Introduction to Operations Research

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Module - 08
Solving LP's Using a Solver
Lecture - 05
Solving an Assignment Problem

In this class we continue the solution of the transportation problem using the Solver. So, we look at another example of a transportation problem. Once again there are 3 supply points and 3 demands points, the 3 supplies add to 40 plus 40 80 plus 50, 130 while the 3 demands add up to 100.

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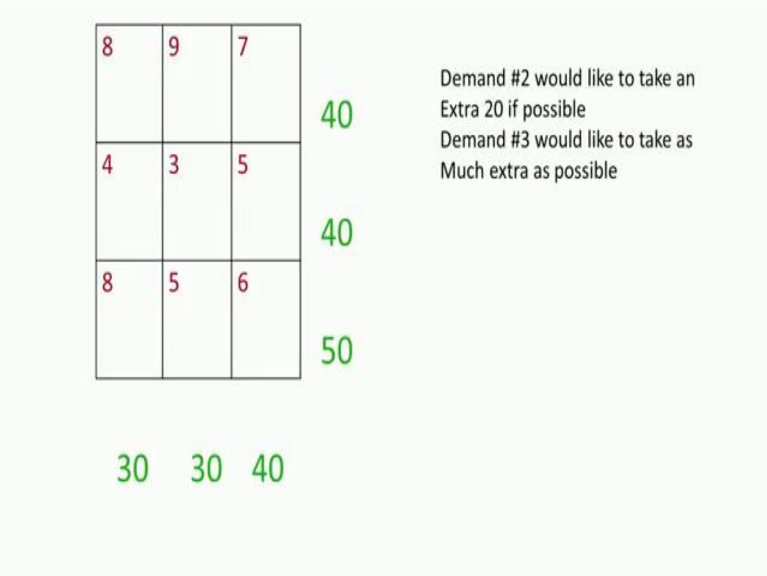

So, this is not a balance transportation problem, this is unbalanced with total supply exceeding total demand. We also bring in another aspect into the problem and say that the demand number 2 which right now requires minimum of this 30 is willing to take an extra 20 is given and demand number 3 would like to take as much extra as possible. So, now the person with demand 1 is going to take exactly 30, the demand 2 let us assume knows that the total supplied exceeds total demand by 30 units and says up to 20 extra I can take.

So, this person can be given any quantity between 30 and 50, the minimum 30 should be given. So, this person can be given anything between 30 and 50 and this person can be given anything between 40 and 70, because this person has said that he or she would like to take as much extra as possible. So now what happened is the maximum demand becomes 30 plus 50 plus 70 which actually seeds the 130 and then we realise that the extra 20 and 30 that they are seeking one of them or both may get less then what is maximum possible for them.

So, also interesting that we would like to use of all these 130 items, because they are willing to take extra. So, this formulation all the 130 have to be used, because we want give us much extra as possible. So, the supply becomes equations, the demand here will be an equation, because the minimum demand has to be met, her it will be too inequalities it is between 30 and 50 here again it is too inequalities between 40 and 7. So, let us look at the spread sheet formulation of this problem.

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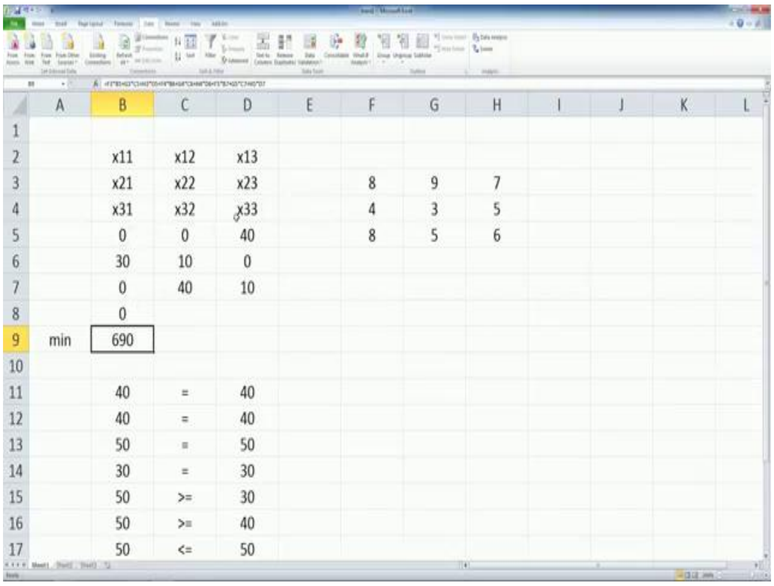

So, let us look at this problem, so the three availabilities, so once again there are nine variables x 1 1, x 1 3, to x 3 3. So, there are nine variables which are written here, I have given some arbitrary values 0, 0, 40 and so on, the objective function coefficients are written, the objective function value is written here. So, this is your X i j, so this will become B 5 into F 3, so B 5 is here into F 3 plus C 5 into G 3 plus D 5 into H 3. So, the objective function is a multiplication of this, so 0 into 8 plus 0 into 9 plus 40 into 7 and so on, so that is written here.

Now, the 3 supply constraint are all equations, because we want to give as much as we can. So, 40, 40 and 50 whatever goes out is equal to what is available here, solve the supply quantities are exhauster, the first demand constraint is an equation we said exactly 30 the first person will get. The second person will get a minimum of 30 which is shown here and a maximum of 50 which is shown here, as the less than or equal to constraint.

Because, you will get a maximum 50, the minimum of 30 shown as a greater than or equal to constraint. The third person you will get a minimum of 40 which is shown as a greater than or equal a constraint and a maximum of 70 which is shown as a less than or equal to constraint. So, there are 3 supply constraints, there are 5 demand constraints one for the first demand to each for the second and third demand, so there are eight constraints here.

So, the problem as nine variables and eight constraints, so let us check what is the solution. So, we go to the solver and I have already define all of them here, so the objective function is given as B 9, the decision variables go from B 5, C 5, D 5 to B 7, C 7, D 7 which is shown here, you will see that 1, 2, 3, 4, 5, 6, 7, 9 constraints have to be there, so let me just check that.

So, the 3 equations are B 11 is equal to D 11, B 12 is equal to D 12 and B 13 is equal to D 13 the first demand is an equations. So, that is also here 14, so second one is greater than or equal to inequality, B 15 is greater than or equal D 15 which is here, B 16 is greater than or equal to D 16 which is here solve the eight constraints are here. So, there are eight constrains 4, 5, 6, 7, 8 and you can see that there are eight constrains are here.

So, I can solve the problem directly, so I solve it I get the solution. So, I get the same solution which is here, just to show that the solver is giving a different solution. So, let me just add another 10 here, so we now realize that it is here and you also realise that it is actually infeasible, because you are getting 50 greater than this is 50 less than equal to 50, here you find that 50, 40 and 50, so the first one is violated so at presents the solution is infeasible.

But, never the less we will see what happens and we solve it, so I am just solving it and finally, get a solution with 690 and solve it again keep the solver solution, so I get 690. Now, I realize that all the 40, 40 and 50 are transported, the first person gets exactly 30, remember the second person should get anything between 30 and 50. So, the person is getting actually 50 here you see 10 plus 40 50. So, 50 greater than equal to 30, 50 less than equal to 50.

The third person should get anything between 40 and 70, third person is getting 50, 40 plus 10. So, the extra 30 was available if we meet the minimum requirement, now 20 is given to person number 2 and 10 is given to person number 3. So, like this we can solve simple transportation as the linear programming problem using the somewhere. Now, we move to one more aspect which is to try and solve the assignment problem. So, we go back to the assignment problem here.

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Example 1

8	6	4	8
6	5	5	8
9	10	11	12
7	6	8	10

And so we solved assignment problem, we started our discussion on the assignment problem with the simple example which I am taking here, I am taking a 4 by 4 assignment problem. If you assume that these are jobs and these are people, so each job goes to exactly one person, each person gets exactly one job, what is the minimum cost of assignment. So, minimize C i j X i j subject to X i j equal to 1 summed over j for all i subject to X i j equal to 1 summed over i for all j.

So, 4 by 4 assignment problem will have 16 variables, 16 terms in the objective function 4 constraints supply constraints and 4 demand constraints or if there are 4 jobs and 4 people, 4 constraints one each for the job and one each for the person. So, each job goes to one person, each person gets one job, so this is the formulation. Now, we will show this formulation using the solver and do that, so this is the solution.

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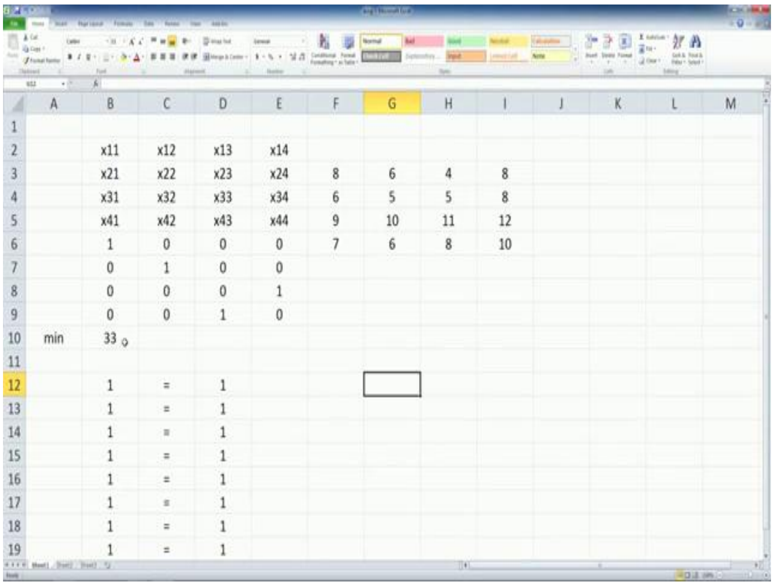

	A	B	C	D	E	F	G	H	I
1									
2		x11	x12	x13	x14				
3		x21	x22	x23	x24	8	6	4	8
4		x31	x32	x33	x34	6	5	5	8
5		x41	x42	x43	x44	9	10	11	12
6		1	0	0	0	7	6	8	10
7		0	1	0	0				
8		0	0	0	1				
9		0	0	1	0				
10	min	33							
11									
12		1	=	1					
13		1	=	1					
14		1	=	1					
15		1	=	1					
16		1	=	1					
17		1	=	1					
18		1	=	1					
19		1	=	1					

So, it is a 4 by 4 assignment problem, so the 16 variables are x 1 1, x 1 2, x 1 4, x 4 1, x 1 2 up to x 4 4, the objective function coefficients are shown here 8, 6, 4, 8, 6, 5, 5, 8 and so on. Now, the value for an arbitrary solution is shown here, so 1, 1, 1, 1, 1 so on. So, I am just creating a different solution, so I am just creating a solution like this. So, this is a feasible solution each job goes to one person, each person gets only ones job, so you see the row sum is equal to 1 for each of these, the column some is also equal to 1. So, each job goes to one person, each person gets one job, the cost associated with just solution is 33, now we want find out the best assignment.

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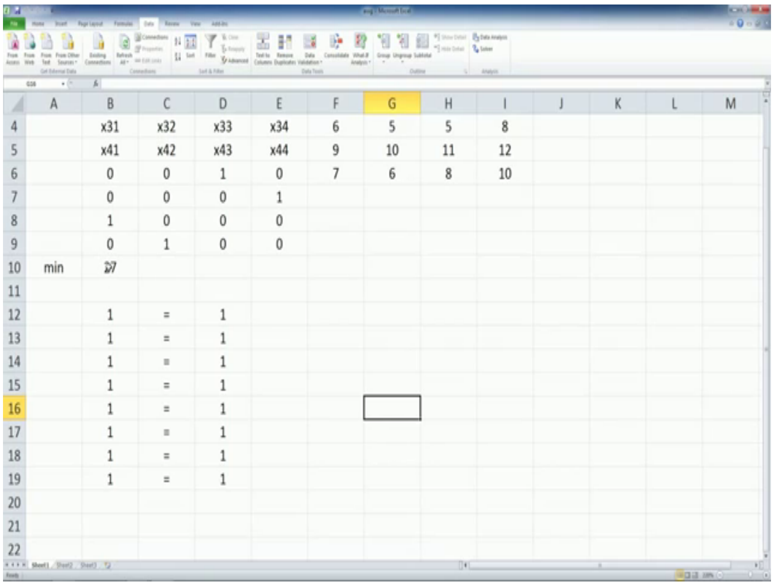

	A	B	C	D	E	F	G	H	I
4		x31	x32	x33	x34	6	5	5	8
5		x41	x42	x43	x44	9	10	11	12
6		0	0	1	0	7	6	8	10
7		0	0	0	1				
8		1	0	0	0				
9		0	1	0	0				
10	min	27							
11									
12		1	=	1					
13		1	=	1					
14		1	=	1					
15		1	=	1					
16		1	=	1					
17		1	=	1					
18		1	=	1					
19		1	=	1					

So, we define this X i j’s as I said we have eight constraints for the jobs and for each for the people so let and their equation, because each job goes exactly one person, so the right hand side values are all 1. So, look at this, this is B 6 plus C 6 plus D 6 plus E 6 which is here, this plus this plus this plus this should be equal to 1, which means if the left side the rows are jobs and the columns are persons. Now, the first person gets only one job, second person gets only one job that is given here, the third person gets one job, the fourth person gets one job that is the fourth constraints, each job goes to one person.

So, look at this constraint this will be B 6 plus B 7 plus B 8 plus B 9, so each job will go to only one person. So, like this the eight constraints are written all of them are equations with right hand side is equal to 1, now let us go back and try to solve this. So, we move to the solver I have already modelled it in the solver, you can see that the objective function is B 10. So, objective function is B 10 which is shown here, the variables are from B 6 to E 9, so B 6, C 6, B 6, E 6 and this E 9, so the sixteen variables are shown here, the feasible solution now has a value 33.

Now, the assignment problem can also be solved as a linear programming problem, even though the X i j's are binaries which we have already seen when we study the assignment problem, we can actually solve it if you are solving it as a linear programming problem, we can solve it with continuous variables, it will still through up binary values that because of the unimodularity property and so on. So, we do not have to explicitly define the X i j has binaries when we solved the assignment problem it can be solved as LP.

So, it is enough to have only these eight constraints which are shown here, the eight constraints are here you can see 4, 5, 6, 7, 8 constraints are here, now we are ready to solve this problem. So, let me go to solve this and when I solve this you realise that I have a solution with 27. So, x 1 3 is 1, x 2 for is 1, x 3 1 is 1, x 4 to is 1 with minimum cost equal to 27.

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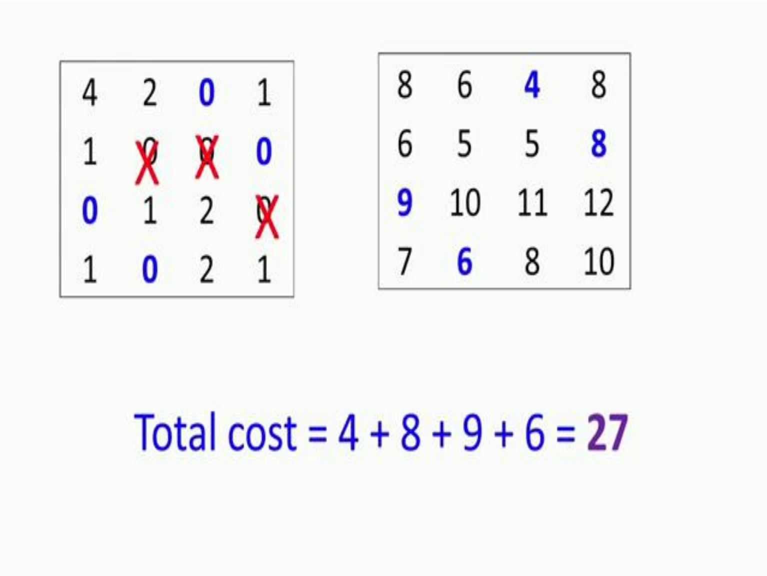

So, let us go back to the assignment and check what we did hear. So, when we actually did all the columns subtraction and so on, from this we did the row subtractions and column subtractions and then we solve them we got a solution with 27. So, we are able to get the same solution by solving it as a linear programming problem which gives us the solution of 27. So, small sized assignment problems we can solve using this solver, if we have access to this; otherwise, we can always solve it by hand understand the theory behind it and solve it by hand using the Hungarian algorithm. So, both the transportation and assignment we have actually solve them as linear programming problems when we used the solver. Now, let us look at one more aspect of the assignment problem.

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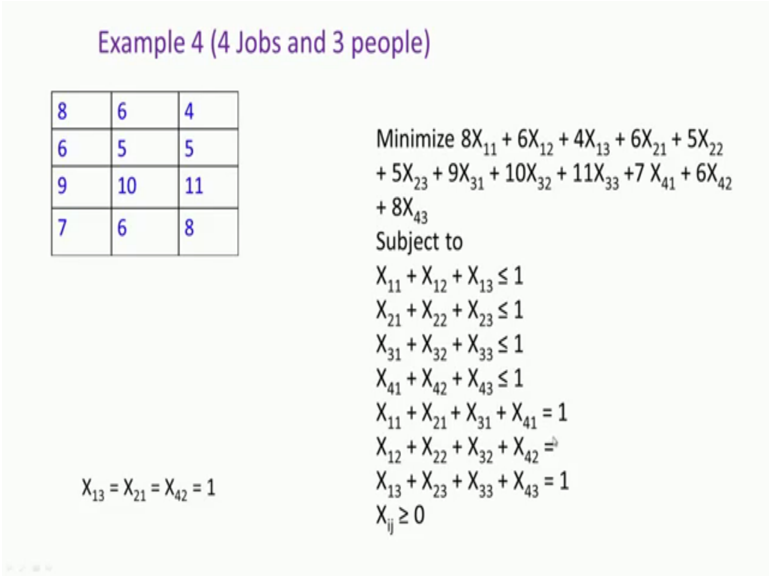

Let us look at this, this is an example of an assignment problem, where I have created a problem which has 4 jobs and there are 3 people. So, this is not a balanced assignment problem, you see there are 4 rows and 3 columns, there are 4 jobs and 3 people. So, if we put a condition that each person gets exactly 1 job only 3 jobs can be assigned, because only 3 people are there. So, 1 out of the 4 is not going to be assigned.

So, when we did this problem by hand introduced a dummy person and then we solved a 4 by 4 assignment problem using the dummy. Now, when we solve as a linear programming problem, we need not introduce a forth, we need not introduced a dummy, we will now say that each of these persons will get exactly one job. So, there will be three constraints which you can see here equal to 1 this also equal to 1 x 1 2 plus x 2 2 plus x 3 2 plus x 4 2 equal to 1, x 1 3 plus x 2 3 plus x 3 3 plus x 4 3 equal to 1.

So, each person will get exactly 1 job, now each job will go there are only 3 people. So, one of the job is not going to be allocated, so we will say that each job will go to a maximum of one person, because one job will not got anybody. So, x 1 1 plus x 1 2 plus x 1 3 is less than or equal to 1, x 2 1 plus 2 2 plus 2 3 is less than or equal to 1 and so on. So, these four constraints for the 4 jobs will be less than or equal to whereas, this 3 will be exactly equations.

So, that way we do not have to add the dummy and we can solve this, in the earlier when we had a balance problem we had all the constraints as equations. Whereas, here we realize that 4 of them are inequalities and 3 of them are equations. So, let us go back and see how we solve this.

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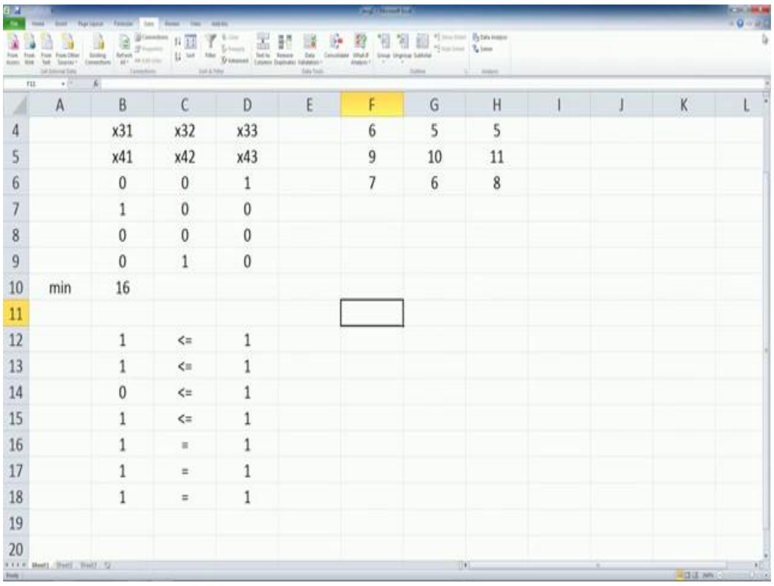

	A	B	C	D	E	F	G	H
4		x31	x32	x33		6	5	5
5		x41	x42	x43		9	10	11
6		0	0	1		7	6	8
7		1	0	0				
8		0	0	0				
9		0	1	0				
10	min	16						
11								
12		1	<=	1				
13		1	<=	1				
14		0	<=	1				
15		1	<=	1				
16		1	=	1				
17		1	=	1				
18		1	=	1				
19								
20								

So, once again I have already set it up, so we realise that these are the twelve variables, x 1 1 to x 4 3 these are the objective function coefficients, these are the values of these variables. So, just to show a different solution I am just looking at 1 here, I have 0 here, I will have a 0 here and let me put 1 here. So, I have a solution with 19, now what happens in this solution there are 4 jobs and 3 people, this job is assigned to the first person, this job is assigned to the second person, this job is not assigned to anybody, this job is assigned to the second person, third person.

So, out of the 4 job 3 jobs are have been given one each to this people the cost is 90. Once again the constraints are there seven constraints. Now, the people constraints each person gets exactly one there equations, the job constraints are inequalities it goes to a maximum of one. So, let me gain set it up on the solver which I have already done, so the objective function is to be minimize the total cost which is given by B 10. So, B 10 is here, so the objective function will be 8 into 1 plus 6 into 0 plus 4 into 0 and so on.

The values of the variables vary from B 6 to D 9, so which are given here B 6 to B 9, there are seven constraints, the three constraints corresponding to the people or equations which are shown here. Here also shown here as equations you will see 18 is a an equations, 16 is an equation, 17 is an equation. So, 16, 17 and 18 are equations, the other four are in inequalities which are shown here. So, now, we are ready to solve it, so just solve the problem.

Once again I am not using binary variables explicitly, I am solving it as a linear programming problem. So, as long as the problem is within the assignments structure I can solve it by defining continuous variables and just solve it without defining the binary. Let us see when we solve this you realize that we are getting binary solutions here with minimum cost equal to 60. So, now, you see that out of the 4 job the first job is going to the third person, second job is going to person number 1, the third job is not going to anybody, the fourth job is going to person a number 2 with the minimum cost of 16.

So, this is how we can solve a simple assignment problem using these spread sheets and solvers, you can go back and check the answer also, the answer happens to be 1 3 2 1 4 2 4 plus 6 the answer is actually 16. So, 6 plus 4 10 plus 6 16 this is 3 2 in this the rows grad change. So, the answer is actually 16 for this problem, so this is how we use this solver, the solve and learnt simple problems linear programming transportation and assignment that can be solved using the solver.

So, with this we come to the end of this course that 8 week course with 5 lessons in each week, in a very quick summary of what we have seen in this 8 weeks. The first week we looked at some problem formulations, we did about 8 different formulations, largely based on continuous variables and linear programming, we did one formulation which was a binary IP which was the bin packing. The second week we introduced simple tools to solve the linear programming problem, the studied the graphical and the algebraic method, graphical for two variables algebraic also for small size problems.

Third week we introduced with simplex algorithm which is the most popular algorithm to solve linear programming problems. So, we solve maximization problems and minimization problems explained how the algorithm works for both maximization and minimizations. The fourth and fifth week we spend some time on the dual, so in the fourth week we define the dual, we also learnt how to write the dual and we saw some simple theorems that related at primal and the dual.

In the fifth week we look at some more things about the dual, the economic interpretation of the dual, the meaning of the dual, complementary slackness theorems, how to find the solution of the dual from that of the primal, went on to say that the simplex actually solves both the primal and the dual and also look at matrix method of solving linear programming problems.

The sixth week we did transportation problem, we define the problem and we defined ways to solve it faster and better, even though it can be solved as a linear programming problem. We saw three methods to get initial solution and two methods to get the optimum solution and finally, said that combination of the Vogel's approximation or penalty cost with the u v method would is a very nice way to get the optimum solution.

The week seven we looked at the assignment problem, we again define the problem and we saw the Hungarian algorithm to solve the assignment problem. And the last week we look at how to solve these problem using an excel spreadsheet solver and we give examples of linear programming, we gave examples for in transportation and assignment and we solved the simple problems using the solver.

So, with this we come to the end of this online courses ((Refer Time: 22:08)) course on operations research. But, with primary emphasis on linear programming and introduction to or fundamental of operations research involving up to linear programming and simple topics. We hope that all the students have registered for this course would benefit from the material that has been presented in the last 8 weeks and let me wish you all the best at the end of this course.

Thank you very much.