Introduction to Operations Research
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Module - 08
Solving LPs using a solver
Lecture - 02
Unboundedness and infeasibility

We will look at some more examples to solve L P’s, before we go to the formulations, and try to solve those problems that we had actually formulated.

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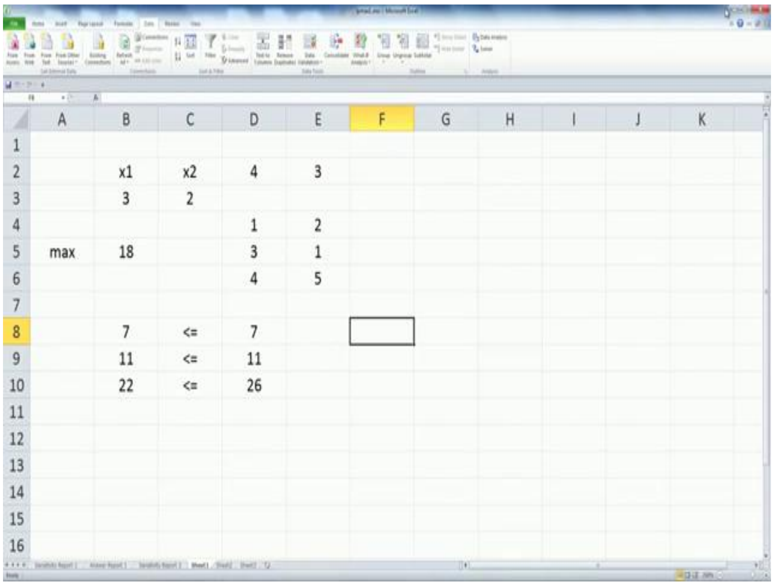

So, we take another maximization problem with two variables. The two variables are X 1 and X 2 that are shown here, X 1 and X 2. Right now, we give some arbitrary values, say 1 comma 1 to the two variables that we have here. The objective function for this problem is maximize 4 X 1 plus 3 X 2. So, 4 X 1 plus 3 X 2, I am just writing the 4 and 3 and I am just writing the objective function is equal to 4 into the value of X 1 plus this 3 into the value of X 2, which happens to be 7.

There are three constraints X 1 plus 2 X 2 less than or equal to 7; that is already written here, X 1 plus 2 X 2, 1 into 1 plus 2 into 1. Second constraint is 3 X 1 plus X 2 is less than or equal to 11 and the third constraint is 4 X 1 plus 5 X 2 is less than or equal to 26, they have all been written here. So, now, we have written the problem in the sheet and we now look at the solver.

So, we go to the solver, and then we set the objective function to this position, which is already set here, it is a maximization problem. The two variables are the B 3 and the C 3, which is already set here and the three constraints are here, which I have already set here. So, the three constraints are B 8 is less than or equal to D 8, B 9 value should be less than or equal D 9 value and B 10 value be less than or equal to D 10 value.

Now, there is also an option here, there is a thing called select a solving method and there is a dropdown, you could use simplex L P as the method that we are going to use to solve this problem. So, select simplex L P and that is what you get here. Now, you solve this problem to get the solution, which is X 1 equal to 3, X 2 equal to 2 with objective function is equal to 18.

We are actually used this problem before, we have actually used keep the solver solution and there we also get the sensitivity report for this problem. So, we have a sensitivity report for this problem, which I have got. We have actually solved this problem before to get the solution of Z is equal to 18. So, we have used this in an earlier situation.

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Maximize $4X_1 + 3X_2$
Subject to
$X_1 + 2X_2 + u_1 = 7$
$3X_1 + X_2 + u_2 = 11$
$4X_1 + 5X_2 + u_3 = 26$
$X_1, X_2, u_1, u_2, u_3 \geq 0$

Feasible solution
$X_1 = 3, X_2 = 2, Z = 18$

Apply complimentary slackness
$X_1 = 3, X_2 = 2, u_1 = 0, u_2 = 0\ u_3 = 4$

Feasible solution
$X_1 = 11/3, X_2 = 0, u_1 = 10/3$
$u_2 = 0\ u_3 = 34/3\ z = 44/3$

Minimize $7Y_1 + 11Y_2 + 26Y_3$
Subject to
$Y_1 + 3Y_2 + 4Y_3 - v_1 = 4$
$2Y_1 + Y_2 + 5Y_3 - v_2 = 3$
$Y_1, Y_2, v_1, v_2 \geq 0$

Apply complimentary slackness
$v_1 = 0, v_2 = 0, Y_1 = Y_2 =$ basic
$Y_1 = 1; Y_2 = 1$; feasible and hence optimum

Apply complimentary slackness
$v_1 = 0, v_2$ basic, $Y_1 = 0\ Y_2 =$ basic, $Y_3 = 0$
$Y_1 = 1; Y_2 = 4/3; v_2 = -5/3\ W = 44/3$
Dual infeasible; primal non optimum

So, this is the problem that we have used. So, this is the problem that had used earlier and you observe that, we have this solution here for this problem. In this case, it is a feasible solution, but the optimum solution is X 1 equal to 3, X 2 equal to 2, Z equal to 18 and we said it is optimum. So, X 1 equal to 3, X 2 equal to 2, Z equal to 18 is optimum in this example.

So, we get the same example here with X 1 equal to 3, X 2 equal to 2 and Z is equal to 18, which is the optimum solution here. Now, what also happens is, we have this report called sensitivity report and if I go back and zoom this report and make it bigger.

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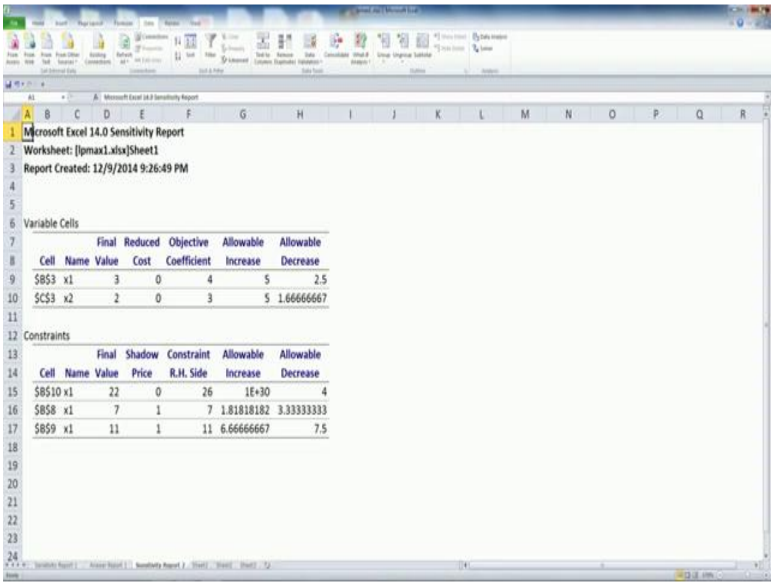
Microsoft Excel 14.0 Sensitivity Report
Worksheet: [lpmax1.xlsx]Sheet1
Report Created: 12/9/2014 9:26:49 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	x1	3	0	4	5	2.5
C3	x2	2	0	3	5	1.66666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B10	x1	22	0	26	1E+30	4
B8	x1	7	1	7	1.81818182	3.33333333
B9	x1	11	1	11	6.66666667	7.5

Now, it shows a few things, it shows the values of X 1 and X 2, which are 3 and 2, which gives us the objective function value of Z is equal to 18. You also have something called allowable increase and allowable decrease, which are actually the right hand side values can go up to an increase of 5 and a decrease of 2.5 for the same variables to be in the basis.

But, more importantly, now these are the right hand side, there are three constraints, now there is a description about the three constraints. Now, the first constraint is here, first constraint the left hand side value is 22, the right hand side value is 26. The second constraint the left hand side value, final value is 7 versus 7 and 11 versus 11 and 22 versus 26. Now, you can see this also here, 7 versus 7, 11 versus 11 and 22 versus 26.

Now, corresponding to this, you see shadow price as 0, 1 and 1, corresponding to this constraint, shadow price is 1, corresponding to this constraint shadow price is 1, corresponding to the third constraint, which is the B 10 position, the shadow price is 0. So, if I go back to the excel sheet, you find corresponding to this, shadow price is 1, corresponding to this, shadow price is 1, corresponding to this shadow price is 0.

Now, this constraint the resource is not fully utilized, only 22 out of the 26 is utilized. So, primal constraint is, the resource is not fully utilized, therefore, the shadow price is 0. In these two cases, the resources are fully utilized, the shadow prices can be seen here as 1 and 1, corresponding to positions 8 and 9. So, those are the positions, you can see corresponding to positions 8 and 9, the shadow prices are 1 and 1, the 7 into 1 plus 11 into 1 is 18, which is the value of the objective function.

So, when we use the simplex L P and go to the sensitivity report, we can also get the dual solution or solution to the dual of this linear programming problem. So, just as the simplex solves both the primal and the dual, you can use this solver to get the solution to the dual also by looking at the sensitivity report. Now, these are with respect to the right hand side values and how much the right hand side can increase for the same basis to remain and so on.

But, right now we restrict ourselves, only to understanding the solutions of the primal and the solutions of the dual, when we use this particular solver. Now, let us move to couple of other examples, where we understand a few more things in linear programming.

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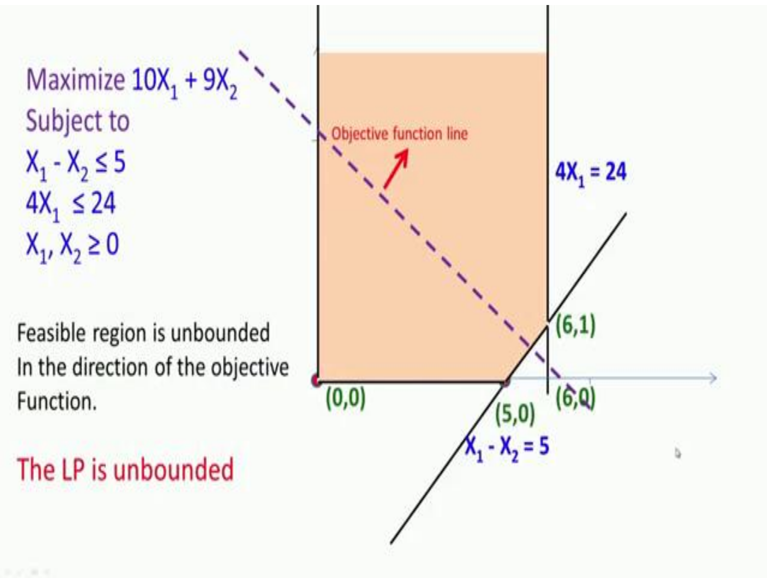

So, to do that, we go back to this, when we introduced, when we looked at the simplex algorithm, we looked at three situations. We said that, we have a linear programming problem can give us an optimal solution or a linear programming problem can be unbounded or the linear programming problem can be infeasible. So, we looked at examples for unboundedness and infeasibility and we showed the graphical solution to this problem and we said that, this is unbounded.

And we also said that, if when we solved it by the simplex algorithm, we also said that, when we actually solved it by the simplex, we said simplex indicates unboundedness by. Being able to identify an entering variable, but there is no leaving variable. So, in this iteration, we showed that, there is an entering variable, but there is no leaving variable, there is an entering variable here, but you cannot divide and do that.

Now, let us take the same example that we used here maximize 10 X 1 plus 9 X 2, subject to X 1 minus X 2 less than equal to 5, 4 X 1 less than or equal to 24 as the problem. And see, what happens, when we solve this problem using the solver. So, we go to this example.

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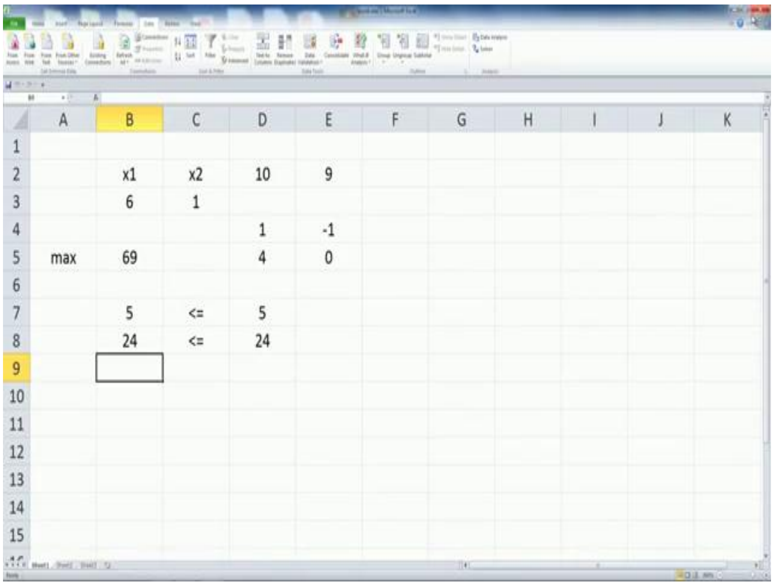

So, let us look at this example. So, we again have two variables X 1 and X 2, the objective function is written as 10 X 1 plus 9 X 2. So, I can show those 10 and 9, I can write here, 10 X 1 plus 9 X 2 and we can write the objective function as equal to 10 into the value of X. So, objective function is equal to 10, objective function is equal to equal to the value of X 1 into 10 plus 9 into the value of X 2, which is given by 69.

The two constraints are X 1 minus X 2, X 1 minus X 2 is less than or equal to 5, 4 X 1 plus 0 X 2 less than or equal to 24. So, these two constraints have already been mapped here and the values are written here. So, now, we call the go to data and we call the solver, the objective function is already said to be 5, the variables are said to B 3 and C 3. The two constraints are written here and we are going to use simplex L P and we solve this.

So, when we solve this problem, you will realize that, when in this box, you read a message called the objective cell values do not converge. It is not given a solution; we will read a message, which says the objective cell values do not converge. So, when you get a message saying the objective cell values do not converge, it means that the problem is unbounded. So, unboundedness is indicated by this message, which says, the objective cell values do not converge.

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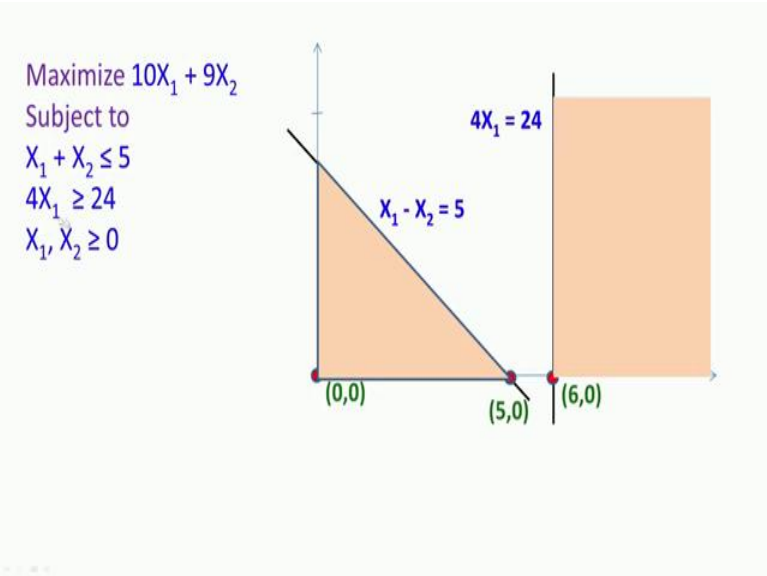

So, now let us go to another example, here when we did this simplex algorithm. We looked at another example, where we looked at this particular example, which is to maximize 10 X 1 plus 9 X 2, subject to X 1 plus X 2 less than equal to 5 and 4 X 1 greater than or equal to 24. And if we drew the graph for this, we said there is no feasible region and therefore, the problem infeasible.

Now, we also said in simplex, simplex will indicate infeasibility by reaching the optimal solution to the new problem with the artificial variables. So, whenever we have such problems, we will have artificial variables. So, simplex will terminate, but the artificial variable will be present with the strictly positive value. Now, let us look at this problem and see, what happens when we use the solver. So, maximize 10 X 1 plus 9 X 2, subject to X 1 plus X 2 less than equal to 5 and 4 X 1 greater than or equal to 24.

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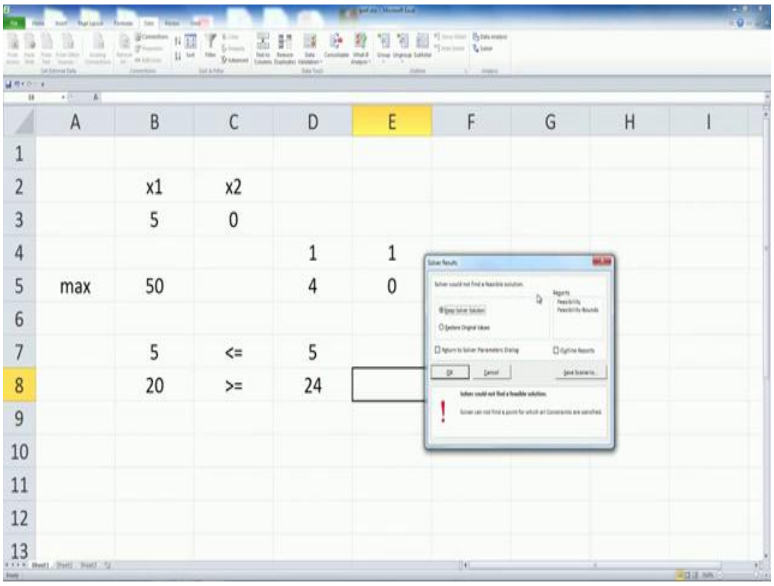

So, we look at this problem, the again let me, so we look at this problem, there are two variables X 1 and X 2. Right now, I have given values 4 and 0. So, we can give any value 2 and 1, let us say 2 and 1 and quickly, we realize that, if we look at this constraint, we realize that, it is infeasible for the present value. The objective function is to maximize 10 X 1 plus 9 X 2, I have already written 10 into this position 2 plus 9 into this position 1, 29.

Instead of writing 10 and 9 here separately and pulling it into the objective function, I have written the objective function is 10 into B 3 plus 9 into C 3, which gives the value 29. The constraints are written here X 1 plus X 2 less than or equal to, X 1 plus X 2 less than or equal to 4 here and 4 X 1 is greater than or equal to 24. So, let us go back to this problem, ((Refer Time: 13:58) the constraint was X 1 plus X 2 less than or equal to 5 and 4 X 1 greater than or equal to 24. So, I write X 1 plus X 2 less than or equal to 5 and 4 X 1 greater than or equal to 24.

Now, we have written this problem. So, we want to solve this. So, we go to data and get the solver, everything is already set. So, the objective function is here, the constraint positions are here, the constraints are drawn and we use simplex L P. When, we solve here, we will get a message that solver could not find a feasible solution. So, when we get a message, solver could not find a feasible solution, it means the given problem is infeasible.

So, this solver will give three messages, if it gives an optimum solution, it will say that optimum solution is found. For example, let us go to that situation, where let us take this example where the optimum solution is here.

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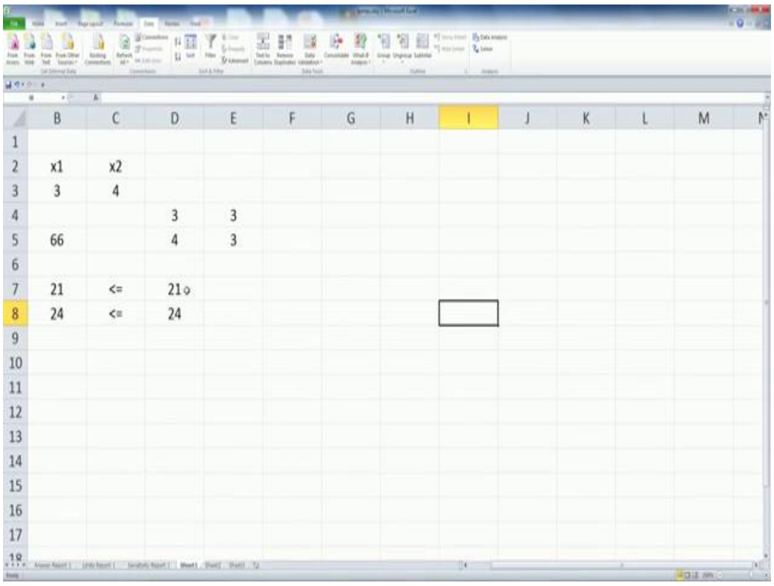

So, sheet 1, suppose I call this problem, again I go data, I go the solver and I solve it, it will say that solver found a solution, all constraints and optimality conditions are satisfied. If it gives that message, and then you have to read the solution to find the best value of the variables and the objective function. If it is unbounded, it will say cell values do not converge, if it is infeasible, it will say, solver was not able to find a feasible solution.

So, the three things that we discussed, every L P either you know terminates with an optimum solution or is unbounded or is infeasible, all the three cases are taken care of this way using this particular solver to solve linear programming problems. And as I also indicated, we could go back to the sensitivity report for this problem, the problem that we have been solving. So, if we write the sensitivity report, the primal as a solution X 1 equal to 3, X 2 equal to 4 with objective equal to 66 and we also see that the shadow prices are 2 and 1, you will realize, shadow prices are 2 and 1.

So, we go back here. So, 2 into 21, which is 42 plus 1 into 24, which is 66. So, the solution to the dual can also be found through this. In the next class, we will go back to some of the problems that we formulated, and then we show how the solver gives the solution to few of these examples. We did eight problems in our formulation, we will try to see a couple of examples and show the solution using this solver.