**Introduction to Operations Research**
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**Module - 06**
**Transportation Problem**
**Lecture - 02**
**North West Corner Rule and Minimum Cost Method**

In the last class, we introduced the Transportation Problem and we considered a numerical example with three supply points and three demand points, and we defined the problem in terms of it is variables. And we also said that it is a linear programming problem which can be solved through linear programming. But, we also mention that there are methods which are faster, quicker compared to solving it as a linear programming problem directly.

So, in this class we will look at some of these methods that can solve the transportation problem faster and efficiently. We would use this three supply three demand problem as a numerical illustration to explain the algorithms and the methods. Now, this problem is shown in the form of a network and to make it little simpler we now show it in the form of a table.

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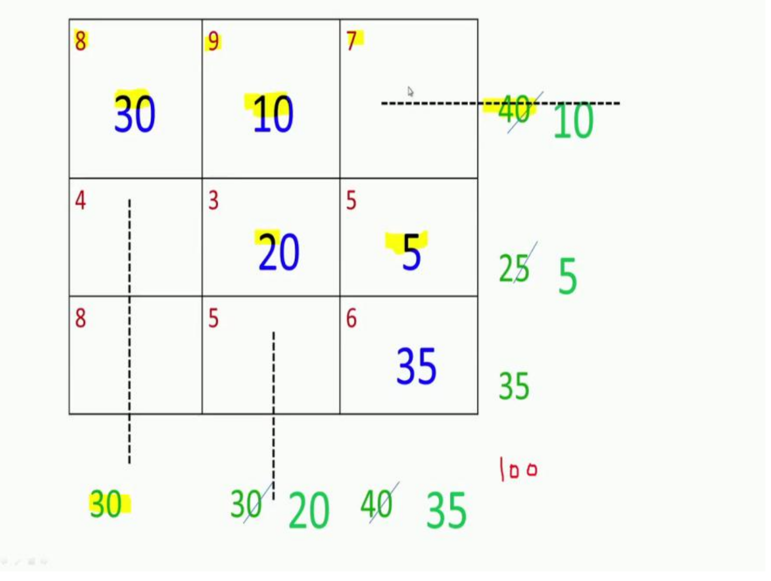

So, this table has 9 positions corresponding to the 9 variables that are there in the problem. So, this corresponds to the variable X 1 1 this is like a 3 by 3 grid, so this corresponds to X 1 1, X 1 2, X 1 3, X 2 1, 2 2, 2 3, 3 1, 3 2 and 3 3. The costs of transportation are shown here in the corner of these positions.

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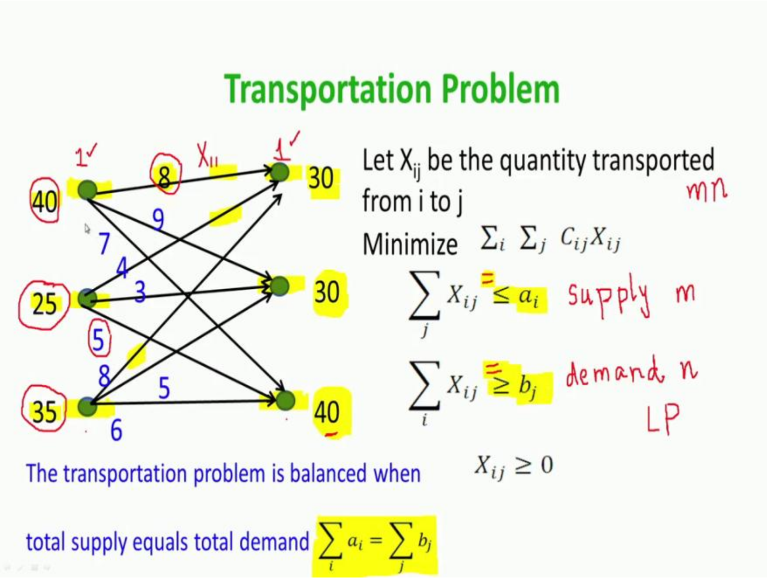


For example, if you look at the previous from the first supply point the three costs are 8, 9 and 7. So, the costs are 8, 9, 7, 4, 3, 5, 8, 5, 6. So, 8, 9, 7, 4, 3, 5, 8, 5, 6 the three supply quantities are 40, 25 and 35, the three demand quantities are 30, 30 and 40 both of them add up to 100, so the problem is balanced with the total equal to 100. So, a balanced transportation problem is often represented in the form of a table that we have shown here with the supply quantity is coming on the right hand side, the demand quantity is coming on the bottom and the costs are shown in the corners of the corresponding positions 8, 9, 7 and so on.

Now, having represented the problem this way, let us try to solve this problem. So, solving the transportation problem is finding the values of the nine variables in this problem, these nine variables are X 1 1, X 1 2, X 1 3, X 2 1, X 2 2, X 2 3 and so on. So, now, when we look at this we observe that the first supply point has 40, second has 25, the third has 35 and the three demands are 30, 30 and 40. So, we can start with any one of these nine positions and start making allocations.

Now, the most comfortable one perhaps is to start with the top left hand which comes naturally. So, we start with the top left hand position and see how much we give to this X 1 1 variable. So, when we look at this position which is the top left hand position, when we look at this position which represents the first. How much I am transporting from the first supply point to the first demand point? Now, the first supply point has 40 items available and the first demand requirement is 30.

So, the maximum we can transport from the first supply point to the first demand point is the minimum of these two numbers which is 30. So, what we do is, we start with this and then we allocate this 30 which is the minimum of the available quantity 40 and the required quantity 30. So, we start with the top left hand position and we try to allocate whatever maximum possible which is the minimum of what is available and what is demanded. So, 30 we will allocate to the variable X 1 1 or to the first position which also means that X 1 1 is equal to 30 in the solution.

Now, once we have allocated this 30, we realize that the entire demand of the first demand point has been met. So, that is shown by this line that we have drawn which also shows now that we are not going to allocate anything here or we are not going to allocate anything here. Because, allocating in this position and this position would increase here and all the 30 has already been met, and therefore we will not put anything here.

So, X 1 1 is equal to 30, X 2 1 will become 0, X 3 1 will also become 0, now when we have allocated 30 out of the 40 available here. So, 30 has been consumed and the 40 will now become 10 which is the remaining quantity available for allocation. Now, having does this, we now move to the next available top left hand corner which is this position, which is the position that is highlighted and shown here, this is the next top left hand corner.

Now, which is which corresponds to the variable X 1 2, now if we go back and see 10 units are available in the first supply point and 30 units are required in the second demand point. So, the maximum that we can give is the minimum of this 10 and 30 which happens to be 10. So, we try to give the maximum which is the minimum of these two quantities and we allocate a 10 here. So, X 1 2 takes a value 10, now with this 10 allocation the entire supply from the first supply point has been consumed, and therefore we will not allocate anything to this position or to this variable X 1 3.

Now, the fact that this has been consumed is now shown by another line which in this case is a horizontal line, which says that this 10 has been completely consumed. Now, by allocating 10 to this position, we have now met 10 out of the 30 requirement, and therefore the remaining requirement that has to be met is only 20 which is shown here. So, we have now made two allocations and we have, at the end of these two allocations we observe that we have completely consumed the 40 supply.

We have completely met the demand of 30 of the first demand point and we have to meet 20 more of the demand of the second demand point. Now, again we look at the top left hand position and now the top left hand position shifts to this position, the top left hand position is this position, because this is the remaining portion of the table at this point, so this is the top left hand position. So, now, we look at this position and then we observe that if we want to make an allocation, the total available from the second supply point is 25, the requirement that has to be met is 20.

Therefore, we try to meet the maximum possible which is 20, which is the minimum of this 20 and this 25, so we allocate 20 to this position. Now, with the allocation of 20 this entire demand has been met, and therefore we will not put anything to this position, X 2 3 will become 0. Now, this entire 20 has been met, and therefore we draw another vertical line to say that this entire 20 has been met. Now, this 20 which we have now allocated has been taken out of the 25 that is available, and therefore since 20 has been consumed, this 25 becomes 5 which is what is remaining, so that is shown here.

Now, again we look at the top left hand corner, this is the area that is remaining, this is the area that is remaining and the top left hand corner is actually this position. Now, once we identify this position, then we realize that 5 is available and 40 is required. We try to put as much as we can which is the minimum between 5 and 40, and therefore we would allocate 5 to this position. So, when we allocated 5 to this position, this 5 has been met entirely, and then out of this 40 5 has been met, so the balance remaining is 35.

Now, we look at the top left hand position of the available thing, so this is the only position that is remaining the availability is 35, the requirement is 35, so we allocate 35 to this. Now, the entire allocations are made, now there is no supply that is available, there is no demand that now has to be met. So, the demands of all the three points have been met, the supplies of all the three points have been consumed.

We can verify that 30 plus 10 is the original 40 that was available, 20 plus 5 is the originally 25 that was available and 35 was originally available. 30 was required 10 plus 20 30 was required, 5 plus 35 40 was required. So, all the supplies have been consumed, all the requirements have been met. We also observed that when we made the last allocation which happened to be here at 35, we realize that the supply was 35 and the requirement was also 35 that happened, because the problem is balanced, a total of 100 is available and a total of 100 is required.

So, now, this gives us an allocation, an allocation and gives us a solution which can be implemented and which is feasible. At the moment we are not in a position to say whether this is the best solution or whether there is another solution better than this. But, at least at this point through a very simple procedure or a simple algorithm we are able to get a solution which can be implemented.

So, a solution which can be implemented or a solution that is feasible is a solution that has greater than or equal to 0 allocations for all the 9 variables 5 out of the 9 variables have positive values, the remaining 4 which is X 1 3, X 2 1, X 3 1 and X 3 2 have 0 allocations. The row sum of the allocations in the row is equal to the supply and sum in the column is equal to the demand represents a feasible solution.

Now, having obtained a feasible solution, the next thing that we ask is how good is this feasible solution. Now, to understand the goodness of this feasible solution we now try to find out the cost associated with this feasible solution.

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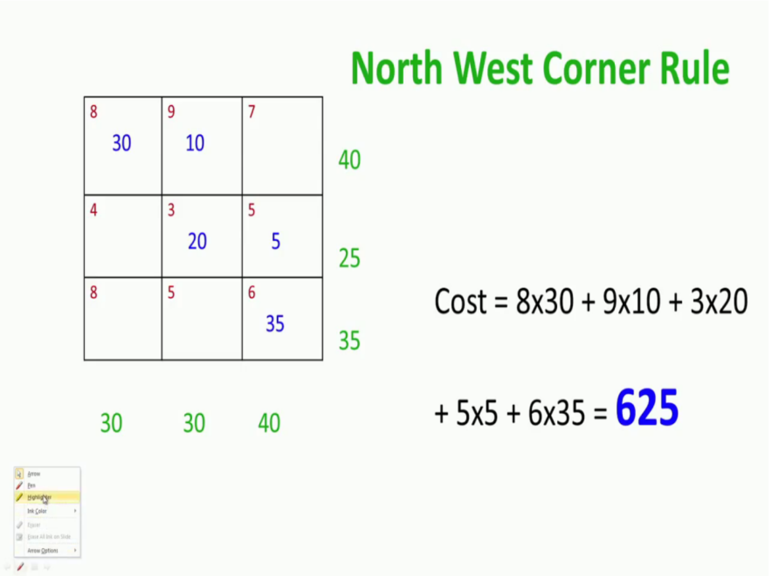


So, the cost associated with this feasible solution I have shown the same solution here, the cost associated is 30 into 8 plus 9 into 10 plus 3 into 20 plus 5 into 5 plus 6 into 35 which happens to be 625. So, 625 is the cost associated with this feasible solution, now at the moment we do not know whether 625 is the best or is there something better than 625. But, nevertheless we have a solution which is going to cost us 625 and if we implement this solution we can meet all the requirements or demands in the three demand positions.

Now, this method which gives us a solution now we are going to call this solution as a starting solution. So, this method which gives us a solution is called the North West Corner Rule as indicated here, it is called the North West Corner Rule. Because, we have consistently used the top left hand corner also called the North West Corner to make our next allocation. The principle in the North West Corner Rule is go to the north west corner and try to allocate as much as you can try to allocate as much as you can is an important rule.

So, allocating as much as we can is allocating the minimum between or minimum between the available supply for that position and the required demand for that position. So, north west corner rule is a very simple procedure, you start with the top left hand corner at any point out of the available allocations go to the top left hand corner and try to put as much as you can in that and you will finally, get a solution and evaluate the cost of it and in this example the cost is 625, now let us look at one more method which tries to do this.

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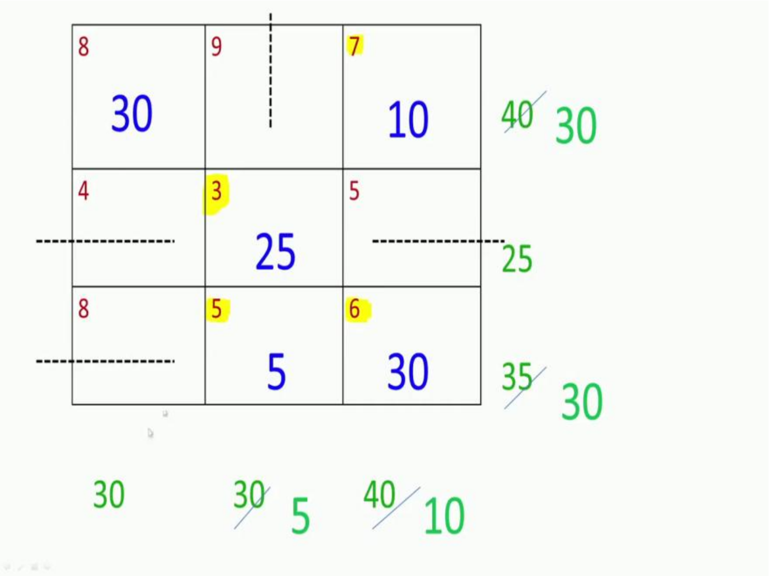


Now, the method that we are going to do is another one, so we go back to the same table, now we know how to understand this table 40, 25, 35 are available 30, 30, 40 are required. Now, instead of choosing the North West corner can we do something else, the obvious thing to do is to choose that position which has the least unit cost of transportation. Now, the least unit cost of transportation is here which is this, so we first choose this position, where the unit cost of transportation is 3.

So, 25 supply units are available as supply, 30 units are required as demand; therefore the maximum that we can put to this position is the minimum between 25 and 30, so we put 25 to this position. Now, the moment we put 25 to this position we realize that all the supply has been exhausted, therefore we would not allocate anything in this position and we would not allocate anything in this position. So, that is shown as two lines where we do not allocate to the other two positions. Now, 25 out of the 30 has been met, and therefore only remaining 5 has to be met.

Now, we look at the remaining portion of this matrix and try to find out that position which has the least unit cost of transportation. Now, the available positions have costs 8, 9, 7 and 8, 5 and 6; therefore, the 5 is the minimum at this point you may observe that there is a 4 here, but we cannot use this, because of this dash, because of the fact that this 25 has been consumed, and therefore we cannot make any allocation in this position.

So, we look at the minimum unit transportation cost from among the available positions for allocation. So, we take this 5 now the requirement is only 5, the supply is 35, so try to put as maximum as you can which is the minimum of 5 and 35. So, we will end up putting 5 here which is shown, now by allocating this 5 we have completely met the demand of the second demand point, and therefore we show this through this dotted line and we say that we will not have any allocation in this position.

Now, by consuming this 5 out of this 35, 35 has now become 30 only 30 units are now available. Now, we repeat this procedure to find out the minimum unit cost and then there are only four positions now with 8, 7, 8 and 6. So, this 6 becomes the one with the minimum unit cost. Now, 30 is available 40 is required we can now put 30 in this position which is the minimum of these two, so we put this 30 here.

Now, by putting all this 30 we have exhausted or consumed all the supply that is available here, which is shown again by this dotted line and it also means that we will not have any allocation in this position. Now, by putting 30 here we have met 30 out of the 40 demand and only 10 more demand has to be met. Now, we go back to the next smallest cost position which happens to be here between 8 and 7, it is 7 and now 40 is available 10 is required, so we will put 10 in that position.

Now, by putting this 10 again this is entirely met and this 40 has now become 30 which is the thing that is available. Now, there is only one position which is this position with cost equal to 8, where the supply is 30 the demand is 30 and we put all 30 here. As we saw in the North West Corner, the last allocation we will observe that the supply is equal to the demand.

So, now we have another solution which is feasible, the way I defined the feasible solution, a feasible solution is 1 where are all the nine variables have greater than or equal to values, 5 out of these 9 have positive value, the remaining 4 have 0 value the row sum is equal to the supply 40, 25, 35, the column sum is equal to the demand 30, 25 plus 5 10 plus 30 equal to 40, so this is another feasible solution.

Now, what is the cost associated with this, the cost associated with this is computed I have shown the same solution again. So, the cost is 8 into 30 plus 7 into 10 plus 3 into 25 plus 5 into 5 plus 6 into 30 C i j into X i j which becomes 590. So, we have this solution which has total cost of 590, now this method is called the minimum cost method or least cost method, it is another method to give us a starting solution, we first saw the north west corner rule, now we have seen the minimum cost method, so let us make a quick analysis of which one is better.

So, the minimum cost gave us a solution with 590 and the North West Corner gave us a solution with 625. Because, it is a minimization problem, the solution with the lower cost is a better solution, and therefore for this particular example we can say that the minimum cost has given us a better solution than the North West Corner. It also now tells us that the North West Corner solution is not optimal, because we were able to find a solution with the lesser cost than that.

Now, is the minimum cost method solution optimal or are there methods such as these which are reasonably simple, reasonably intuitive and reasonably simple and gives good solutions, minimum cost method did that. So, are there other methods which can give good solutions and starting solutions, we will look at one more such method in the next class.