Introduction to Operations Research
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Module – 01
Linear Programming – Introduction and formulations
Lecture - 03
Media selection problem and Bicycle problem

In today's class we will look at two more formulations in linear programming. The first one is called the media selection problem and the other is called the bicycle problem. We will first look at the media selection problem which is a very popular formulation in linear programming the problem is as follows. A company wants to advertise their product in four different media which are television, newspaper, websites and radio.

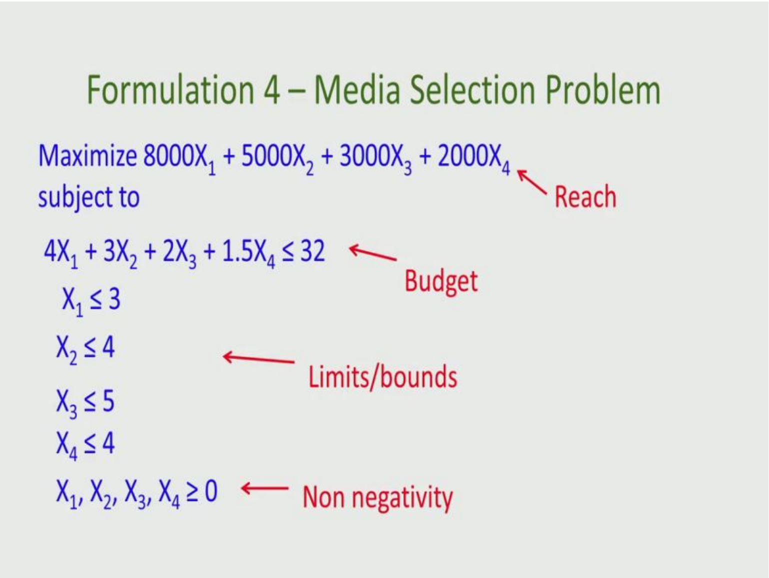
(Refer Slide Time: 00:55)

The reach per advertisement in these four media is, 8000, 5000, 3000 and 2000 respectively. Each advertisement is going to cost and the costs are given as rupees 4 lakhs, 3 lakhs, 2 lakhs and 1.5 lakhs respectively in each of these four different media namely television, newspaper, website and radio.

The company does not want to have a large number of advertisements in a single media and therefore, it restricts the number of advertisement that it wishes to have in each media as 3 4 5 and 4 respectively. There is also a budget restriction and the budget restriction is rupees 32 lakhs which is the maximum amount that can be spent in these advertisements. So, the problem is how many advertisements does the company, decide to have in each of the media so, that we maximize the overall reach. Now we are going to formulate this problem as a linear programming problem and we first define the decision variables.

Now, the decision variables are x 1 to x 4 where x 1 is the number of advertisements placed in television, x 2 in newspaper, x 3 in websites and x 4 in radio. There are four different media in which these advertisements can be placed and there are 4 decision variables one corresponding to each of these.

(Refer Slide Time: 02:50)



We now move on to write the objective function the objective function is to maximize the total reach. The total reach is the sum of the reach associated with each of these media. So, if x 1 advertisements go to tv then 8000 x 1 is the reach that we get similarly 5000 x 2 is the reach that we have through the advertisements in the newspaper, 3000 x 3 is for websites and 2000 x 4 is for radio.

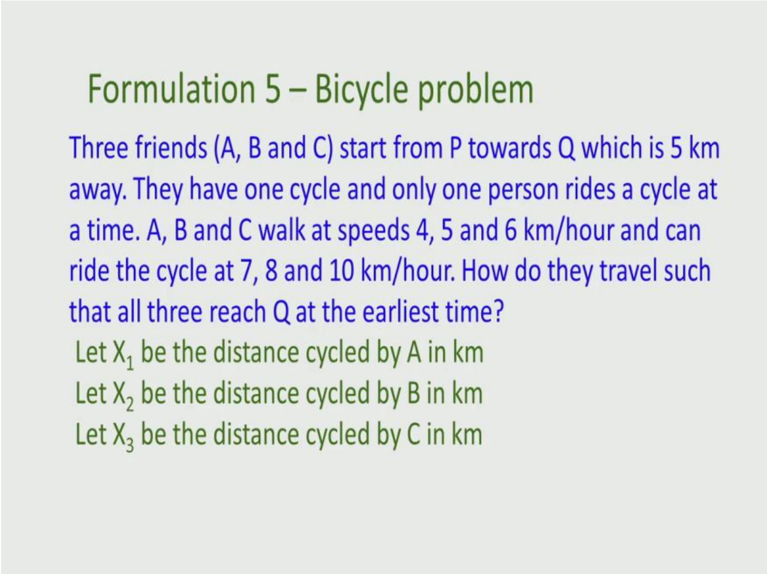
So, the total reach is a sum of the reach obtained through advertisements in the 4 media and therefore, we wish to maximize 8000 x 1 plus 5000 x 2 plus 3000 x 3 plus to 2000 x 4. There are several constraints, the first constraint is the budget restriction. So, x 1 advertisements in television would cost us 4 x 1 lakhs, x 2 advertisements in newspaper would cost us 3 x 2 lakhs, x 3 advertisements in the website would cost us 2 x 3 and x 4 advertisements in the radio would cost us 1.5 x 4. So, the total money that we will be spending is 4 x 1 plus 3 x 2 plus 2 x 3 plus 1.5 x 4. And that cannot exceed 32 lakhs which is the budget restriction.

We also have restrictions on the number of advertisements that we would place in each of these media and they are given by x 1 less than or equal to 3, x 2 less than or equal to 4, x 3 less than or equal to 5 and x 4 less than or equal to 4. We also have the non-negativity restriction which is x 1, x 2, x 3, x 4 greater than or equal to 0. So, this completes the formulation of the media selection problem where the objective is to maximize the total reach. There is a budget restriction and there are restrictions on the maximum number of advertisements that can be placed in each of the media. So, this formulation has 4 decision variables and it has 5 constraints, one is the overall budget constraint and the other 4 are restrictions on the number of advertisement that we would like to place in each of this media.

I have also indicated that these 4 constraints are called limits are bounds and if we have a constraint of the form x 1 less than equal to 3 or x 2 less than equal to 4 which means, if you are restricting the value that a variable can take and if that comes in the form of a constraint like this 4 constraints that we have these constraints are also called bounds on the variables. At the moment we will define these 4 as 4 separate constraints, but they also called bounds on the individual variables. So, this formulation has 4 variables, it has 5 constraints 4 of which are bounds and it has the non-negativity restriction. Once again since we have defined x 1, x 2, x 3 and x 4 as the number of advertisements we have this question whether x 1, x 2, x 3, x 4 should be defined as continues variables or should be defined as integer variables.

At the moment we are going to define them as continues variables because we are going to formulate a linear programming problem. So, we restrict this variable to be continuous and at the moment we do not restrict them to have only integer values.

(Refer Slide Time: 07:09)



We now move on to this second formulation for this class which is called bicycle problem and it is a very interesting problem, and you will see how interesting it is as we move along. Now, the problem statement is as follows three friends let us call them a b and c start from p and want to reach q, p and q are different places or different points and p and q are 5 kilometers away. They have only 1 bicycle and only 1 person rides the bicycle at a time.

So, we will assume that the remaining 2 people will be walking at different speeds, while 1 person will be riding the bicycle. Now, the 3 people can walk at speeds 4, 5 and 6 kilometers per hour and they can ride the bicycle at 7 8 and 10 kilometers per hour when they get to ride the bicycle. How do they travel such that all the 3 reach the destination q at the earliest time? This is a very interesting problem that also has a simple linear programming formulation. Now, we will also observe that when we tray and solve this problem the first thing we observe is the individual cycling speeds 7, 8 and 10 kilometer per hour are faster than the individual walking speed the 4 5 and 6 kilometers per hour. The slowest among the cycling speed is still higher than the fastest among the walking speed and therefore, at any point in time one of them will be riding the bicycle while, the other 2 will be walking.

We also have to understand that when they are walking they are walking at different speeds and therefore, there not going to walk together. We will also make one assumption that it actually does not matter who starts the cycle first, whoever riding the cycle will go to a certain distance stop leave the cycle there and start walking while, the other two who have been walking the one who reaches a cycle first between the two of them will now take the cycle and ride a certain distance leave the cycle at some point and walk towards the destination. For the third person when the person reaches there will now take the cycle and come towards the destination.

So, we will try to model this situation as a linear programming problem and our assumption is valid because the fastest among walking speed which is, 6 kilometer per hour is still smaller than the slowest of the cycling speed which is, 7 kilometers per hour. So, with this assumption letter start for this problem. So, there are three of them. So, we will say that out of the distance of 5 kilometers, let person a ride a distance 1 in the cycle let person ride x 2 in the cycle and let person c ride x 3 in the cycle. It also means that person a is going to walk 5 minus x 1 person b will walk 5 minus x 2 and person c will walk 5 minus x 3. So, there are 3 variables which are called x 1, x 2, x 3 which represent the distance cycle by a b and c respectively. Now we have to find out the time taken by a.

(Refer Slide Time: 11:11)

Time taken by A = $\frac{X_1}{7} + \frac{(5 - X_1)}{4}$

Time taken by B = $\frac{X_2}{8} + \frac{(5 - X_2)}{5}$

Time taken by C = $\frac{X_3}{10} + \frac{(5 - X_3)}{6}$

$X_1 + X_2 + X_3 = 5$

All three reach when the last person reaches. We minimize the maximum of the three times

Minimize u

Subject to

$$u \geq \frac{X_1}{7} + \frac{(5 - X_1)}{4}$$

$$u \geq \frac{X_2}{8} + \frac{(5 - X_2)}{5}$$

$$u \geq \frac{X_3}{10} + \frac{(5 - X_3)}{6}$$

$X_1, X_2, X_3, u \geq 0$

Now, the time taken by a is the time taken when a is using the cycle, and the time when a is walking. So, a is travelling distance x 1 and a is riding at 7 kilometers per hour. So, the time taken by a when a is riding the bicycle is x 1 by 7. The time taken by a when a is walking is a walks a is the distance of 5 minus x 1 because a cycles a distance x 1 a walks a distance 5 minus x 1 and a walks at speed of 4 kilometer per meter hour. So, the time taken by a in walking 5 minus x 1 is 5 minus x 1 by 4.

So, the total time taken by a is x 1 by 7 plus 5 minus x 1 by 4. Now, we have to find out the time taken by b and the computations are similar. So, b cycles a distance x 2 therefore, b would take time to x 2 by 8 on the cycle, b would walk distance of 5 minus x 2 and would take 5 minus x 2 by 5 hours to walk and the total time taken by b will x 2 by 8 plus 5 minus x 2 by 5 in hours. Similarly, we can compute the time taken by c which is the sum of the time taken when c is riding the cycle and the time taken when c is walking so, c rides at a speed of 10. So, c would take time x 3 by 10 to cover a distance of x 3, c would take 5 by 6 because c walks at 6 kilometers per hour and covers a distance of 5 minus x 3.

So, the total time taken by c is equal to x 3 by 10 plus 5 minus x 3 6. Now these three are not constraints by themselves, we have to write constraint out of this. At the moment the only constraint we can write x 1 plus x 2 plus x 3 d equals 5, the distance cycled by a, the distance cycled by b and the distance cycle by c is equal to 5. So, x 1 x 2 plus x 3 equal to 5 where we also do two things we restrict the total distance to 5 and we also now say that only one person ride the cycle at a time which is implied, implicitly assumed in the formulation. We write an equation x 1 plus x 2 plus x 3 equals 5 under the assumption that one of them is using the bicycle right through the distance and that come from the assumption that, the fastest of the walking speeds is slower than the slowest of the cycling speed.

This would mean that, one person would be using the bicycle at any point in time. So, the one constraint that we have written is x 1 plus x 2 plus x 3 is equal 5. And we are yet to write other constraints and the objective function let us now see that. Now, we all know that we wish to minimize the time at which all three reach the destination. So, the objective is to find out the time at which all three reach the destination at the earliest. So, all the three would have reached the destination when the last person out of a b and c reaches the destination.

Therefore, we minimize the maximum of the 3 types, the time taken by a, the time taken by b and the time taken by c. The maximum of the 3 times is going to decide when all the 3 have reached the destination. So, the maximum of the three is the time in which all three have reached the destination and we now wish to minimize the maximum of the 3 times. So, let us called the maximum of the 3 times to be u which is a variable. So, we minimize u which is our objective function. Now, the corresponding constrains are since u is the maximum of the three times, u have to be greater than or equal to the 3 times. Therefore, the u greater then you x 1 by 7 plus 5 minus x 1 by 4 u greater than or equal to x 2 by 8 plus 5 minus x 2 by 5. And u greater than or equal to x 3 by 10 plus 5 minus x 3 by 6.

Now, we realize that we have written the objective function and we have also 3 constraints where we relate the variable u which is in the objective function to the three times taken by a b and c respectively. We also have the constraints x 1 plus x 2 plus x 3 equal to 5 which is the fourth constraint and x 1 x 2 x 3 and u greater than or equal to 0. So, this problem has now 4 variables, the 3 variables x 1 x 2 x 3 which are the distance cycle by a b and c and u which is the maximum of the time taken by a b and c respectively.

So, the objective is to minimize u, u greater than or equal to time taken by a, u greater than or equal to time taken by b, u greater than or equal to time taken by c and x 1 x 2 x 3 and u greater than or equal to 0. So, the formulation has 4 variables, 4 constraints the 3 constraints that are shown here for u and the constraint x 1 plus x 2 plus x 3 equal to 5 and then all the 4 variables are greater than or equal to 0.

Now, this formulation also teaches us to have objective functions which, essentially try to minimize the maximum of certain function or in certain other formulations we will have situations where we have to maximize the minimum of certain function. So, if we have to minimize the maximum of certain functions in this case minimize the maximum of these three functions now defined another variables u which represent the maximum of the three minimize the u and then say that since u is the maximum u has to be greater than or equal to each 1 of the 3. So, this formulation helps us to understand and helps us do the modelling when we have situation where we minimize the maximum of certain given functions.

In the next class, we will look at two more formulations related to linear programming.