Introduction to Operations Research
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Module - 06
Transportation Problem
Lecture - 03
Penalty Cost Method

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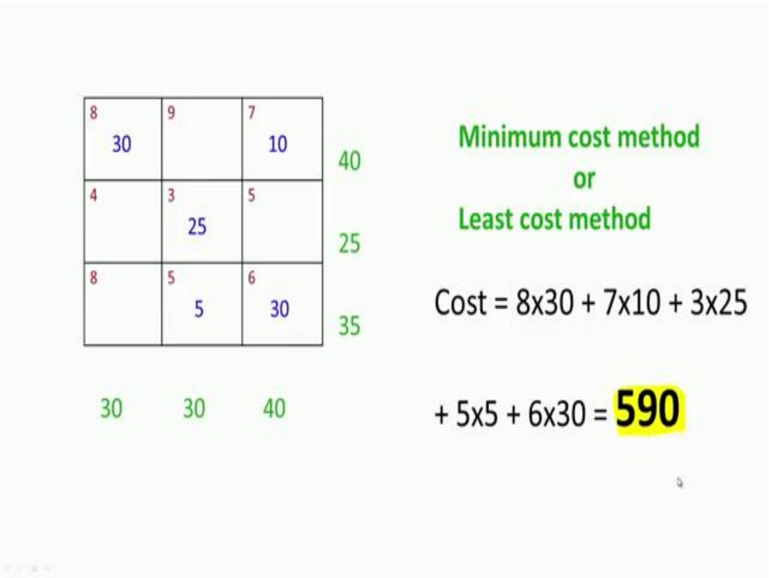

In this class, we will see another method that gives us a good starting solution. So far we have seen two methods, the North West Corner rule and the least cost method or minimum cost method. The North West Corner rule gave us a solution with cost equal to 625, the minimum cost or least cost method gave us a solution with cost equal to 590.

Now, between the two we observed that the minimum cost or least cost method gave a better solution with the lower cost. If we compare those two methods, the North West Corner and the minimum cost, the minimum cost is expected to give us a solution with lower cost than the North West Corner rule. Because, the minimum cost method takes into consideration, the unit transportation cost every time it makes an allocation. Whereas, the North West Corner rule was concentrating more on the position, rather than what was the cost associated with that position.

So, between the two methods we could say that least cost is preferred, it most of the times it gives us the solution with the cost lower than that of what is given by the north west corner rule. Now, we will look at another method where using which we will get a starting solution.

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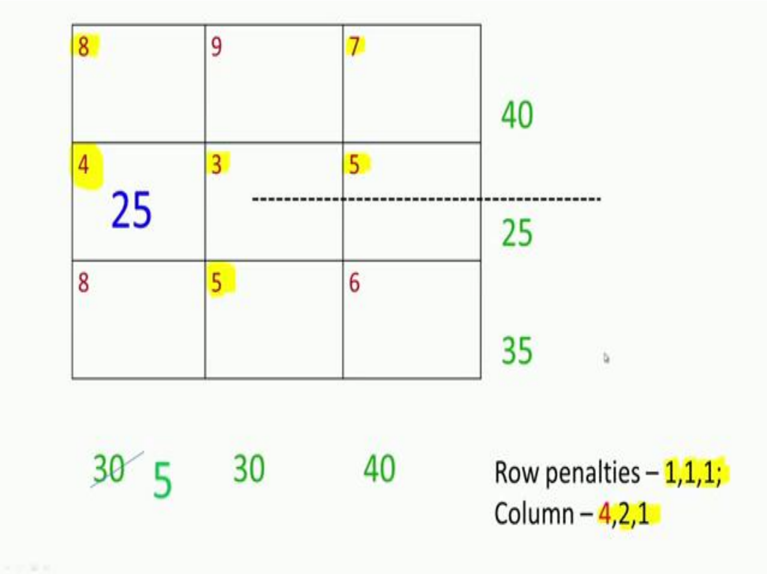

Now, let us go back to this table and we are familiar with this table, the three supplies are shown in the three right hand side, the demands are shown 30, 30 and 40. Now, let us look at the first supply point of 40. Now; obviously, in the solution at the end of it when we get a solution all the 40 has to be consumed. So, all the 40 has to be distributed between the value here, the value here and the value here X 1 1 plus X 1 2 plus X 1 3 is equal to 40.

So, if we want to allocate this 40 without at present looking at the demands, we would ideally put all the 40 to it is least cost position which happens to be 7 which happens to be here. Ideally we would like to put it here or try to put as much as we can in the least cost position. Similarly, if we look at this 25, we would like to put everything here in this or we would like to put as much as we can in the least cost position.

In the same manner, if we take this 35 we would try to put as much as we can here and so on. So, if we take any supply point or any row, we would like to allocate as much as we can in the least cost position corresponding to that row. Under the assumption that if we are able to do that and we are able to get a feasible solution the cost will obviously, be less. Let us now look at the demands, the demands are 30, 30 and 40. Now, when I sum, how I have to meet the demand of 30?

So, whatever I am allocating to this X 1 1 plus X 2 1 plus X 3 1 is equal to 30, so this 30 has to be distributed to values in these three positions, non negative values in these three positions. So, ideally I would like to get or put as much as we can in this position which is the least cost position, which means ideally I would like to get as much of this 30 as I can from this 25 that is available in this position.

Similarly, as much of this 30 I would like to meet from this 3 or the second position and as much of this 40 from 5 or from this position. So, given a row or given a column which means given a supply or given a demand, we would like to have as much allocation as possible in the corresponding least cost position, but now let us look at another situation. Now, if we take this 40 we just now said that we would like to put as much of this 40 in the least cost position.

But, if we are not able to allocate in the least cost position for example, if we take here this 40 maximum we would like to get from this 25, this 30 maximum we would like to get from this 25. So, it may not be possible to have this least cost allocation in every row or every column. So, in the event of not being able to allocate in the least cost position, our next alternative would be to try and allocate in the next costlier position.

Example, if we take this 40 as much as we want to have a large number coming here, because this 7 represents the least cost position. In the event of us not being able to allocate in the least cost position, we would try to put as much as we can is the next position which is 8. So, we now say that the cost of not or the increased cost of not being able to allocate in the least cost position is the difference between the least cost and the next higher cost. If we are not able to put in that 7 we would then like to put as much as we can in the position, where the cost is 8.

So, we now define something called a penalty which is the increased cost that we would incur by not assigning or if we are not able to put the maximum in the least cost. And this penalty, therefore is the difference between the next least cost and the least cost. The next least cost is higher than or equal to the least cost, a row or a column can have two positions with the same least cost or one position which is smaller and the next one would be a little higher.

So, in the first row the positions the least the costs are 7 and 8, 7 is the least cost, the next least cost is 8, therefore the penalty is 8 minus 7 which is 1. So, the penalty for the first row is shown here which is 8 minus 7 which is 1. For the second row, the minimum cost is 3 next minimum cost is 4, therefore the penalty is 4 minus 3 which is 1. For the third row, the minimum cost is 5, the next minimum cost is 6, therefore the penalty is 6 minus 5 which is 1.

Now, we compute the column penalties. For the first column, the minimum cost is 4 the next minimum cost is any of the 8, so 8 the penalty is 4. For the second column, the minimum cost is 3 the next minimum cost is 5, the penalty is 5 minus 3 which is 2. For the third column, it is 6 minus 5 which is 1. Again, if a row or a column has the minimum cost coming in two positions, the penalty is 0. Now, we have computed the 6 penalties, and then we now look at the 6 penalties and understand that if I take the first row and if I am not able to allocate in the least cost position, my penalty is 1.

Whereas, for the first column if I am not able to allocate in the least cost position my penalty is 4. Therefore, this penalty higher penalty of 4 makes me look at the first column first and not the first row. So, choose the row or column that has the largest penalty, because that large penalty we want to avoid by being able to allocate in the least cost position in the chosen row or column that has the largest penalty. So, you choose that row or column that has the largest penalty which happens to be 4.

And now we understand that if we are able to allocate as much as we can in the least cost position in that row or column, then we will avoid this penalty of 4 as much as we can. Therefore, choose the row or column that has the largest penalty and in that row or column that has been chosen, allocate as much as you can in the least cost position, so that the penalty can be avoided. So, we choose the first column which has the largest penalty of 4. So, this is the column and we would like to look at this least cost position in the chosen column which is this position.

Now, this position the availability is 25, the requirement is 30, so we will allocate the maximum possible which is the minimum of 25 and 30 and the first allocation is 25. Now, when the first allocation is 25 we have met the entirely we have met the demand of, we have consumed the supply from this position and that is shown by this dotted line.

Now, by meeting 25 out of the 30 only an additional 5 has to be met and that 5 is shown here.

Now, having done this now the matrix that we have has only two rows, the first row and the third row, and then it has three columns. Now, we have to find out the row penalty for the remaining two rows and the column penalty for the three columns. Now, we try to do that.

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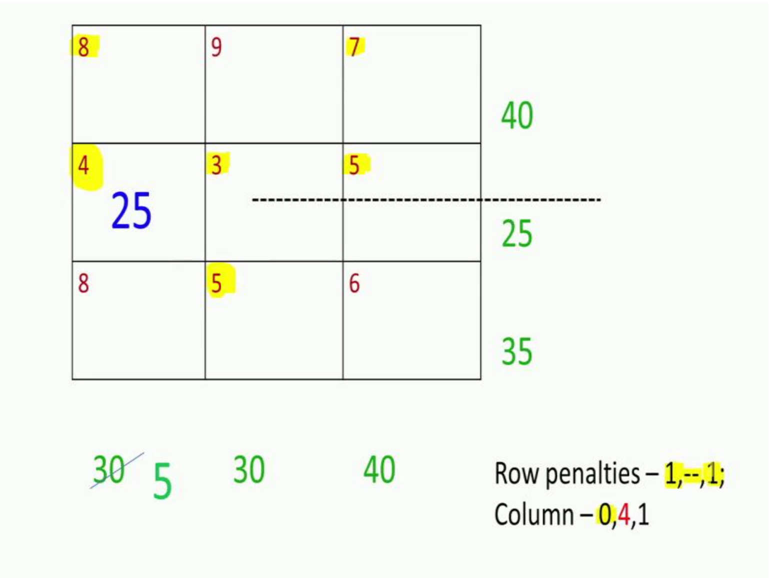

So, now let us look at the two rows that we have, the first row the minimum cost is 7, the next minimum is 8, therefore the penalty is 8 minus 7 which is 1. The penalty was the same as that in the last calculation. The second row we are not going to look at it, therefore I have shown it using a dash. The third row, the penalty the minimum is 5 the next minimum is 6, so 6 minus 5 is 1 which happened to be the same as in the previous calculation.

But, now let us look at the column penalties for the three columns, now if we take the first column, the minimum now is 8. Because, already there is an allocation in this, therefore this 4 is not considered. Only the unallocated positions the minimum is 8, there is another 8, therefore the penalty is 0. The next minimum is also 8, therefore the penalty is 0. For the second column the minimum is now 5, because we cannot put anything in this 3, the minimum is 5. Because, we cannot put anything in this 3, because all this 25 has already been consumed, the minimum is 5 the next minimum is 9, therefore the penalty is 4.

For the third column the minimum is 6, because we are not going to allocate anything in this position, the next minimum is 7, so 7 minus 6 is 1. So, we have now calculated the penalties again, the two row penalties and three column penalties, now find out that row or column that has the largest penalty. Now, the second column has the largest penalty, now having identified the second column with the largest penalty try to put as much as you can in the least cost position in corresponding column, so that the penalty can be avoided.

So, take the second column, take the least cost position which happens to be this 5 and try to put as much as you can now 35 is available, 30 is required. So, minimum of 30 and 35 which is 30 can be put in this position, so 30 has been put in this position. Now, by putting 30 in this position, we have completely met the requirement of the second demand point and that is shown through a vertical line which is there, it also means that we are not going to make any allocation in this position.

Now, by putting this 30 here 30 out of the 35 available has been consumed and only 5 is remaining and that is shown this way with 5 being available. Now, we have to look at the rest of the matrix and now calculated the penalties again, so to do that we have to calculate the penalties again, so we do this.

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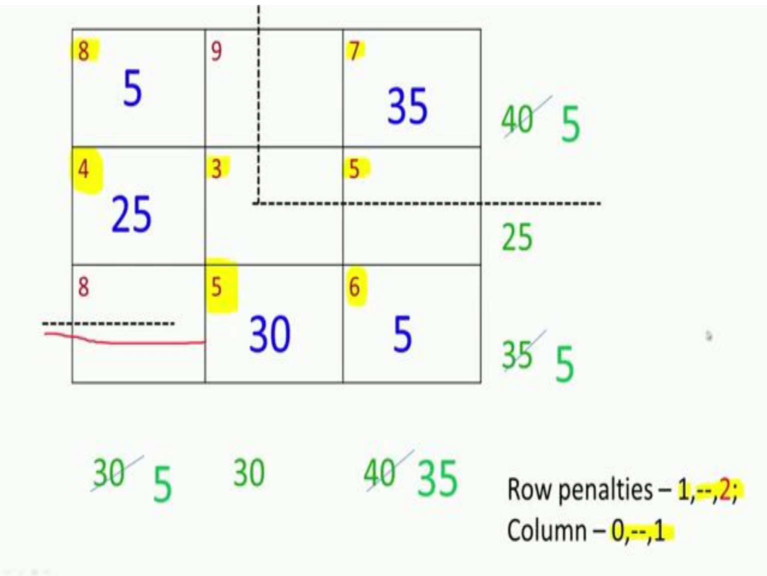

Now, the remaining part of the matrix has only two rows and two columns left, the second row and the second column are not going to be considered. So, we have first row and third row where supplies are still available and first column and third column, where requirements have to be met. So, the row penalty for the first row is 7 is the least, 8 is the next, so row penalty is 1, second row we are not going to calculate, for the third row the minimum is 6 the next minimum is 8, so a penalty is 2.

For the first column it is again 8 and 8, so 0 second column we do not calculate, third column 6 and 7, so the penalty is 1. Now, row number 3 has the maximum penalty, identify the minimum position in that and try to put as much as you can. So, that you avoid this penalty as much as you can. So, now, the available supply is 5 and the requirement is 40, and therefore you put the minimum between 5 and 40 which is 5.

Now, by putting this 5 this entire supply is consumed, therefore this becomes 35 this is entirely consumed. So, we would not put anything in this position, so this is entirely consumed, so I am just going to show this with a red line and we will not have anything is this position. Now, the remaining part of this matrix has only the first row remaining and it has two columns remaining. So, first row again the penalty is 1, so that shown.

So, now, again we do not have to now calculate for the penalty for the first row is 1 and since we have only one row, now penalty for the first row is 1. So, 35 is remaining here, the first column will have a penalty of both are used up there is only an 8 here. So, we could assume there is only one position, so the cost of not doing it, you could take it as a large value or 8 similarly this 7. So, we could now look at the 1, the penalty here is 8 minus 7 which is 1 and we do not actually find column penalties. So, we try to put as much as we can in the 7 position which happens to be the remaining 35 which is available, so 35 is put in this position.

And then we now realize that this 40 will become 5 and 5 will go to the last position, because only one position is available, so 5 will go to the last position. So, once again when we came to the other one, there is only one row that is available and other there are two columns that are available. Actually when we come to a situation, where only one row is available we do not have to compute the penalties, we have to put as much that is available in that row to the allocation in the various columns that would also lead us to putting 35 in this position and 5 in the other position.

Now, we have another feasible solution which has row sum is equal to 5 plus 35 40, 25 30 plus 5 35 column sum 30, 30 and 40 all values are greater than or equal to 0, 5 positions have positive values 4 of them 0 value and let us find the cost associated with this.

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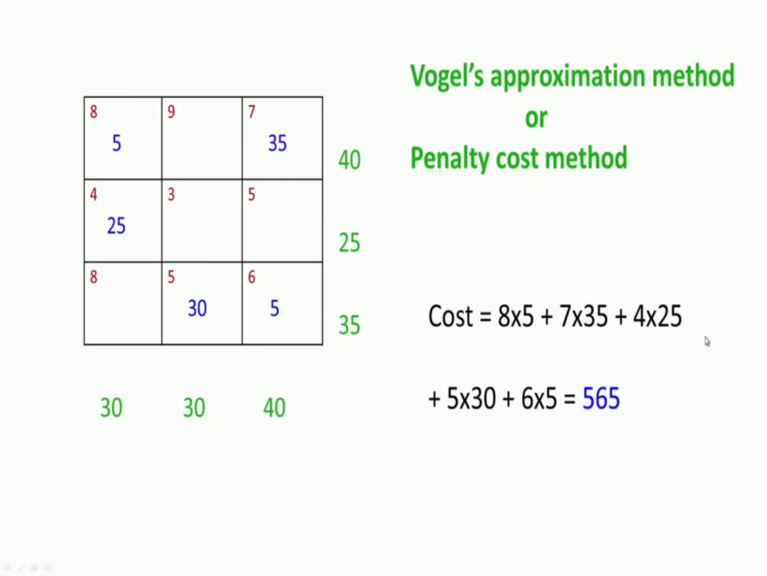

So, the cost associated with this method I have shown the solution again is 8 into 5 plus 7 into 35 which is from this 4 into 25 which is from this 5 into 30 plus 6 into 5 which is 565, so the cost is 565. Now, North West Corner gave 625, min cost gave 590 and this method which is called penalty cost method or Vogel's' approximation method gives us a cost of 565.

Now, we have found out three methods that give us good starting solutions, north west corner which was essentially based on position, the min cost which is essentially based on the least cost available at that point and the penalty cost which is basically centered around identifying the penalties and choosing based on maximum penalty and putting as much as we can in the minimum position.

In terms of effort required North West Corner is very simple and intuitive, minimum cost requires a little more effort to identify the minimum cost and the penalty cost requires more effort to identify the place, where we have to make the allocation. In terms of solution generally Vogel's approximation does better than the other two and minimum cost does better than the North West Corner. All three of them are centered around a principle that when we make an allocation, we put as much as we can which means we either consume all the supply or exhaust all the demand.

Now, having seen these three we now have to answer the question, now where is the optimum or where is the best or these three close enough to the optimum or the best and how do we get the optimum from any one of the three that we may choose to solve the transportation problem. So, far we have seen methods that have given us what are called good starting solutions, now from these we have to go back and look at methods that would give the best or the optimal solution, we will see those methods in the next class.