel to this for example, this could represent 10 X 1 plus 9 X 2 equal to 60 and so on.

So, as we increase 10 X 1 plus 9 X 2 to a constant and keep increasing this constant, the objective function lines become parallel to each other and move in this direction. You can see that the objective function is moving in this direction and as it moves the objective function line. Finally, it touches the feasible region at the point 3 comma 4, the corner point where the objective function lines last touch before leaving the feasible region is this point 3 comma 4, which is the optimum solution.

So, there are two ways of getting to the optimum solution, one of which is to evaluate the 4 corner points. In this example there are 4 corner points, in other examples there could be a different number of corner points. Evaluate the objective function at all the corner points and find out which corner point has the best value. The other is to draw a series of objective function lines, where the objective function is a constant and if it is maximization, increase the constant the lines move in this direction.

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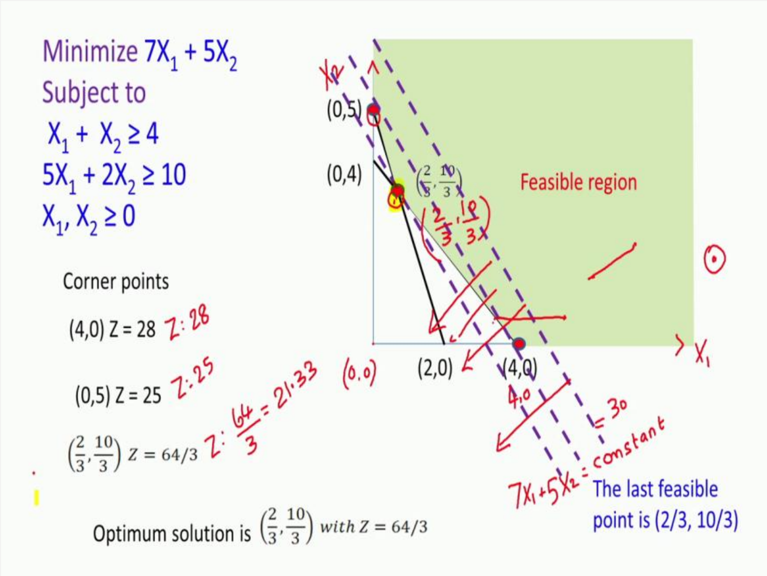

And because we are maximizing the lines finally, leave the feasible region at this corner point which happens to be the optimum solution. Now, we will understand how we solve a minimization problem using the graphical method, the method is similar; however, we would explain it through an example. Now, we look at this minimization problem which minimizes 7 X 1 plus 5 X 2 subject to two constraints X 1 plus X 2 greater than or equal to 4 and 5 X 1 plus 2 X 2 greater than or equal to 10, X 1, X 2 greater than or equal to 0.

Now, we draw the graph as we did for the maximization problem and then as since X 1 and X 2 are greater than or equal to 0, we will look at only the first quadrant, we will not look at the other quadrants. Because, in these quadrants which are here or here or in this place either X 1 or X 2 or both will become negative. And therefore, we will be looking at only this quadrant which is the first quadrant. And we also have X 1 here as the X axis and we have X 2 here as the Y axis.

We now draw X 1 plus X 2 greater than or equal to 4, to do that we first draw X 1 plus X 2 equal to 4. So, X 1 plus X 2 equal to 4 is this line which passes through 4 comma 0 and 0 comma 4 which are shown here. So, this is the first line is X 1 plus X 2 equal to 4 which passes through 4 comma 0 and 0 comma 4 which are here and this line is shown here X 1 plus X 2 equal to 4. Now, this line divides the graph into two regions, one of which is less than 4 and the other is greater than 4.

So, to find out which one it is we look at a convenient point which is the origin, you look at the origins which is here and then check whether 0 comma 0 satisfies X 1 plus X 2 greater than or equal to 4 or not. Now, when we substitute 0 comma 0 into the left hand side of this inequality, we realize the left hand side value is 0 and therefore, it does not satisfy the inequality. Therefore, the region that is X 1 plus X 2 greater than 4 is the region on this side. So, all this region is the region X 1 plus X 2 greater than or equal to 4.

We now draw the next constraint which is 5 X 1 plus 2 X 2 greater than or equal to 10. So, we first draw the line 5 X 1 plus 2 X 2 equal to 10 and that line is shown here, so that line is shown here. So, this is the line 5 X 1 plus 2 X 2 equal to 10 and to draw this line we have two points which is 2 comma 0 and 0 comma 5. Now, once again this line divides a graph into two regions, one of which is less than 10 and the other is greater than 10. So, once again we look at the origin which is here and check whether the origin satisfies the condition or violates the condition.

So, when we substitute 5 X 1 plus 2 X 2 to the left hand side of this inequality, we get 0 on the left hand side which is less than 10 therefore, the other one is the one that satisfies greater than or equal to 10. So, now, looking at this you will realize that the region goes like this, so all of these is the region that we are talking about. So, the entire thing is the region that we are talking about and that is shown the line is 5 X 1 plus 2 X 2 equal to 10 and this is the region which is now shown in a different color, which now this is the region which is shown in a light green color which is the feasible region for this particular example.

We also observed that points in this space satisfy one of the constraints, they satisfy the first constraint. But, they violate the second constraint and therefore, they do not lie in the feasible region. Now, our feasible region has 3 corner points, the first corner point is given here which is 4 comma 0, the other corner point is here and the third corner point is 0 comma 5. So, at 4 comma 0 we evaluate the objective function and the objective function value is given as z is equal to 28.

The next corner point is 0 comma 5 which is here and when we evaluate the objective function 7 X 1 plus 5 X 2 we get z is equal to 25, 7 into 0 plus 5 into 5 gives us 25. We now look at the third corner point which is 2 by 3 comma 10 by 3, now this 2 by 3 comma 10 by 3 can be found in the graph, if we draw this graph to scale and we draw the graph correctly as we have attempted to do, now you realize that this point has an x coordinate of 2 by 3 and has a y coordinate of 10 by 3.

So, this point is 2 by 3 comma 10 by 3 that I have shown here and the objective function value when we substitute we get 7 into 2 by 3 which is 14 by 3 plus 10 into 5 by 3 which is 50 by 3, so 14 by 3 plus 50 by 3 is 64 by 3 which is what we have here. Now, we have 28 for this corner point and we have 64 by 3 which is 21.33 for the other corner point which is 2 by 3 comma 10 by 3.

Therefore, the optimum solution to this minimization problem is that corner point which has the smallest value of objective function and out of these 3 we realize that the point 2 by 3 comma 10 by 3 with 64 by 3 has the smallest value and therefore, is the optimum solution. So, this point is the optimum solution with value 64 by 3, like we did for the maximization problem we can also draw what are called I so objective function lines, where we fix 7 X 1 plus 5 X 2 to a constant.

So, we fix 7 X 1, so these are 7 X 1 plus 5 X 2 equal to a constant for example, this would mean 7 X 1 plus 5 X 2 would be equal to about 30. And since it is a minimization problem, we draw the objective function lines in the other direction, where 7 X 1 plus 5 X 2 gets progressively reduced. So, the direction of movement of the objective function is this way for a minimization problem unlike in a maximization problem. So, it moves in this direction and as it moves downwards or in the direction in this direction it finally, touches this corner point before it leaves the feasible region.

So, the last feasible point is given by 2 by 3 comma 10 by 3 which is the optimum solution to the minimization problem. So, this way we solve minimization problems using the graphical method. Now, the only difference between the maximization and the minimization is many times it will happen that in a minimization problem, the feasible region will not be bounded as in this case.

Even though, I have shown a certain portion of the feasible region one would realize that a point here is also feasible and a feasible region is not bounded or it is not a closed region whereas, in a maximization problem many times we would get a feasible region, that is a closed region. So, that is one of the things that we need to understand, the other most important thing where the reason for which we evaluate only the corner points is that we have shown that every point inside the region there will be a corner point which will have a better value of the objective function.

Now, that is happened due to what is called the convexity property of the feasible region, if we take two points in the feasible region and join them by a line, we will realize that all the points in the line will lie in the feasible region. And because this feasible region is convex, we can easily show that for every point inside the feasible region there is always a point on the boundary and then there is always a corner point which has a better value of the objective function.

So, these are the basic ideas and principles that we use to solve the linear programming problem using the graphical method. While the graphical method is simple and easily understandable we are limited by the fact that we can only represent two variables in the graphical method the X 1 and X 2. We have already seen formulations that involve more than two variables and several linear programming problems have a large number of variables.

So, we need other methods other than the graphical to solve linear programming problems when we have more than two variables, we will see the algebraic method in the next class.