Introduction to Operations Research
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Module – 01
Linear Programming – Introduction and formulations
Lecture - 02
Manpower and Production planning formulations

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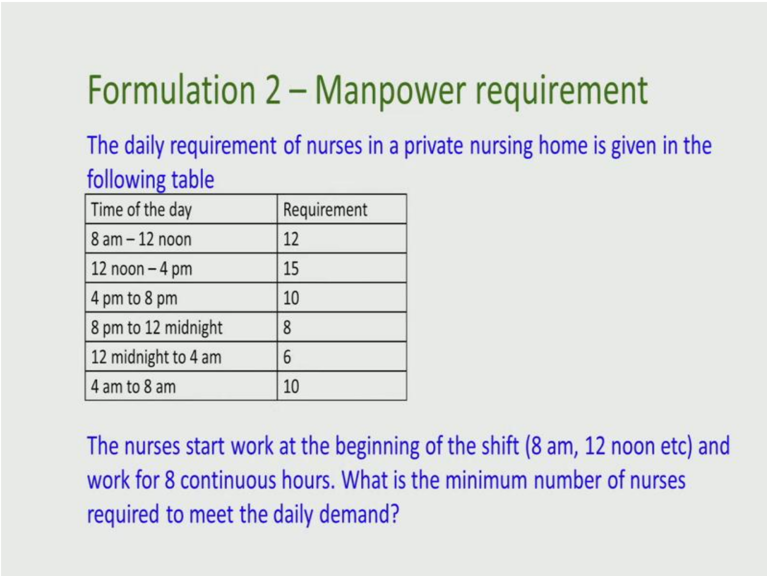

In this class, we look at some more aspects of formulating linear programming problems. We now look at the formulation of manpower requirement problem. The problem is as follows, the daily requirement of nurses in a private nursing home is given in the following table. Now, this is given as 6 time slots in a day each time slot being for 4 hours. For example, 8 am to 12 noon we require 12 people, 12 noon to 4 pm we require 15 people and so, on. The nurses who come to work can start work at the beginning of any of these 6 slots which means, they can start working at 8 am or 12 noon or 4 pm or 8 pm 12 midnight and 4 am. They can start working on any of these hours and they work for 8 continuous hours.

What is the minimum number of people required to meet the daily demand? We also make an assumption that a person who starts working at 8 am works in the slot 8 am to 12 noon as well as in the slot well noon to 4 pm. Again, a person who starts working at 8 pm would work in the 8 pm to 12 midnight slot as well as in the 12 midnight to 4 am slot that, is how the 8 consecutive hours are coming. So, the problem is how many people start working at the beginning of these 6 time slots, such that we have minimum total number of people working and we are able to meet the requirement or demand of each of these time slots. Since there are 6 times slots we have 6 variables and we define this variable as x 1 to x 6.

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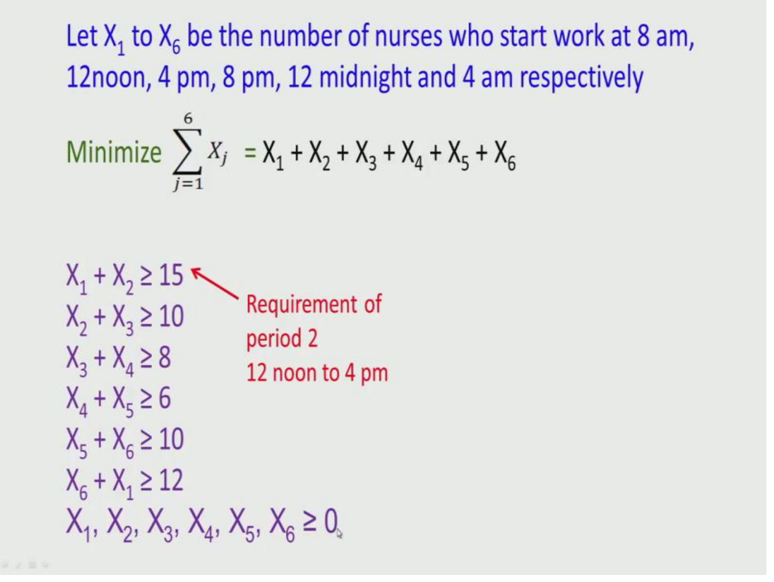

So, x 1 to x 6 let it be the number of nurses who start work at the beginning of the 6 slots which is 8 am, 12 noon and so on. Now, the objective function will be to minimize the total number of people who are working. And therefore, it is to minimize x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6. Now, that is shown here as the objective function here x 1 to x 6. It is also a shown in a form of a summation where this sigma j equal to 1 to 6 x j when expanded will give x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6. So, the objective function will minimize the number of people who are working in who come to work in the beginning of these 6 slots. Now, the constraints are to meet the requirement of these 6 slots. For example, x 1 plus x 2 is greater than or equal to 15.

Now, x 1 plus x 2 the number of people who actually work in the slot 12 noon to 4 pm. If we see 12 noon to 4 pm 15 people are required, but we also observed that people who start working at 8 am work for 8 hours and there 8 hours includes 12 noon to 4 pm. People who start working at 12 noon also work for 8 consecutive hours, 4 hours between 12 noon and 4 pm and another 4 hours between 4 pm and 8 pm. Therefore, the number of people who are working in this slot which is between 12 noon and 4 pm are those who start working at 8 am and those who start working at 12 noon.

So, x 1 plus x 2 are the number of people who actually work between 12 noon and 4 pm and therefore, x 1 plus x 2 should be greater than or equal to 15 and we should be able to meet the requirement of period 2 which is given by 15 as the requirement. So, similarly x 2 plus x 3 the number of people who actually work between 4 pm to 8 pm that should be greater than or equal to 10, x 3 plus x 4 should be greater than or equal to 8, x 4 plus x 5 is greater than or equal to 6, x 5 plus x 6 is greater than or equal to 10 and x 1 plus x 6 is greater than or equal to 12.

Because to meet the 8 am to 12 noon those who come to work from 4 am to 8 am will be working in this slot people who come to work at 8 am also work in this slot. So, this 12 requirement is to be met with x 1 people who start work at 8 am and x 6 people who start work at 4 am. So, we have these 6 constraints which represent the requirement of the 6 periods to be met. What is important is to understand that both x 1 and x 2 are the number of people who are working in the second period, x 2 and x 3 working in third period and like that x 6 and x 1 work in the first period. Now, all these variables x 1, x 2, x 3, x 4, x 5 and x 6 have to be greater than or equal to 0. Of course, one may ask that this x 1 to x 6 representing the number of nurses should also take only integer values. So, is it necessary to introduce these variables also as integer variables.

Now, at the moment we choose not to represent them as integer variables. We are interested in formulating linear programming problems. Therefore, we expect and define the variables to take continuous values. If we put explicit integer restriction on the variables then the formulation becomes linear integer programming. But at the moment since we are doing only linear programming we are defining these variables to be continuous variables. So, this simple formulation, have taught us a few things.

In this formulation, the objective function is a minimization function unlike the maximization function in the last example that was done in the last class. Here, the 6 constraints are greater than or equal to inequalities unlike less than or equal to inequalities which we saw in the previous example. So, based on the two examples we will be able to say that the objective function is either a maximization function or minimization function. The constraints are either less than or equal to type inequalities or greater than or equal to type inequalities they can also be equations there is a non-negative restriction on the variables.

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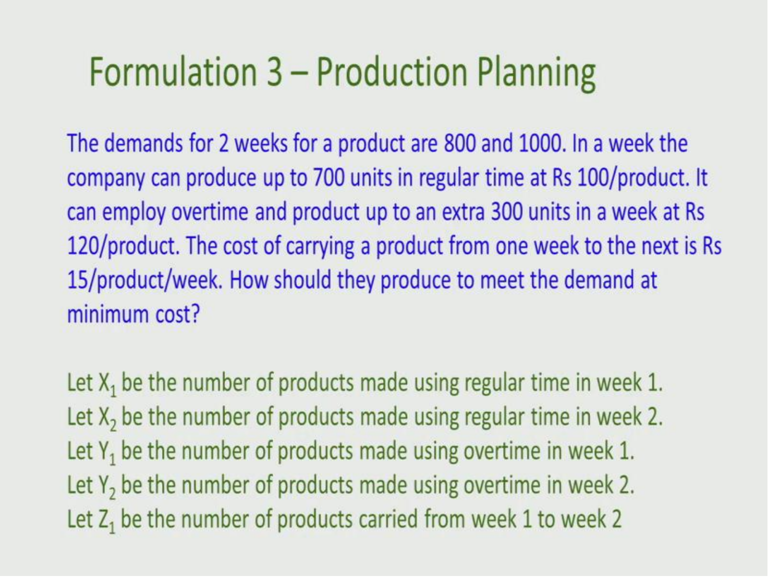

Now, we look at the third formulation where we look at production planning situation. We consider a single product and a company making this product the demand for 2 weeks for this product are 800 and 1000 respectively. In a week, this company can produce up to 700 units using their regular time production and it cost rupees 100 for product made or per unit made. The company can also employee overtime and produce up to an extra 300 units in a week. Now, the cost of making that unit through over time is 120 per product. The company can also produce a little more in the first month if it is possible and use the excess from one week to another and the cost of carrying this excess inventory from one week to another is rupees 15 per product per week. How should they produce to make the demand of the 2 weeks at minimum cost?

Now, we introduce 5 variables x 1 be the number of products made using regular time in week 1, x 2 be the number of products made using regular time in week 2. Week is the unit of time that we are looking at in this example y 1 is the number of products made using overtime in week 1 and y 2 be the number of products made using overtime in week 2. Now, z 1 is introduced as a number of products carried from week 1 to week 2 after meeting week ones demand so, there are 5 variables in this formulation. The objective function is to minimize the total cost 100 x 1 is the cost of producing x 1 units in week 1 using regular time, 100 x 2 is the cost of producing x 2 in week 2 using regular time. Y 1 is the quantity produced in week 1 using overtime and therefore, cost 120 y 1 similarly 120 y 2 is the cost of producing in week 2 using overtime and 15 z 1 is the cost of carrying the z 1 items from week 1 to week 2. There are 5 terms 3 distinct costs regular time cost for 2 weeks, overtime cost for 2 weeks and cost of carrying the excess inventory from week 1 to week 2.

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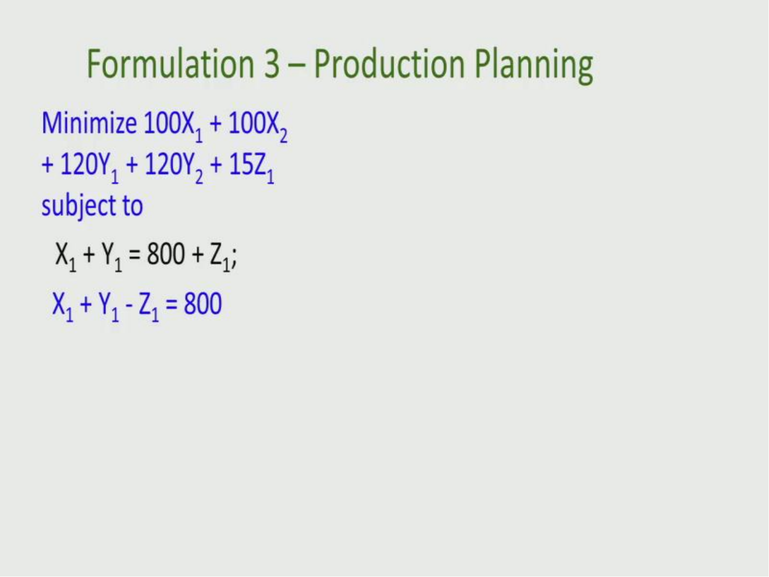

Now, we look at the constraints the first constraint is to satisfy the demand of the first week. So, this is written as x 1 plus y 1 equals 800 plus z 1, 800 is a demand for the week which is to be satisfied. If we produce more than 800 the quantity z 1 is carried to the next week. So, whatever is produced in the first week should meet the demand of the week and the excess that is carried to the second week therefore, x 1 plus y 1 is equal to 800 plus z 1. Now, this is rewritten as x 1 plus y 1 minus z 1 equals 800. It is customary to write the constant in the right hand side of the equation or the inequality on the left hand side of the equation of inequality contains the variable terms. Now, this particular constraint is an equation. So, the actual constraint written in a standard form is shown in different color here.

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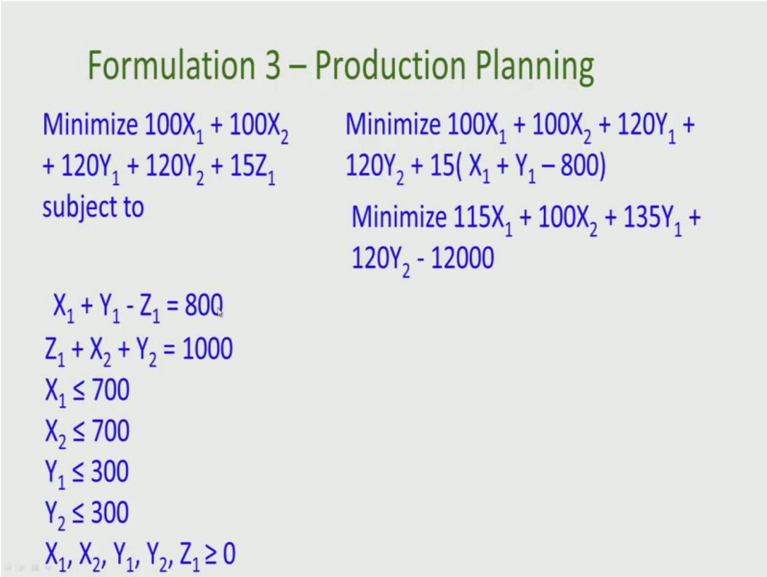

Now, the constraint for the demand of the second month or second week is x 2 plus y 2 represent the quantity produced using regular time and overtime in week 2 z 1 is the quantity carried from week 1 to week 2. Therefore, what is available to meet the demand of week 2 or the 2 production quantities x 2 plus y 2 and the excess inventory that is carried which is z 1. So, z 1 plus x 2 plus y 2 equal to 1000 is the constraint to meet the demand of the second week. We also have capacity restrictions where x 1 is restricted to 700 less than or equal to 700 x 2 is less than or equal to 700 represents the capacity regular time capacity in the 2 weeks.

Y 1 less than or equal to 300 y 2 less than or equal to 300 represents the overtime capacity in the 2 weeks. So, there are 6 constraints 2 constraints to meet the demand of the 2 weeks which are a expressed as equations the remaining 4 constraints are inequalities less than or equal to inequalities, because they represent the limit on regular time production and overtime production in the 2 weeks. There is an explicit non-negativity restriction given by x 1 x 2 y 1 y 2 and z 1 greater than or equal to 0. So, this completes the formulation of this problem.

Now let us look at the same formulation in a slightly different way. Now the objective of function is once again written in a slightly different way. It is now written as 100 x 1 plus x 100 x 2 plus 120 y 1 plus 1 20 y 2 plus 15 into x 1 plus y 1 minus 800. Now, z 1 has been left out and z 1 has been replaced as x 1 plus y 1 minus 800 from this equation. So, I am leaving out or eliminating the variable z 1 and writing it as 15 into x 1 plus y 1 minus 800.

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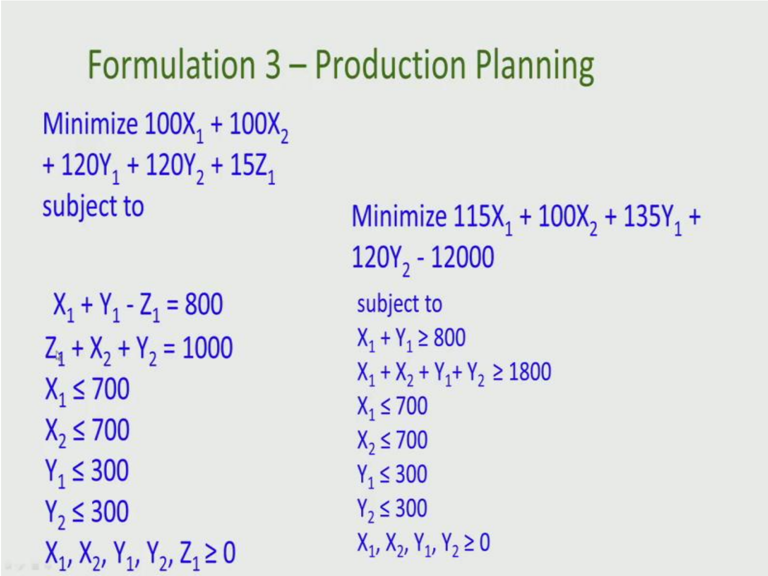

Now, this is obtained from this equation. So, this is rewritten on simplification as 115 x 1 plus 100 x 2 plus 135 y 1 plus 120 y 2 minus 12000. Rest of the constrains are x 1 plus y 1 is greater than or equal to 800 what is produced in the first week is either to 800 or more, if it is more the excess carried to the second week. So, what is carried to the second week its x 1 plus y 1 minus 800. So, this z 1 is now written as x 1 plus y 1 minus 800 and on simplification will give x 1 plus x 2 plus y 1 plus y 2 is greater than or equal to 1800 because the z 1 is x 1 plus y 1 minus 800. The minus 800 goes to the other side of the equation to give us x 1 plus x 2 plus y 1 plus y 2 is greater than or equal to 1800.

Now 4 capacity constraints remain the same, now there are only 4 variables because the variable z 1 has been left out. So, all the 4 variables are greater than or equal to 0. Now, if we compare these two formulation we may also wish to say that x 1 plus x 2 y 1 plus y 2 could be equal to 1800 or it could be greater than or equal to 1800. For example, if we substitute z 1 as x 1 plus y 1 minus 800 into the second equation here, the second equation would become an equation instead of inequality.

However, since the first constraint is returned as an inequality. We simply maintain the consistency and write this as greater than or equal to 1800 because x 1 plus x 2 y 1 plus y 2 would represent the total production in the 2 weeks. This represents the total demand on the 2 weeks and because there is no third week in this particular example we will not end up making more than 1800, if we look at the production of the 2 weeks taken together. So, it is alright if we write a greater than or equal to inequality here instead of an equation.

So, the new formulation now has eliminated 1 variable and has 4 variables which represent the 2 variables which are regular time production quantities in the 2 weeks, an another 2 variables which are over time production quantities in the 2 weeks. Now, there is also a constant term in the objective function and when we actually solve this we can leave out the constant both from the formulation and to begin with, in the solution and can add that. So, this will be rewritten as minimize 115 x 1 plus 100 x 2 plus 100 and 351 plus 20 y 2 and we can leave out the minus 12000 and add it as an when required.

There are again 6 constraints 2 for the demand constraints and 4 for the capacity constraints now, when this formulation, the 2 demand constraints are greater than or equal to constraints, and the 4 capacity constraints are less than or equal to constraint. So, in second formulation we have 1 variable less and the constraints are all inequalities instead of equation. In the next class, we will see some more examples in formulating linear programming problem.