Introduction to Operations Research
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Module - 06
Lecture - 26
Introducing the transportation problem

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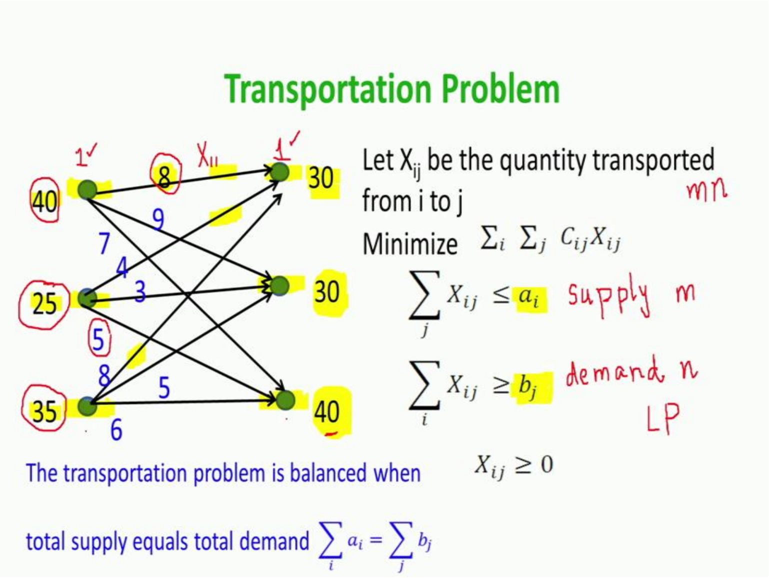

In this module we study the transportation problem the transportation problem is a very important topic in linear programming and in operations research. Let me explain the transportation problem a network is shown here which has some nodes or vertices which are connected by arcs. Now, the transportation problem talks about transporting a commodity or a single item from a set of supply points or points where the item is available to destination points or demand points where the item is required. So, this network shows a transportation problem where the item is available in three supply points which are shown here. It is available in three supply points this is the supply point one, this is the supply point two, this is the supply point three. So, it is available in three places the quantities available are given by these numbers 40, 25 and 35. Units of this item are available in these three supply points.

Now, the other three points are the requirement points or the demand points. Now, these three are the demand points and the requirement is 30 another 30 and 40 in these three places. Now, the problem is to transport the available items from the places where they are available to the places where they are required, obviously there is going to be a cost of transportation. Now, this cost of transportation is given as a unit cost for example, this 8 is the cost of transporting one unit from supply point one to the demand point one. We can call this as supply point one this is demand point one. So, this 8 is the cost of transporting a unit from supply point one to demand point one. Similarly, this 5 can be taken as the cost of transporting one unit from this supply point to this demand point. So, since there are three supply points and three demand points there are nine ways of transporting them and you are able to see nine costs that are written here. Unless otherwise stated the transportation problem assumes that it is possible to transport from every supply point to every demand point or destination point directly.

You will also observe that in a transportation problem while you can supply directly from supply point one to demand point one. You do not transport them from one supply point to another or from one demand point to another the transportation problem does not permit us to model such a situation. So, transportation is permitted from supply points to demand points directly. So, there are set supply points in this example there are three supply points and there are three demand points. We should also note that it is not necessary that the number of supply points should be equal to the number of demand points. We could have another transportation problem where there could be five supply points and eight demand points unequal number of supply points and demand points. We could even have a situation where there are eight supply points and six demand points and so, on. Now, the problem is how do I transport this 40, 25 and 35 to meet the demand of 30, 30 and 40 such that the total cost of transportation is minimized this is called the transportation problem. Now, let us understand the transportation problem further and let us try to formulate this as O R problem.

So, let X i j be the quantity transported from i to j. Now, in this notation ‘i’ represents the set of supply points and j represents the set of demand points or destination points. It’s customary to represent the supply points on the left hand side and the demand points on the right hand side and an arrow indicates that there is transportation from a supply point to a demand point. So, let X i j be the quantity transported from supply point i to demand j. Now, obviously we want to minimize the total cost of transportation. So, the objective function will be to minimize summation over i summation over j c i j x i j. Now, since there are three supply points i is equal to 1, 2, 3 since there are three demand points of destination points j is equal to 1, 2, 3. So, there are 9 x i j variables there are also 9 c i j. Or cost, unit cost coefficients. The unit cost coefficients would be 8, 9, 7, 4, 3, 5 and so, on. So, if x i j is transported from this transported from this one supply point to this each unit transportation is going to cost us 8t therefore, the total cost will be 8 into x 1 1. Since, this transportation can happen in nine ways.

So, the objective function will be expanded to minimizing 8 x 1 1 plus 9 x 1 2 plus 7 x 1 3 plus 4 x 2 1 plus 3 x 2 2 plus 5 x 2 3 plus 8 x 3 1 plus 5 x 3 2 plus 6 x 3 3. So, there are nine terms and these two summation signs i equal to 1, 2, 3 j equal to 1, 2, 3 c i j x i j will give us these nine terms and that summation is to be minimized now what are the constraints. The constraints to this problem will be sigma x i j summed over j is less than or equal to a i. Now, i equal to 1, 2, 3. So, let me explain this constraint now a i are the supply quantities that are available. So, a I are the supply quantities that are available. So, a 1 is this 40 a 2 is this 25 and a 3 is this 35. So, what does this constraint tell us? Summed over j x i j is less than or equal to a i.

Suppose I take the first supply point which has 40. Now, from this supply point I am transporting something to this I may transport, I may not transport, but whatever is available in this 40 only can be transported. So, if x 1 1 is going here from this point. From 1 to 1 x, 1, 2 is going here and x 1, 3 is going here then x 1 1 plus x 1, 2 plus x 1, 3 is the total quantity that is going out of the supply point one and that should be less than or equal to the available quantity which is 40.

So, there are three constraints here one for each supply point for the first point the constraint will be x 1 1 plus x 1 2 plus x 1 3 is less than or equal to 40 for the second supply point which has a 2 equal to 25 the constraint will be x 2 1 plus x 2 2 plus x 2 3 is less than or equal to 25 for the third point it will be x 3 1 plus x 3 2 plus x 3 3 is less than or equal to 35. So, there are three constraints here which are represented in the form of a single expression and there are three constraints for each of the supply points. Now, continuing we have the second set of constraints which says sigma x i j summed over i is greater than or equal to b j. Now, b j represents the demand in these demand points. So, b j represents the demand there are demand points which are here the values are 30, 30 and 40. Now, if you take the first demand point with 30. Now, the demand of 30 has to be met by sending things from these 40, 25 and 35. So, x 1 1is sent from this 40 x 2 1 is sent from the second supply point x 3 1 is sent from the third supply point. So, for this x 1 1 plus x 2 1 plus x 3 1 here is x 1 1. This would be our x 2 1 and this is x 3 1 in this arc.

So, x 1 1 plus x 2 1 plus x 3 1 comes to the first demand point and that has to meet the demand therefore, it has to be greater than or equal to b 1 which is equal to 30. So, again there are three constraints one for each demand point the first demand point constraint will be x 1 1 plus x 2 1 plus x 3 1 greater than or equal to 30. So, a summation i is equal to 1 to 3 the second one will be x 1 2 plus x 2 2 plus x 3 2 is greater than or equal to this 30 which is the demand in the second demand point and the third constraint will be x 1 3 plus x 2 3 plus x 3 3 is greater than or equal to 40. So, there are 3 demand constraints and then we have the non-negativity restriction x i j greater than or equal to 0 at this point we may ask a question should we transport integer quantities. Or can the quantities be continuous variable and we are now going to assume that the x i j's which are the quantities transported are continuous variables to begin with. So, this is the formulation of the transportation problem.

So, if I have m supply points customary to use this if there are m supply points. If there are n demand or destination points then the number of variables x i j is equal to m n. So, number of variables is m into n in our example we have taken m is equal to n both are equal to 3. So, we had nine variables otherwise it is m n and then there are m supply points. So, there will be m constraints here one for each supply point there are n demand points there will be n constraints here. So, it will have m into n m n variables and m plus n constraints and a non negativity restriction. Now, the important thing that we need to look at is the transportation problem is balanced when total supply equals total demand. So, what do we convey through this now there is a demand of 30, 30 and 40 hundred units there is a demand. Now, we are going to assume that we have enough quantities in the supply points. So, as to meet this demand of hundred so, obviously, this problem when we solve. If we should meet the demands of all the destination points and we should meet all these supplies or total availability should be greater than or equal to the total requirement.

Only then we will be able to send sufficient quantities to meet all the requirements let me come back to this formulation again and the formulation talks about minimizing sigma c i j x i j double sigma c i j x i j subject to summed over j x i j is less than equal to a i summed over i x i j is greater than or equal to b j x i j greater than or equal to 0. Now, this formulation has a linear objective function in terms of the variables x i j constraints are linear inequalities and there is a non negativity restriction therefore, this problem is a linear programming problem. So, the transportation problem is a linear programming problem. That is the first thing that we observed is that the transportation problem is a linear programming problem. Now, let us come back now for this transportation problem to have a solution which means for us to meet all the demands and satisfy all the demand constraints the total supply should be greater than or equal to the total demand. So, for this problem to have a feasible solution in the first place then sigma a i should be greater than or equal to sigma b j total supply is greater than or equal to total demand. A further look into the problem will also tell us that if the total supply is greater than the total demand because the total demand is 100 or in any quantity sigma b j.

We will not transport more than what is required for every demand point because transporting an additional quantity over and above the demand is only going to increase the transportation costs. This is under the assumption that all the unit transportation costs are non-negative or positive therefore, even if the total supply is greater than or equal to the total demand we will not supply more than the demand for an individual demand point and therefore, the problem becomes nice when we have sigma a i equal to sigma b j which means when the total supply is equal to the total demand we will have a feasible solution. We will be able to meet the demand and the problem will not be different if the individual supplies are actually greater than the total demand. So, even if the total supply is greater than or equal to total demand it is enough to have a problem where the total supply is equal to the total demand to solve and get a solution to this. So**,** when the total supply equals total demand which is shown here this problem is called a balanced transportation problem. So, sigma a i is the total supply sigma b j is the total demand. So, this problem is called a balanced transportation problem. Now, first let us check whether our problem is balanced the total supply is 40 plus 25, 65 plus 35 100 30 plus 30, 60 plus 40 100. So, sigma a i is equal to sigma b j. So, our problem that we are looking at as a numerical example is a balanced transportation problem, but we have also seen that this transportation problem is a linear programming problem. So, if the transportation problem is a linear programming problem involving m into n variables and m plus n constraints then we could solve the linear programming problem directly, but before that let us also understand what is the implication when total supply is equal to the total demand.

Finally, we will have a situation where we use up all the supplies that are available and we also met the demands exactly because total supply is equal to the total demand. So, when the total supply is equal to the total demand the balanced transportation problem can be written by replacing these inequalities with the equation that is written here so replacing it with the equation. So, the constraints become sigma x i j is equal to a i and sigma x i j summed over i is equal to b j. So, the linear programming problem for a balanced transportation problem will have equations instead of inequalities it is a linear programming problem and we could think of solving this problem as a linear programming problem using the simplex algorithm and so, on. But if we look at this example that we are talking we will have 9 variables and 6 constraints and it would become difficult for us to solve it using the simplex table of course if we are using solvers and other ways to solve it is a little easier to do that, but then over a period of time people who have studied the transportation problem, also understood that there are good efficient and faster ways to solve the transportation problem than solving it as a linear programming problem.

Now, the transportation problem therefore, has special methods and techniques which are used to solve the transportation problem in a faster, quicker manner compared to solving it using the simplex table even though it can be solved using the simplex table. Now, these faster and quicker methods also use ideas from linear programming and duality that we have seen earlier in this course.

In the next class, we will look at what are these special methods which can solve the transportation problem nicely and solve it fast faster than perhaps solving it using the simplex table.