Introduction to Operations Research
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Module - 08
Solving LPs using a solver
Lecture - 04
Solving a transportation problem

In this class, we continue the discussion on problem formulation. We look at the formulation and solution of the Bin packing problem. The previous classes, we have solved other examples that we have seen at the formulation stage of this course. So, we look at the bin packing problem, briefly look at the formulation, and then explain how we solve this. If you would remember, this is a formulation, where we define binary variables or 0 1 variables.

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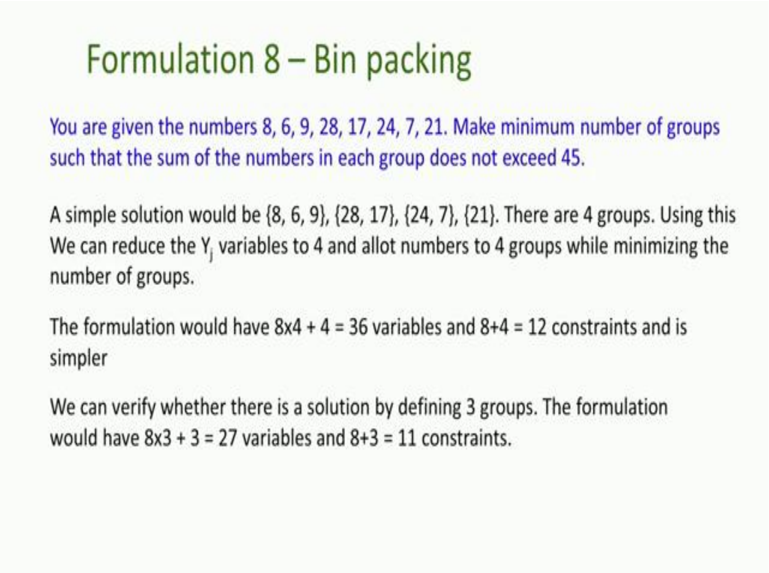

The bin packing problem is as follows, we are given 8 numbers in this example, these 8 numbers are the number 8, 6, 9, 28, 17, 24, 7 and 21. Make minimum number of groups, such that the sum of the numbers in each group does not exceed 45. This is like saying these 8 numbers are like eight items, which have to be packed in a one dimensional bin of length 45 and we want to use minimum number of bins. And that is how this problem gets it is name of one dimensional bin packing problem.

We started by giving a simple solution and said that a simple solution would have to the solution given here as 4 groups. The lengths 8, 6 and 9 can go to 1 group, 28 and 17 can go to 1 group, 24 and 7 can go to 1 group, 21 is a group by itself. So, this simple solution has 4 bins. Now, the moment we have a simple solution with 4 bins, the question that we have is, can we solve this with three bins.

We also mention that a formulation for this problem would have 36 variables and 11 constraints and simpler than the previous formulation.

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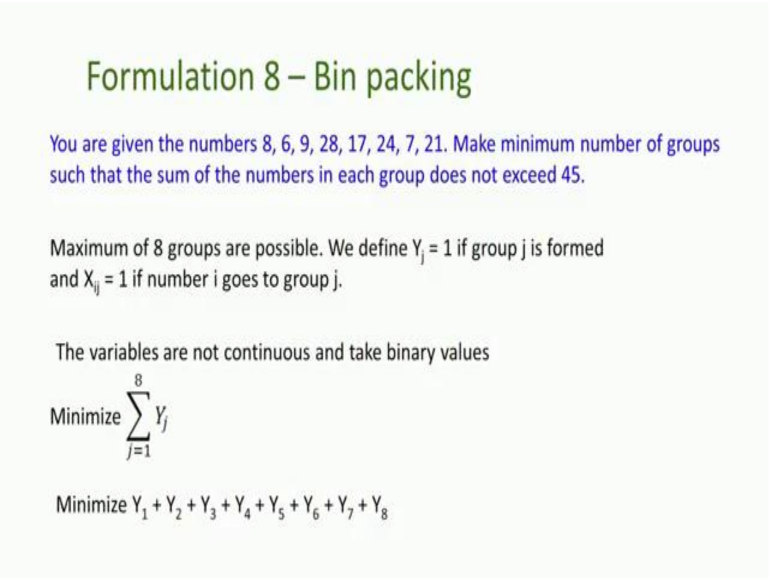

Because an earlier formulations, we could start the bin packing problem by saying that, we can think of maximum of 8 groups, because there are 8 element and we would like to minimise the number of groups into which these 8 elements can be packed. So, if we use 8 possible groups and 8 possible bins, the objective will be to minimise Y 1 up to Y 8.

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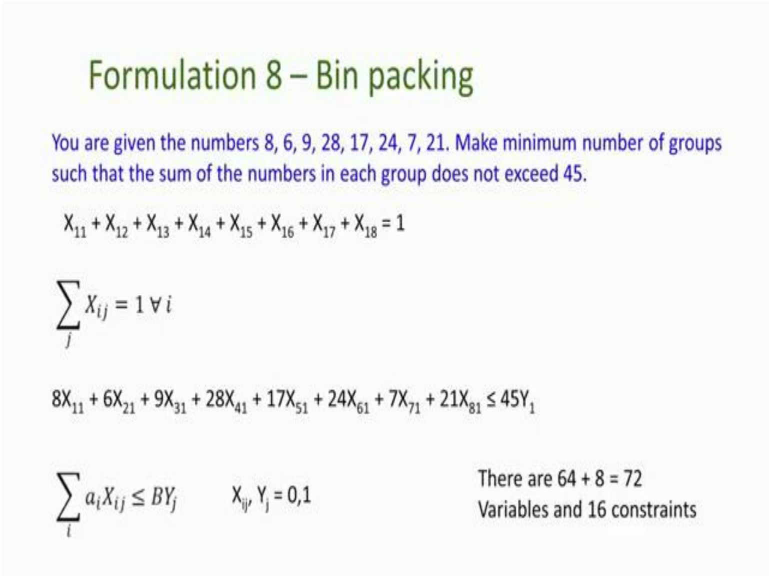

Each number should belong to only 1 group. So, sigma X i j summed over j is equal to 1. So, X 11 is 1,if the first number or the first items is allotted to the first group, X 21 is equal to 1, if the second item is allotted to the first group and so on. So, all those items that are allotted to the first group, the total should be less than or equal to 45. So, the constraints will have a right hand side of less than equal to 45 Y 1, Y 1 is equal to 1, if bin 1 is chosen or group 1 is chosen.

So, whatever is allotted to that group should have a link less than or equal to 45. So, general form is a i X i j less than or equal to B into Y j, where B is the bin capacity. Both X i j and Y j are binary variables, 0 1 variables. So, if we start with 8 possible groups, and then try to reduce it, there will be a 64 X i j variables and 8 Y j variables. We can try and reduce it a little bit by looking at the simple solution with 4 groups. Therefore, if we have a solution with 4 groups, then we could think of 18 into 432 X i j variables and 4 Y j variables.

Now, we will formulate this problem to see whether a solution with 3 groups is possible. The formulation with 4 groups would have 8 into 4 X i j, 32 X i j variables and 4 Y j variables, so it will have 36 variables and 12 constraints. So, let us show this formulation on the excel sheet, and then show the solution to the problem.

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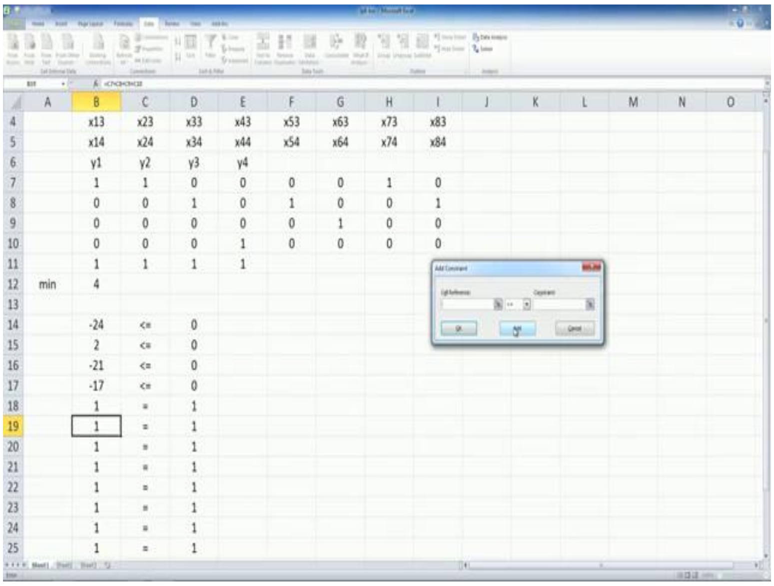

So, this is a formulation I have shown X 11 to X 81, 1 4 to X 8 4 here indicating that, these are variables that represent item going to bin. So, X 8 4 represent item 8 going to bin number 4, Y 1 to Y 4 are the 4 bins that we have, because we are right now looking at a solution with 4 bins. Now, as we did before we give values to the X variables and Y variables, so at the moment there are some fractional values, all this have to be integers. So, I am just giving them binary values. So, I am just giving them 0 and 1 and those kind of values.

So, this is a solution that I have indicated, it may be feasible, it may not be feasible. Now, we will check whether it is feasible or not feasible at the moment, it is not feasible, because I have put item 6 to 2 bins, which is not feasible. So, I change this, I change this. So, now, we observe that, here is the solution, where 4 bins are there. The first bins is having items, this item, this item and the third item. The second bin is having this third item, item number 5 and item number 8. The third bins is having item number 6 and the 4th bin is having item number 4, therefore, there are 4 bins, which is given this solution.

Now, there are the constraints. So, for every bin, the length of the items allotted to that bin should be less than or 45 and if that bin is chosen. So, let me just show the first left hand side of the first constraints. So, this we will realize 8 into B 7 plus 6 into C 7 and so on minus 45 into B 11. So, we let me explain this. So, B 7, C 7, D 7, E 7, F, G, H up to I 7 represent the values, whether this item is going to the first bin are not.

So, in the feasible solution, I have given this item is going to the first bin, this item is going to the first bin, this item is going to the first bin. So, some of these three lengths we need to check, the three lengths are 8 for this 1, 6 for this 1 from C 7 and H 7 is 7. So, 8 plus 6, 14 plus 7 is 21 is less than 45. So, you have a minus 24 here. Because, I have put minus 45, B 11 minus 45, this bin is chosen.

So, this bin is chosen. So, 45 it can give us a capacity of 45, 21 has been put. So, the balance 24 is showing as minus 24. Similarly, if we take the second bin, we be realize that items number this item and this item are in this bin. So, let us look at this. So, this item has a length of this is D 8. So, D 8 has a length of has a length of 9 and E, F 8 has a length of 17. So, 17 D 8 has a length of 9 plus 17 plus 9, 26 and the last one has 21. So, 47 is a length, 45 is the length that is available.

So, at the moment this solution is infeasible, because this bin is using of more than 45 a positive value. So, even though I have shown a solution with binary values for all the variables at the moment this solution is not feasible, but it actually does not matter. Only when we optimize, we have to check whether we will get a feasible solution. So, 4 constraints one for each bin are written here, these 4, is all should be less than or equal to 45 times the bin capacity.

Since, bin 45 Y 1, 45 Y 2 are variables, they are taken to the left hand side. So, right hand sides are 0 and they are less than or equal to variables. Each item will go to exactly 1 bin, so this is for example item 1, it is says a B 7 plus B 8 plus B 9 plus B 10. So, if we take the first item, it will go to either this bin or this bin or this bin or this bin. So, similar this will be C 7 plus C 8. So, these set of 8 constraints make sure that, each item is going to exactly 1 bin, so that is written here.

Now, we go to the solver, I have already mapped it into the solver. So, the objective function is this, the number of bins, which is B 12, you realize B 12 is the objective function. The variables are listed here, the variables are all from here, they are from B 7 to I, I 7, 8, 9, 7, 8, 9, 10 to I 10. So, you see that B 7 to I 10 and B 11 to E 11. So, the variables are listed here. All the constraints are listed here, the 8 constraints are written here.

Now, in addition all the X i j's in the Y j have to be binary. So, I have to define 36 variables as binary variable and these 36 binary variables are defined here to save time, I am just showing, you can see a bunch of binary variables that are all define here. So, now, we are ready to solve this problem. So, we go to the solver and we try to solve this problem and we get this solution.

So, this is a solution that is obtained by the solver, we realise that only 3 bins are used, you can realise that Y 1, Y 2, Y 3 is equal to 1, so three bins are used. And the allocation is this item, the 1st item is going to the third bin, 2nd item is going to the third bin, 3rd item is going to the third bin, 4th item is going to the first bin, fifth item is going to the first bin, 6th item is going to the second bin 7th item is going to the third bin and the 8th item is going to the second bin.

So, we are able to accommodate all the 8 items into 3 bins and now, the left hand side also tells us, how much the bins are utilised. The first bin is utilise fully, so the left hand side is 0, the second bin is utilized fully, the left hand side is 0, the third bin has minus 15, which means out of the 45 length; that is available in the third bin, only 30 has been used and 15 is unused.

Now, every bin is assigned to, every item is assigned to only 1 bin. So, that is how the 4th bin is not use at all, so we have 0 equal to 0 from this. So, that is how the solution to the bin packing problems. So, this is an example, where we define variables as binary variables. For example, if you go back to the solver, I have defined these variables as binary.

Just to illustrate one of them, all the variables X i j's BY j's are binary, so how to add it in the solver, just go click here. So, if you click here B 7, B 7 is a X i j variable. Now, if you want to add, I have already done it. So, I have just demonstrating in it, I can put add a constraint. Now, go to this B 7 is a binary variable. So, B 7 and click this down, you get binary, you get will binary here than you can add.

So, like this for the 36 variables, you have to add all the X i j and Y j's have to be added as binary variables into the formulation. So, this is an example of how we solve a problem that involves a binary integer programming problem. So, we define some of them as binary variables. In this course, we did not see the methods in details is how to solve binary integer programming.

Though, we introduce a bin packing as an example, just to illustrate that not all formulations are linear programming problem with continues variables. Large number of formulations involves integer variables and binary variables, so just to illustrate that, we had used the bin packing formulation. And for the sake of completion, I have demonstrated, how to solve the binary 0 1 problem using this solver.

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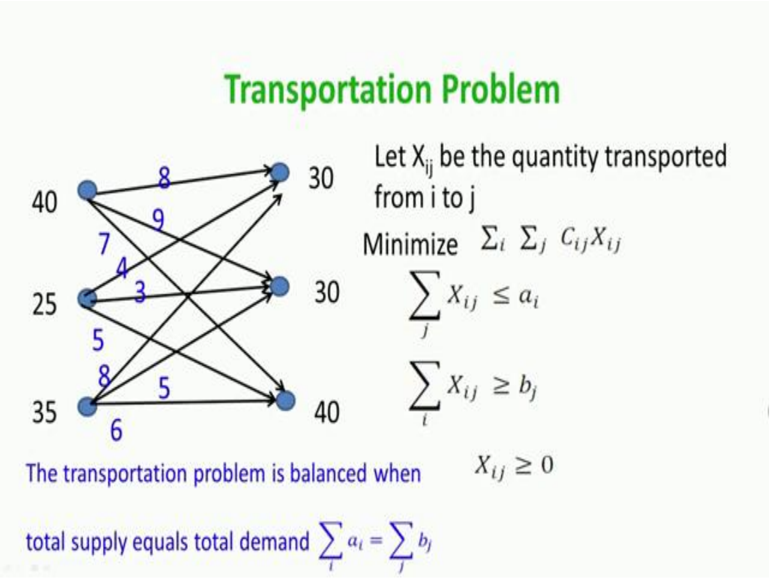

Now, we move to solving the transportation problem and the assignment problem that we have seen as part of this course. So, we go to the transportation problem. So, this is the transportation problem that we had solved. So, there are three supply points and three demand points, it is a balance transportation problems.

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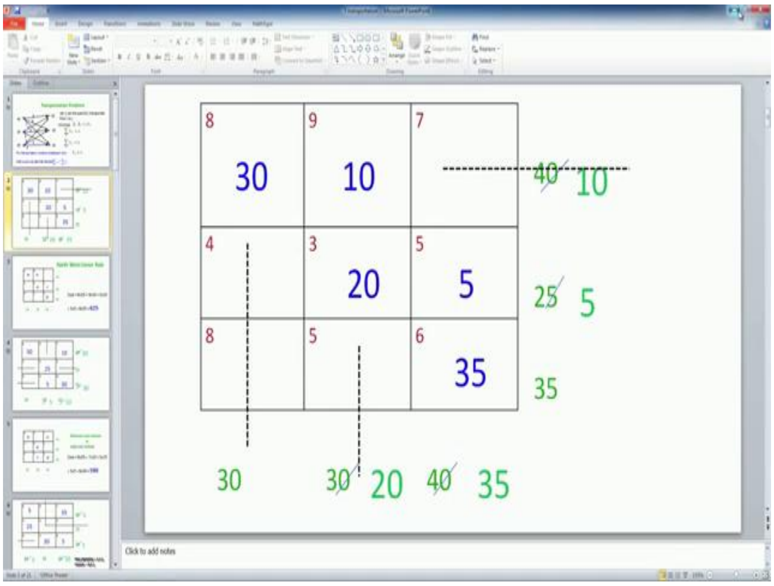

So, this is the transportation problem that we solve, three supply point with 40, 25 and 35 with the total supply of 100 and 3 demand of 30, 30 and 40 with demand of 100, transportation costs are given here 8, 9, 7, 4, 3, 5, 8, 5, 6 and so on. So, let us now try and solve this problem using this spreadsheet.

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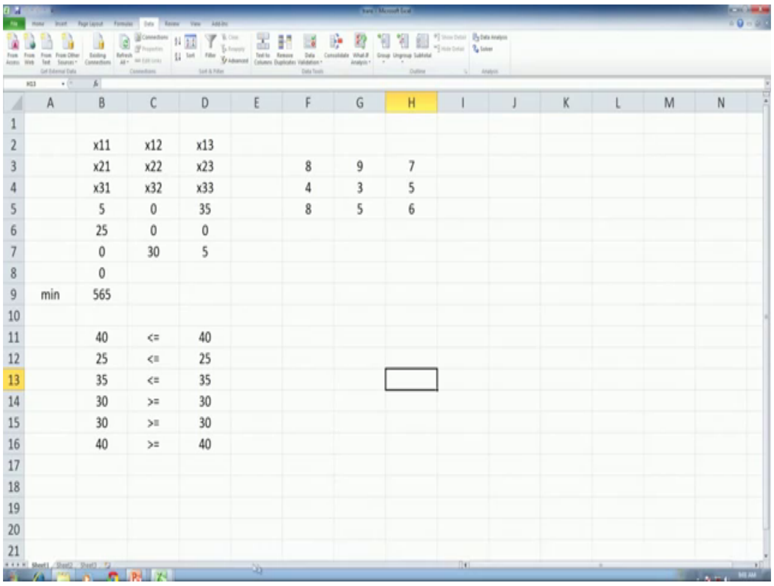

So, we go back to this and I have already created this problem on the spreadsheet. So, it is a 3 by 3 transportation problem. So, we have variables X 11, X 13, X 31 to X 33, 9 variables, where X i j represents the quantity; that is transport. The cost of transportation are shown here 8, 9, 7, 4, 3, 5, 8, 5 and 6, these are the transportation costs. So, the objective function tries to minimise the total transportation cost C i j, X i j.

The X i j variables I am setting here, these 9 variables starting from B 5, C 5, D 5, B 6, C 6, D 6, B 7, B 7 and D 7, these are the 9 variables. The objective function is shown as C i j, X i j, which is put here. For example, F 3 into B 5, so F 3 is the first value 8, B 5 as a quantity that is transported like this, I multiplied that 9 terms added them and return the objective function, which is like this.

Now, the variables are defined here, the values are defined here, the objective function is defined here. Now, this transportation problem has 6 constraints, 3 supply constraints and 3 demand constraints. So, the 3 supply quantities are 40, 25, 35; the demand quantities are 30, 30, and 40. So, whatever that goes out of the first supply point should be less than or equal to 40.

So, now, I have written what goes out B 5 plus C 5, D5, B 5 plus C 5 plus D5 is what goes out of this 40; that should b less than or equal to the right hand side, which is 40. Similarly, the second constraint which is B 6 plus C 6 plus D 6 goes out of the second supplier of 25. So, that should be less than or equal to 25 and so on. So, there are three supply constraints, three demand constraints, this is what reaches the first destination. So, it is B 5 plus B 6 plus B 7, the requirements is 30, so what reaches should be greater than or equal to the requirements.

So, there are 6 constraints, three supply constraints and three demand constraints. Now, we are also aware that the problem is a balanced transportation problem, therefore, instead of putting less than equal to inequalities for the supply and greater than or equal to inequalities for the demand assuming that the minimum demand has to be met. We can substitute all of them by equations, because the problem is a balanced transportation problem. Either, we can have all constraints as equations or we could have the supply constraint being less than equal to and constraints be greater than or equal to.

Now, the present’s solution that is shown is not feasible, because if you see the second supply 25 is available, but 35 is going out. Similarly, if you see here the demand is actually exceeding, but in a balance transportation problem, we will meet exactly the demand, because the total supply is equal to the total demand. But, as is id it actually does not matter or it matter only when we are going to solve.

So, we now put the problem into the solver, which I have already done here. So, I am just showing you. Now, the objective function is B 9, which is shown here B 9, we have already seen the objective function as the some of this 8 into 5 plus 9 into 0 like that, the objective function is written. E 6 constraints are written here, you can see three of them having less than or equal to and the other three demand constraint having a greater than or equal to.

Now, in a transportation problem, the variable X i j need not be defined as integer variables, they can stay as continuous variables and as long as the supplies and demands are integers, they will give integer values. So, we are now solving it as a linear programming problem, we are not solving it using any Vogel’s approximation or any such method. So, we are solving it as a linear programming problem.

So, we define the variables as continues variables and use simplex L P to solve this. So, when I solve this, right now for this solution, which is not feasible as I mentioned 35 is going out of a supplied point, which has only 25 and as I said, it does not matter at this stage. So, when we solved this, we realise that we are getting a solution with. So, we are once again we solve this. So, we are getting a solution with this getting the solution with cos equal to 565.

Now, we also realise that the constraints are all satisfied nicely, 40 is available, 40 goes 25 is available, 25 is goes 35 is available, 35 goes, 30 is required, 30 is given, 30 is required, 30 is given, 40 is required, 40 is given. Transportation quantities are all shown here with cost equal to 565. Now, If we go back to the transportation problem that we solve, we realise that, the optimum solution for this came to 565, which we have here showing the solution with all the three methods, which we did by hand.

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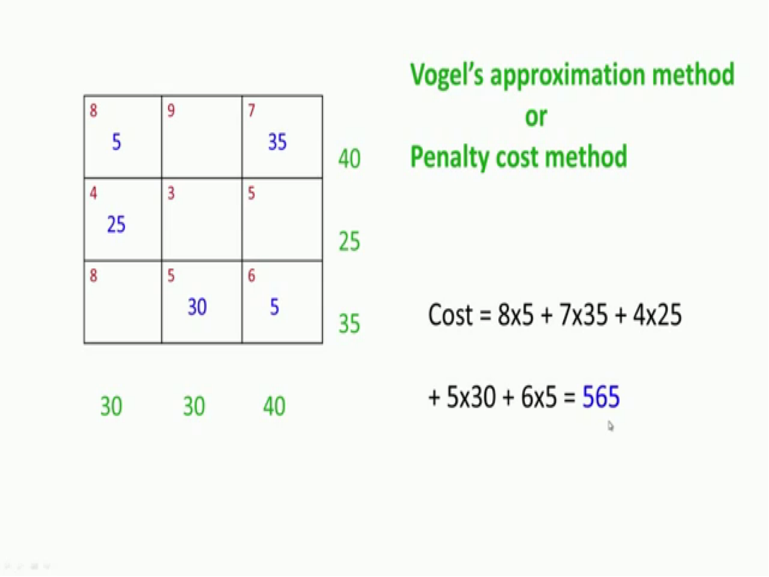

So, Vogel's approximation or penalty cost gave 565, we also said later, when we use the stepping stone method, we said no gain and hence optimum.

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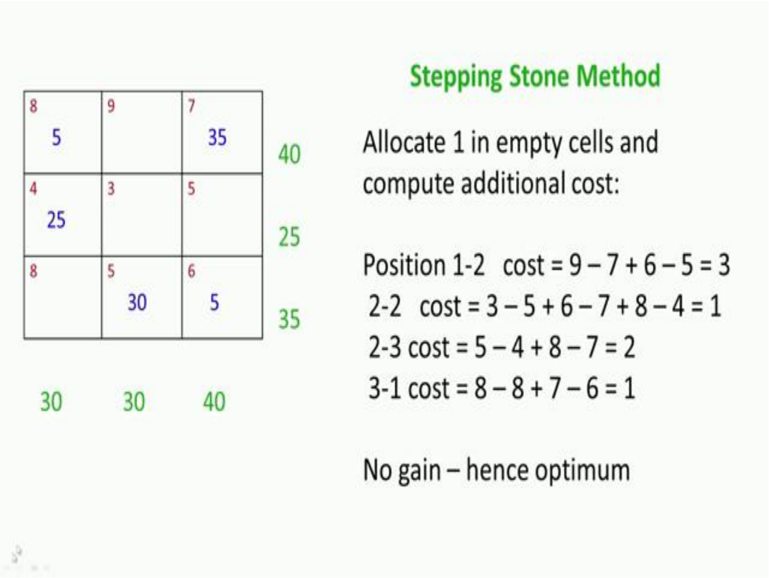

So, the optimum cost is 565 for this transportation problem, which we got from our solution. So, we look at one more example of the transportation problem and we will look at couple of examples in the assignment problem in the next class.