Introduction to Operations Research
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Module - 01
Linear Programming - Introduction and Formulations
Lecture - 04
Caterer Problem

In today's class we will look at another formulation for Linear Programming which is called the Napkins problem. This napkins problem also sometimes called the Caterer problem is a very important formulation in linear programming, the problem is as follows.

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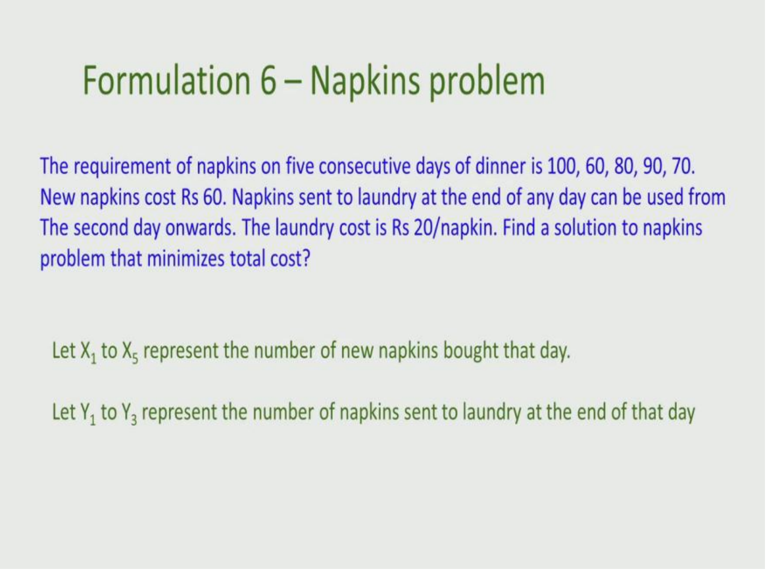

The requirement of napkins on 5 consecutive days of dinner is 100, 60, 80, 90 and 70, which means that, we are going to have dinner on 5 consecutive days and for each guest, there is a napkin that is given for them to use and the requirement of these napkins are given as 100, 60, 80, 90 and 70. A new napkin costs rupees 60, we can also send some of the used napkins to a laundry and these napkins sent to a laundry at the end of any day, which means after the dinner can be used from the second day onwards which means, if we send a bunch of napkins at the end of the first day's dinner to the laundry.

We will assume that these napkins that have come from the laundry can be used for the dinner on the third day and so on. We can actually use these napkins even on days 4 and

5. Now, we have some use napkins are sent to the laundry at the end of the second day, then they can be used from day 4 onwards and so on. The laundry cost is rupees 20 per napkin to wash and give it back. Find a solution to the napkins problem that minimizes the total cost.

Now, we first introduce the decision variables, now there are 5 days it is possible to use new napkins on each of these 5 days. So, we can define X 1 to X 5 as the number of new napkins bought on days 1 to 5. We can also define X 1 to X 5 the same variables as the number of new napkins used on days 1 to 5, they mean the same. So, X 1 to X 5 would represent the number of new napkins used in day 1 to 5.

Now, we can also use napkins that have come from the laundry and we can put it to use on certain days. Because, this problem has demand only for 5 days and the laundry takes 2 days, we are not going to consider putting the used napkins to the laundry on days 4 and 5. Because, when they come back they cannot be used therefore, we are going to define that we are going to send used napkins to the laundry only on the first 3 days and therefore, we introduce Y 1 to Y 3 as the number of used napkins sent to the laundry at the end of days 1, 2 and 3, let me again explain the variables.

There are 5 days in which we are going to have the dinner and; obviously, we can use new napkins for the dinner. So, X 1 to X 5 will represent the number of new napkins used in days 1 to 5, I have indicated here that they are bought on that day, they are bought and used they can be used. So, the number of new napkins used on that day, Y 1 to Y 3 represents the number of napkins sent to laundry at the end of days 1, 2 and 3.

Now, we do not define Y 4 and Y 5, because if some used napkins are sent at the end of day 4 they will arrive on day 6 and can be used from day 6 onwards. But, the problem talks only about 5 days therefore, we do not introduce Y 4 and Y 5 and we introduce only Y 1 to Y 3. So, we have 8 variables, 5 variables representing the new napkins for days 1 to 5 and 3 variables representing the number of napkins sent to the laundry at the end of days 1, 2 and 3. So, there are 8 variables in this formulation, we will now look at the constraints.

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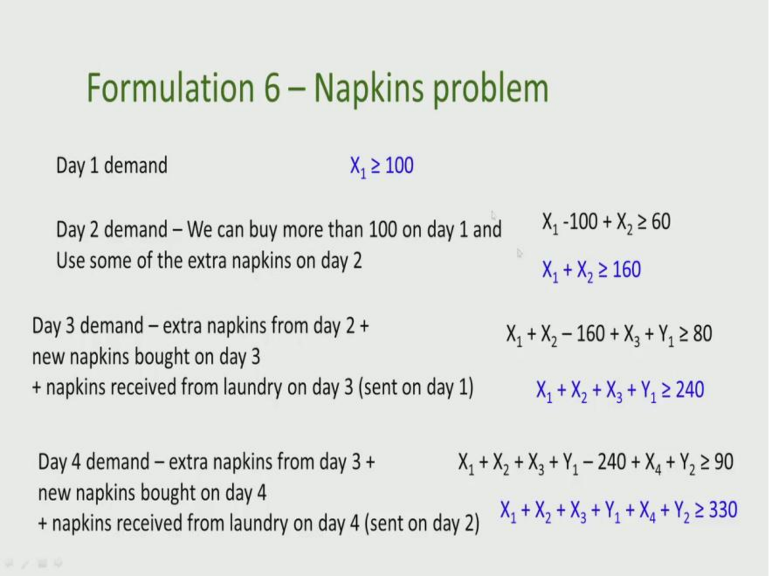

Now, the important constraint is that the demand for napkins on each day has to be met. So, day 1 demand X 1 is the number of new napkins used in day 1, X 1 should be greater than or equal to 100, because 100 is the demand for day 1. The first day we have only one way of meeting the demand which is by using new napkins, napkins that come from the laundry can be used only on days 3, 4 and 5. Because, we have defined variables as the napkins that go to the laundry at the end of days 1, 2 and 3 and these Y 1, Y 2 and y 3 number of napkins which go at the end of days 1, 2 and 3 can be used to make the demand of days 3, 4 and 5 because they take two days to come back.

So, the first two days we have to use new napkins and therefore, to meet the first day's demand we have X 1 greater than or equal to 100. Second day's demand we have to use new napkins, but we can also do something like this. We can either buy exactly the required number of napkins on day 2 or nothing prevents us from buying more than 100 on day 1 and using the remaining ones to meet the demand of day 2.

So, we can buy more than 100 on day 1 and use some of the extra napkins on day 2. So, this becomes X 1 minus 100 plus X 2 is greater than or equal to 60. Now, this comes because we have X 1 greater than or equal to 100, the number of napkins bought on day 1 is greater than or equal to 100. So, X 1 minus 100 napkins can be carried to day 2, so X 1 minus 100 napkins that are carried to day 2 plus X 2 which are the number of napkins bought at the beginning of day 2. Now, these two act as supplies for day 2 and this should be greater than or equal to 60.

So, this is rewritten as X 1 plus X 2 greater than or equal to 160, because you take this 100 to the other side of the inequality. So, X 1 plus X 2 greater than or equal to 160 which is shown in blue color and the two constraints shown in blue color are the actual constrains for days 1 and 2 respectively. Now, we move onto the third day, now the third day we have to meet the demand and we can do it in 3 ways.

One is, if we could carry some extra new napkins which have not been used at the end of second day, we can buy some new napkins on the third day and some napkins that we have put to the laundry at the end of the first day are going to come back as usable napkins. And therefore, there are 3 ways in which we could meet the demand of day 3. So, extra napkins from day 2, new napkins bought on day 3 and napkins received from laundry on day 3 which were sent at the end of day 1.

So, this gives us… Now, from the previous constraint X 1 plus X 2 greater than or equal to 160, now the extra new napkins that are available are X 1 plus X 2 minus 160, so that act as some kind of a supply for day 3. So, X 1 plus X 2 minus 160 is the number of new napkins that are available at the end of day 2 and have not yet been used. So, they can be used to meet the demand of day 3, so X 1 plus X 2 minus 160 plus X 3 napkins are bought in day 3.

So, they can be used plus Y 1 napkins have come back from the laundry, Y 1 was sent at the end of day 1. So, Y 1 napkins would come back from the laundry. So, X 1 plus X 2 minus 160 plus X 3 plus Y 1 should be greater than or equal to 80 which is the demand for day 3. Now, this is again simplified and the 160 is taken to the other side of the inequality to give us X 1 plus X 2 plus X 3 plus Y 1 greater than or equal to 240.

You can also see a pattern emerging in the constraints, the first constraint had only X 1 and the first day’s demand. The second constraint has X 1 plus X 2 and the sum of two days demand. The third constraint has X 1 plus X 2 plus X 3 plus another Y 1 the sum of 3 days demand. So, you can see a pattern emerging and you will see that this pattern will continue. Now, day 4 again has 3 sources, now at the end of day 3 if we have unused new napkins they can be used on day 4, new napkins bought on day 4 napkins received from laundry on day 4 that were sent on day 2.

So, there are 3 sources and we also have to bear in mind that the extra napkins that are available unused napkins available at day 3. Now, can be new napkins can also be some of the napkins that have come on the earlier day and have not been used. So, I have written here day 4 is demand can be met in 3 ways, extra napkins available for use at the end of day 3. Now, we do not really worry at this point whether they are new or they have come from the laundry, the inequality will tell us that these number are extra napkins.

New napkins bought on day 4 and napkins that have come on day 4 morning or before the dinner and these have been sent at the end of day 2. So, again there are types of supplies and these three are written as from the previous constraint X 1 plus X 2 plus X 3 plus Y 1 are the napkins available for use on day 3 that has to be greater than or equal to 240. So, if there is an excess, so X 1 plus X 2 plus X 3 plus Y 1 minus 240 is what is carried from day 3 to day 4 which represents the extra napkins from day 3 to day 4.

So, X 1 plus X 2 plus X 3 plus Y 1 again repeating is the number of napkins available for use in day 3. Now, that is greater than or equal to 240 and with that we meet the demand of day 3 in addition to day 2 and day 1. Now, this inequality if X 1 plus X 2 plus X 3 plus Y 1 exceeds 240, then a balance of X 1 plus X 2 plus X 3 plus Y 1 minus 240 is carried over to the 4th day and that represents the extra napkins from day 3. X 4 is bought on the 4th day and Y 2 are received before the dinner on the 4th day and Y 2 are the number of napkins sent at the end of the second day to the laundry.

So, these available napkins for the day 4 are X 1 plus X 2 plus X 3 plus Y 1 minus 240 plus X 4 plus Y 2 should be greater than or equal to 90 which is the demand for the 4th day. Now, this is rewritten as X 1 plus X 2 plus X 3 plus X 4 plus Y 1 plus Y 2 greater than or equal to 330. Now, consistent with the pattern you can now see the first constraint as X 1 and first day's demand, the second constraint has X 1 plus X 2 and it has sum of two days demand.

The third constraint has X 1 plus X 2 plus X 3 and a Y 1 and 3 days demand, the fourth constraint has X 1 plus X 2 plus X 3 plus X 4 which is new napkins. But, on 4 days plus Y 1 plus Y 2 napkins that are coming in and then greater than or equal to sum of the 4 days demand, so you can see that pattern emerging.

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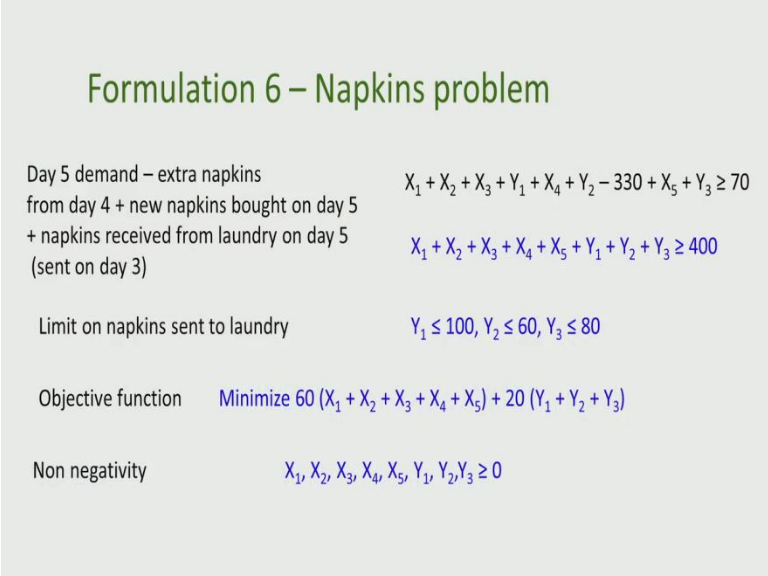

We now move to the 5th day, so again the 5th day will have three types of supplies, the excess napkins available at the end of the 4th day. The new napkins bought on the 5th day and the napkins received from the laundry just before dinner on the 5th day and these have been sent to the laundry at the end of the 3rd day. So, we write this the expression becomes longer, so we could go back to the previous one and say that now this is the day 4 constraint.

So, X 1 plus X 2 plus X 3 plus X 4 plus Y 1 plus Y 2 minus 330 are the extra napkins carried from day 4 to day 5. So, X 1 plus X 2 plus X 3 plus X 4 plus Y 1 plus Y 2 minus 330 is what is carried this long expression is what is carried from day 4 to day 5. And then we buy another X 5 on day 5 and then we have the other supply of Y 3 which has come before dinner on the 5th day and this Y 3 had been sent to the laundry at the end of the 3rd day. So, this should be greater than or equal to 70 which is the requirement for the 5th day.

Now, this is again simplified to the 330 is taken to the other side of the inequality. So, X 1 plus X 2 plus X 3 plus X 4 plus X 5 plus Y 1 plus Y 2 plus Y 3 is greater than or equal to 400 and consistent with the pattern you realize that X 1 to X 5 is appearing in the 5th constraint, Y 1 to Y 3 is appearing in the 5th constraints and the sum of demands of all 5 days are appearing. Now, this is happening largely because we could carry the excess napkins on a certain day for use on a subsequent day.

Because, we are modeling this we get the constraint of this type where for each day finally, the constraint is the sum of the demands of up to that day. So, we have now written all the 5 constraints to meet the demand on all the 5 days. Now, we also have some limits on the number of napkins that can be sent to laundry, now these are Y 1 less than equal to 100, Y 2 less than equal to 60 and Y 3 less than or equal to 80 this is because the demand for day 1 is 100.

And therefore, at the end of day 1 we can have a maximum of 100 soiled napkins and we will assume that all these 100 are used by the people and therefore, poses a limit on what can be sent to the laundry at the end of day 1. So, Y 1 less than equal to 100, Y 2 less than equal to 60 and Y 3 less than or equal to 80. So, these three constraints give the limit on the napkins sent to the laundry and the objective function now will be to minimize the costs, the new napkin cost 60 and we are buying X 1 plus X 2 plus X 3 plus X 4 plus X 5 amount of new napkins on days 1 to 5.

So, the new napkin cost is 60 into X 1 plus X 2 plus X 3 plus X 4 plus X 5, Y 1 plus Y 2 plus Y 3 is the number of napkins sent to laundry taking into account all the 3 days, the laundry cost is 20 per napkin. So, 20 into Y 1 plus Y 2 plus Y 3, so the objective function is to minimize the total cost, which is the cost of new napkins and also the cost of sending these napkins to the laundry. We also have the non-negativity restriction, where we have all the X variables and the Y variables to be greater than or equal to 0.

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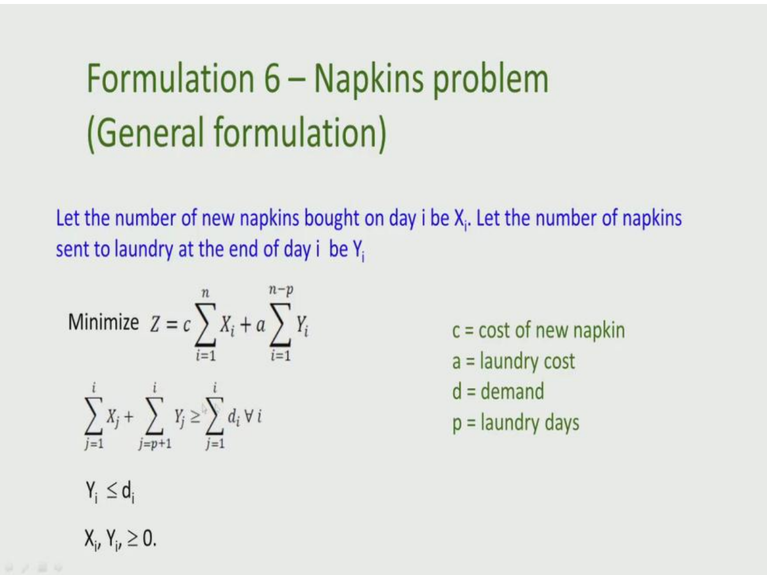

We also summarize this by giving a general formulation, so X 1 be the number of new napkins bought on day i X i be the number of new napkins bought on day i. The number of napkins sent to laundry at the end of day i be Y i, c is the cost of the new napkin, a is the laundry cost, d is the demand and p is the number of days it takes in the laundry. So, our objective function will be the sum of the new napkin cost and the laundry cost now X i is the new napkins bought on day i i equal to 1 to n. So, X1 to X n and c is this cost.

Now, the y variable will be from 1 to n minus p in our example we looked at 5 days total and 2 days for the laundry. So, only the first 3 days we set therefore, we will have Y i i equal to 1 to n minus p. A typical constraint would look like on a particular day i the sum of the new napkins up to i plus the sum of the napkins that have come from laundry up to that day is greater than or equal to sum of the demands up to that day.

So, this was our 5 constraints that we had for each of these 5 days, the limit on putting the napkins to laundry is Y i less than or equal to d i. So, each day I can put only the maximum demand to the laundry X i Y i greater than or equal to 0 is a general formulation of the napkins problem. We have seen a slightly simpler version of this problem in many instances you would observe that there will be two types of laundries there will be a fast laundry and a slow laundry, but just to make the problem simple I have used only one type of laundry. So, this napkins problem or the caterer problem is a very important formulation in linear programming.

In the next class we will look at two more formulations related to linear programming.