Introduction to Operations Research
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Module - 08
Solving LP's using a Solver
Lecture - 03
Solving Other Formulations

We now show the solution to some of the problems that be formulated right at the beginning of this course. So, one of the problems that we formulated is the caterer problem also called the napkins problem.

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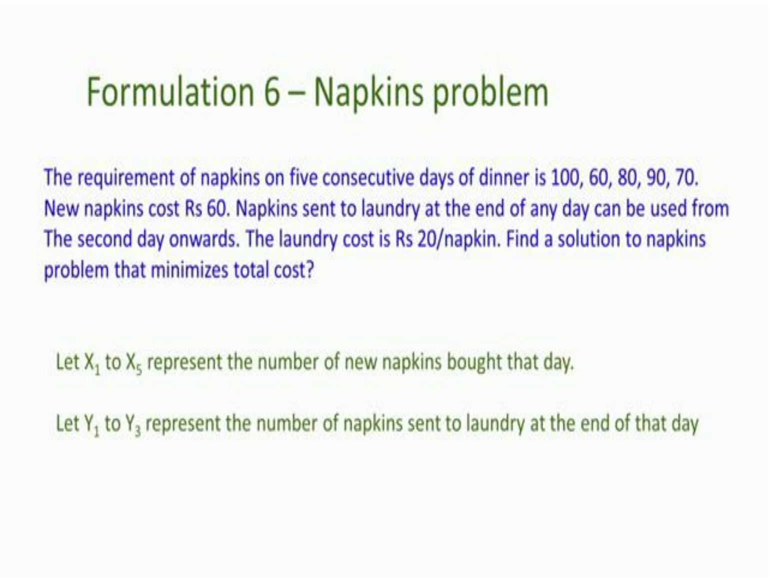

So, the requirement of napkins for 5 consecutive days are shown 100, 60, 80, 90 and 70. The new napkin cost 60, they can be send to a laundry and they come back after couple of days laundry cost 20 how do we optimise this.

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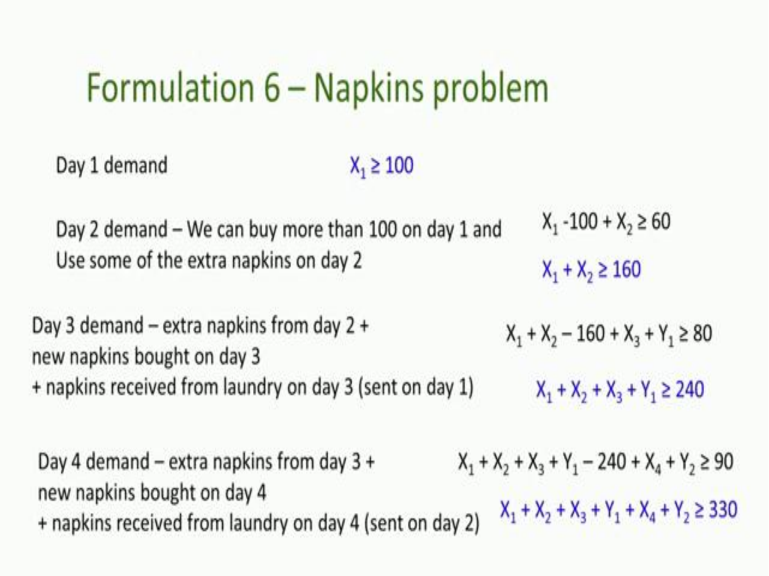

So, the constraints that we wrote where day 1 demand is made out of new napkins, day 2 constraint resulted in X 1 plus X 2 greater than or equal to 160, day 3 was X 1 plus X 2 plus X 3 plus Y 1 greater than or equal to 240 day 4 had a similar constraint.

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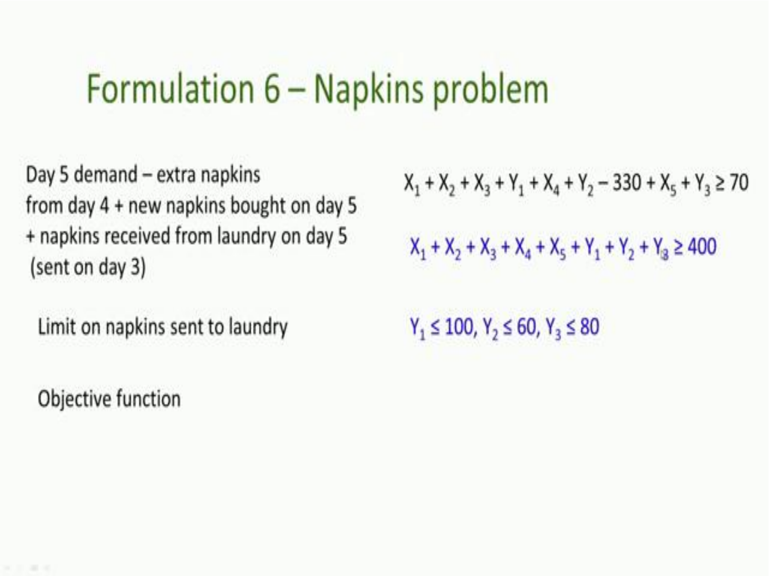

And day 5 also had similar constraint, so there were five constraints for the 5 days and then the napkins are sent on 3 days to the laundry, because it takes 2 days to come back they are send on days 1, 2 and 3. So, there are eight constraints and there are eight variable X 1 to X 5 or the new laundry is used from day 1 to day 5, Y 1, Y 2, Y 3 are the laundries sent or the napkins sent to laundry at the end of days 1, 2 and 3, X 1 to X 5 are the new napkins used from the 5 days. So, there are eight variables and there are eight constraints in this problems. So, let us now right this formulation and try to solve this problem using the solver. So, let me go to this and let me also zoom it a little bit.

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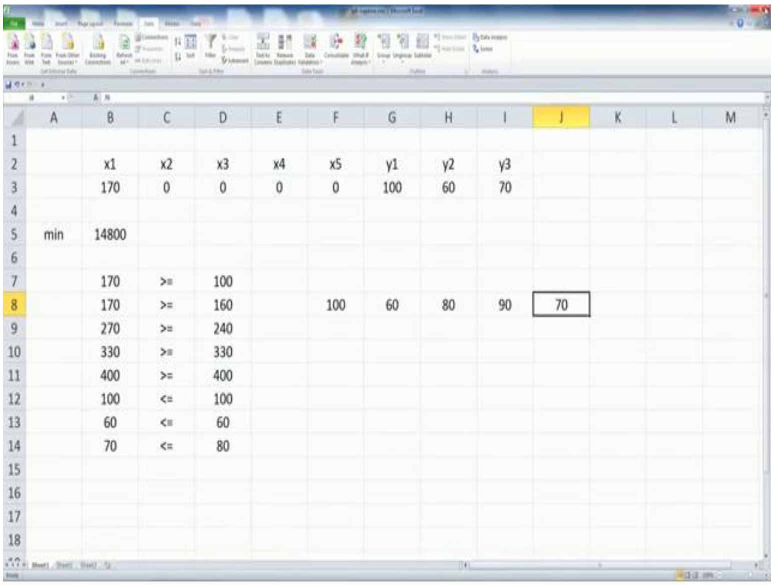

	A	B	C	D	E	F	G	H	I	J
2		x1	x2	x3	x4	x5	y1	y2	y3	
3		170	0	0	0	0	100	60	70	
5	min	14800								
7		170	>=	100						
8		170	>=	160		100	60	80	90	70
9		270	>=	240						
10		330	>=	330						
11		400	>=	400						
12		100	<=	100						
13		60	<=	60						
14		70	<=	80						

So, now, you see that here I have defined the eight variables which are X 1 to X 5 which are the new napkins that are used on days 1 to 5 Y 1 to Y 3 are the number of the napkins that are send to the laundry. So, I am just let me use some arbitrary values here, so 80, 60 let me use that 20 here, let me use at 10 here and so on. Now, the objective function is to minimize the cost of buying the new napkins which are bought at 60 and the cost of sending it to the laundry the cost being 20 per napkin.

So, if you see this objective function carefully what I have written is 60 into B 3 which is 60 into the new napkins, 60 into B 3 plus 60 into this plus 60 into this 10 plus 20 plus 10. This is a solution, where I am assuming that 80, 60, 10, 20 and 10 napkins or new napkins are used on all the 5 days and the number of napkins sent to this are 100, 60 and 70 I have just used some values that are here, the objective function is 60 is a prize of each new napkin. So, 60 times X 1 plus 60 times X 2 plus 60 times X 3 plus 60 X 4 plus 60 X 5 plus 20 Y 1 plus 20 Y 2 plus 20 Y 3.

And for the given values that cost is 15400 and we are eight constraints X 1 is greater than or equal to 100, X 1 plus X 2 is greater than or equal to 160, X 1 plus X 2 plus X 3 plus Y 1 is greater than or equal to 240, X 1 plus X 2 plus X 3 plus X 4 plus Y 1 plus Y 2 is greater than or equal to 330 and X 1 plus X 2 plus X 3 plus X 4 plus X 5 plus Y 1 plus Y 2 plus Y 3 greater than or equal to 400. Y 1 less than or equal to 100, Y 2 less than or equal to 60, Y 3 less than or equal to 70 these are the constraints.

So, all these are written here, so let me go to the solver and I have done everything all ready all I need to do is to solve it, the objective function is set as B 5 position which is here, the variable positions happen to be B 3to I 3 which are set here and the eight constraints are all ready written here. You can see the three less than equal to constraints are coming here they corresponds to the positions 12, 13 and 14.

So, position 12, 13 and 14 they are the less than or equal to constraints, rest of them all are greater than or equal to constraints. So, all the eight constraints are written here, so simplex L P and solve this problem to get 14800 as the solution. So, use 170 new napkins buy 170 new napkins on the first day and then keep the solver solution, so buy 170 napkins on the first day.

Now, the demands on all the 5 days are 100, 60, 80, 90 and 70 it also says put 100 napkins to laundry on the first day. So, use 100 out of by 170 use 100 on the first day, now 70 is available I am just writing down the 5 demands for reference. So, the 5 demands are 100, 60, 80, 90 and 70, these are the 5 demands the solution says buy 170 use 100 on the first day and send 100 to laundry on the first day. So, second day use 60, because that is the demand use 60 and send it to laundry at the end of the second day.

So, 10 new napkins are there, the 100 that has been send on the first day will come on the third day. So, third day will have 110 napkins, now use 110 and send 80 to the laundry now for the fourth day, this 60 that has been put that has been send to laundry on the second day will be available on the fourth day as 60, the 100 that was put out of that 20 are also available. So, this 60 plus 20 80 plus another 10 napkins will make it 90 for to meet the requirement of the fourth day and the fifth day now this 70 that is put at the end of the third day can be used to meet the requirements of the fifth day which is also 70.

So, this is how the solution has to be interpreted, so let me again interpret this solution for you, it says buy 170 of the beginning of the day one. So, 170 bought at the beginning of the day 1, 100 used on the first day to meet the requirement of the first day. So, balance available is equal to 70, now the second day use 60 of them, so second day use 60, so balance available for the third day is 10.

Now, put this 100 to the laundry on the first day, so you will get 100 at the beginning of the third day, there is all ready new 10 napkins remaining. So, available 110 for the third day, the demand for the third day is 80, so 13 more napkins are available the 60 that is put a at the end of the second day or going to arrive at the beginning of the fourth day 30 napkins are already available. So, 30 plus 60 will give us 90 to meet the requirements of the fourth day.

The third day 70 napkins have been send the demand on the third day for napkins is 80 out of which 70 is send to the laundry, the 70 arrive are the morning on the fifth day and they can be use to meet the demand of the fifth day. So, this how the solution is interpreted, so the formulation had eight variables and eight constraints and we are able to show the solution to this problem. Now, we look at one more example here we solve what is called the bicycle problem, so the bicycle problem is as follows which here.

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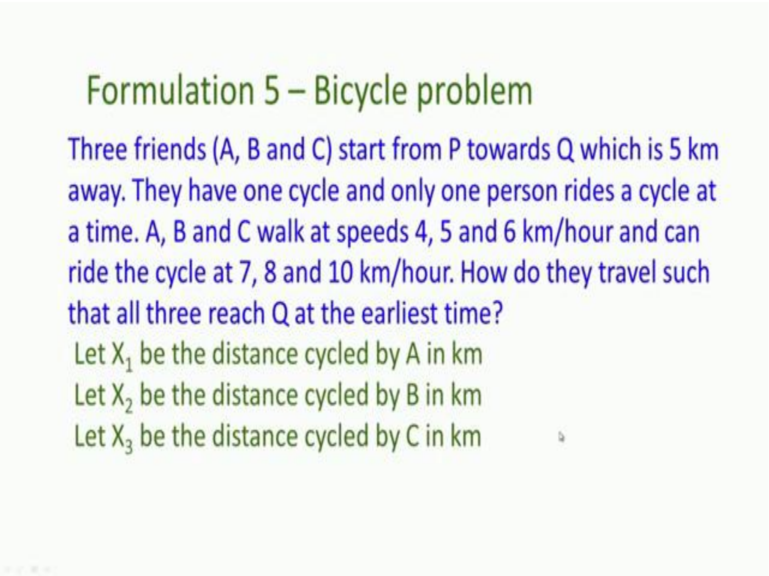

So, the bicycle problem is as follows here, so we said 3 people start and they can walk at speeds 4, 5 and 6 and they ride the bicycle at speed 7, 8 and 10, and then we formulated the problem with three variables X 1, X 2, X 3 as the distance cycled by A, distance cycle by B and distance cycle by C.

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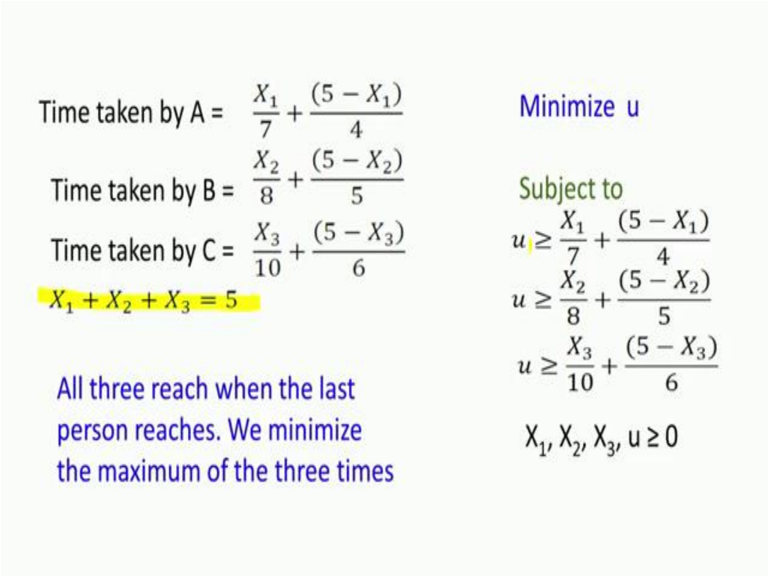

And we also finally, wrote the formulation which minimises u subject to these three constraints and the constraint that X 1 plus X 2 plus X 3 equal to 5. So, there are four variables X 1, X 2, X 3 and u, there are four constraints this is one constraints and these three constraints are there. Now, these three constraints have to be rewritten, so this constraint will be written now as u is greater than or equal to take the LCM 4 X 1 plus 7 into 5 minus X 1 divided by 28.

So, 28 u is greater than or equal to 4 X 1 plus 35 minus 7 X 1, so 28 u is greater than or equal to 35 minus 3 X 1. So, 3 X 1 plus 28 u is greater than equal to 35 that is the constraint. So, 3 X 1 plus 28 u is greater than equal to is 35 is the constraints, similarly these constraints have to be written. Now, these three constraints have to be written accordingly, so let us leave look at this, let us go to the excel sheet corresponding to this problem.

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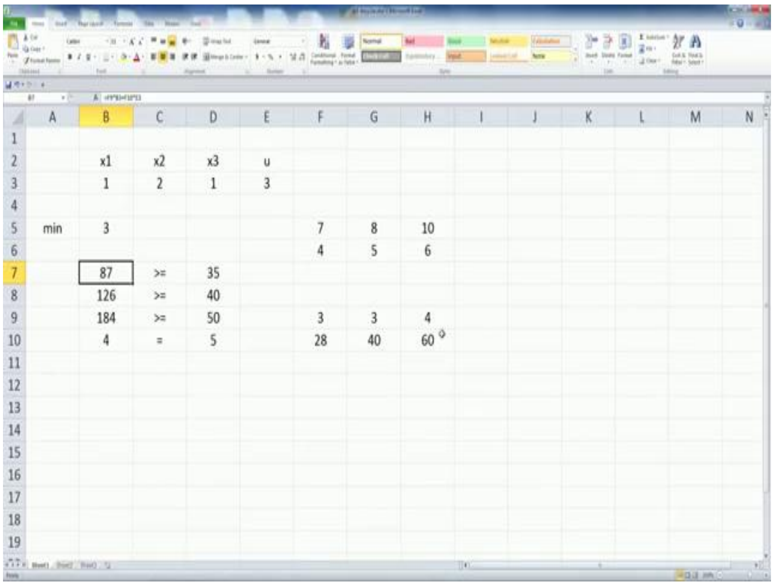

So, this is the excel sheet corresponding to this problem that I have written here, you can see there are four variables X 1, X 2, X 3 and u there are four constraints you can give some arbitrary values here. So, you can give value like 1, 2, 1 and some 3 given some arbitrary values, the objective function minimizes u. So, you can see this minimizes the E 3 positions, there are four constraints this is the constraint X 1 plus X 2 plus X 3 equal to 5, so you can see 1 plus 2 plus 1 4 here instead of 5 this is only a solution, this is not the optimum solution.

Now, these are the three positions that I have, remember that these constraints have to be simplified and have to be written. So, what I have done here is I have simply written the three given values of walking and a cycling. So, 7, 8 and 10 are the cycling speeds 4, 5 and 6 are the walking speeds and if we use this we will get 3 and 28 the difference and the products, the difference and the product, the difference and the product. So, these values will become the values that come here.

For example, this will become F 9 into B 3 plus F 10 into E 3, so 3 into this X 1 plus 28 into u is greater than or equal to 35. The other one 3 into X 2 plus 40 into u will be greater than or equal to 40, this 40 will come out of 8 into 5 40, when we simplify the constraints and bring it to the farm where we have the variables on the left hand side and the constant value on the right hand side the constraint will become like this.

So, the third one will become 4 into X 3 plus 60 into u will be greater than or equal to 10 into 5 50 when we take the LCM you will get that 10 into 5 that is 50 which will come not the LCM, the LCM will becomes 60 which will go to the other side and you will get that coefficient and when we take the LCM and bring it to a single fraction you will realise that the constant becomes 10 into 5 50.

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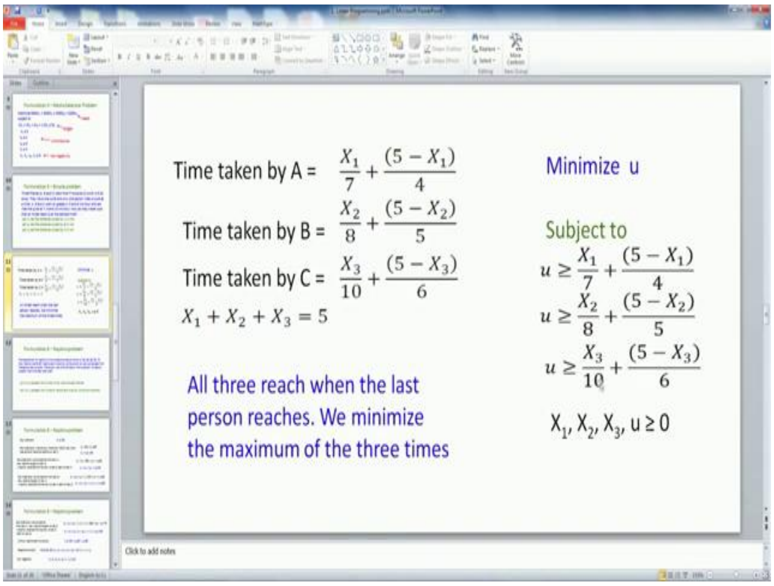

For the first constrain the constant was 35, because when we took the LCM 7 into 4 with 28 u this 7 and 5 is 35, second constraint is 8 into 5 will come, third constraint this 10 into 5 will come.

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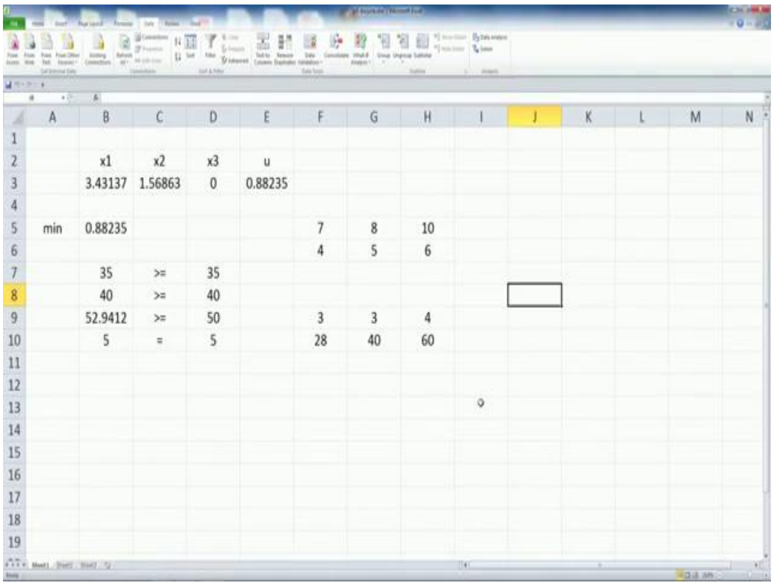

So, the constraints are written here and then we need to solve this, so we go to the data and we solve go to the solvent it is already said here, the objective function is set here, the four constraints are return as I have explain the variables are here. So, we solve and then we get the solution, now we also observe from this solution interestingly that the person X 3 is actually not cyclic, the person is walking the whereas persons corresponding to X 1 and X 2 are going to do the cycling with 3.43 kilometres and 1.57 kilometres which add up to the 5 kilometre.

So, the person X 3 is not cycling at all is walking that larger, because of the speeds and so on and the time at which they reach would be 0.88 hours. Now, this formulation is made in a slightly flexible mode, so if I simply change some of the walking speeds something can also happen. For example, if I change is to 3, changes is to 4 and change this to 3, let me see something happens if coefficients change, because the difference and the products also changed now I go to this solver and if I solved this.

Now, I realise that all three of them are actually using the bicycle for certain amount of time. So, by putting the coefficients in a certain manner and by simply changing it we can solve the problem for various values of the right hand side that is give. So, when we change it we observe that when we change the walking speed 3, 4 and 3 we got a solution where all the three people are using the bicycle.

So, person 1 is using the bicycle for about 2.5 kilometres and is going to walk for the remaining 2.5 kilometres. So, person 2 is going to use it roughly for about 0.5 roughly 0.46 and is going to walk for the remaining 4.54 or whatever quantity. Person X 3 is going to use it roughly 2 or 2.03 and will walk for 2.97 and they will reach at 1.19 hours all three of them will reach at the same time here, because all the three values are the same which happens here.

So, they will reach at the same time in the other, when some one person was actually not using the bicycle we realise that for example, if I change this back to 4, 5 and 6 which was the original situation and if we solve this, where this person does not use you would realise that the third constraint is actually becomes greater than which means actually the third person now takes a different times. So, two of them will reach at 0.88235, the third person would actually have reached a little ahead and little faster. So, that the maximum of the 3 is minimize and so on.

So, when all of them ride the bicycle at least for some time, all of them will reach at exactly at the same point, the two who are walking in the third who is coming in the cycle. But, if the solution shows that one person is not using the cycle at all, the these speeds are such that the person not using the cycle actually reaches first and the others come slightly late and it is maximized. So, this way we can actually solve this small problem with three variables, the other problem had eight variables and eight constraints. In the next class, we will see one more example, we will see the example of the bin packing problem which used binary variables, and we will see the example of the bin packing problem in the next class.