Introduction to Operations Research
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Module - 03
Simplex Algorithm
Lecture - 02
Tabular Form of Simplex (Maximization)

In this class, we introduce the simplex algorithm in the form of a table and it is called the Tabular Form of the Simplex Algorithm. What we saw in the previous class is called the algebraic form of the simplex algorithm. Essentially, the tabular form and the algebraic form are the same, except that in the algebraic form we write the equations explicitly as we did, we will observe that in the tabular form we represent all these in the form of a table.

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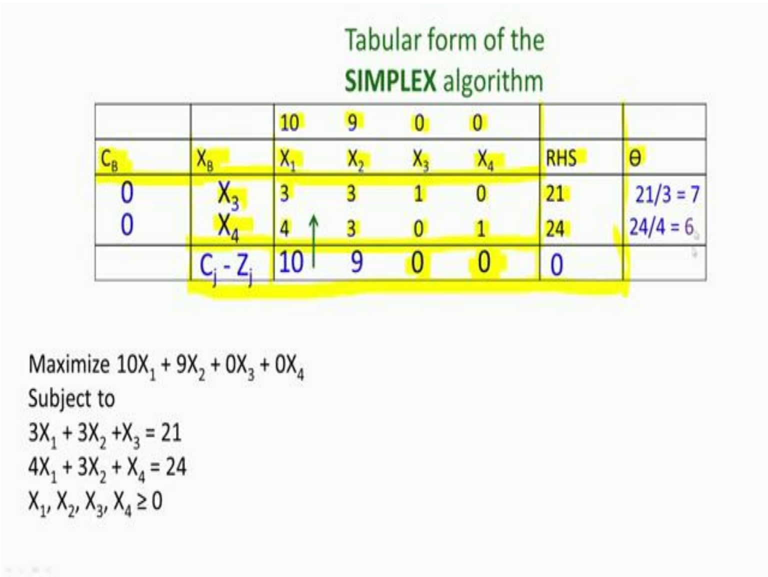

So, to explain the tabular form we consider the same example which after the addition of the slack variables is to maximize 10 X 1 plus 9 X 2 plus 0 X 3 plus 0 X 4 subject to 3 X 1 plus 3 X 2 plus X 3 equals 21, 4 X 1 plus 3 X 2 plus X 4 equals 24 all the four variables are greater than or equal to 0. So, we first set up what is called the simplex table, so let me explain this table. So, first we create a table, where we write the problem has four variables X 1, X 2, X 3 and X 4. So, I write the variables X 1, X 2, X 3 and X 4.

I also write the objective function value of these variables 10, 9, 0 and 0 above these variables 10, 9, 0 and 0 above the variables. So, I first start with writing X 1, X 2, X 3, X 4 and then I write 10, 9, 0 and 0 above the four variables. I also write this thing called RHS or Right Hand Side here. Now, my first equation is 3 X 1 plus 3 X 2 plus X 3 plus 0 X 4 is equal to 21. So, that is written as 3 X 1 under X 1, 3 X 2 3 under X 2, 1 X 3 1 under X 3, 0 X 4 0 under X 4 and equal to 21 which is written under the right hand side.

So, this equation can be read as 3 X 1 plus 3 X 2 plus 1 X 3 plus 0 X 4 is equal to 21, the second equation is 4 X 1 plus 3 X 2 plus 0 X 3 plus 1 X 4 is equal to 24. So, this is written as 4 X 1 plus 3 X 2 plus 0 X 3 plus 1 X 4 equal to 24. So, the two equations are written this way in the form of a table. So, what we have done is, we have seen this part of the table, we first started with creating a table like this, we first you draw this line and then you draw this line and then you say X 1, X 2, X 3, X 4 are the four variables.

Then, you draw this line, then above the four variables you write the objective function value 10, 9, 0, 0. Then, you write the two equations this way 3 X 1 plus 3 X 2 plus 1 X 3 plus 0 X 4 is equal to 21, 4 X 1 plus 3 X 2 plus 0 X 3 plus 1 X 4 equal to 24. Now, just like in the algebraic method it is easy to begin with X 3 and X 4 as the basic variables and keep X 1 and X 2 as the non basic variables. Now, one can also see that you can see this idea that X 3 and X 4 are good basic variables, because if you put X 1 equal to 0 and X 2 equal to 0 in this equation, you will get X 3 equal to 21, X 4 equal to 24.

For example, if we do the same thing, if we treat X 1, X 2 as the non basic variables, we are trying to solve 1 X 3 equal to 21, 1 X 4 equal to 24. You can now see the identity matrix which is under X 3 and X 4 and the other one is not there, because X 1 and X 2 are 0. So, the identity matrix straight away gives a solution X 3 equal to 21, X 4 equal to 24. So, we begin with X 3 and X 4 as the starting basic variables, so I am writing X 3 and X 4 here under a heading which is called X B which represents the set of basic variables. So, X 1, X 3 and X 4 are the set of basic variables that we are trying to solve.

So, what we do is, we draw this extend this line, we draw this line, and then we say X 3 and X 4 are the basic variables. Now, we extend it a little bit on this side and write something called C B which is the coefficient of the objective function of the basic variables. So, X 3 and X 4 have coefficients 0 and 0 and therefore, we write 0 and 0 which are the coefficients of the objective function.

Then, we write something called C j minus Z j and let me explain this C j minus Z j. Now, j represents the variable, j equal to 1 is the first variable X 1, j equal to 2 is the second variable and so on. So, C j minus Z j whatever I am going to write here is C 1 minus Z 1, now Z 1 I am going to define as a dot product or product of this and this. So, the product of this and this will become 0 into 3 plus 0 into 4 which is 0 that is your Z 1, Z 1 because it is under the variable X 1.

So, Z 1 is 0 into 3 plus 0 into 4 which is 0, C 1 is the objective function coefficient of X 1 which is 10. Therefore, C 1 minus Z 1 will become 10 minus 0 which is 10, similarly Z 2 will be 0 into 3 plus 0 into 3 which is 0, C 2 is 9 therefore, C 2 minus Z 2 is 9 minus 0 which is 9, Z 3 is 0 into 1 plus 0 into 0 which is 0 and C 3 is 0, because the objective function coefficient of X 3 is 0. Therefore, C 3 minus Z 3 will become 0 minus 0 which is 0.

In a similar manner C 4 is 0, Z 4 is 0 into 0 plus 0 into 1 which is 0 therefore, C 4 minus Z 4 is 0 minus 0 which is 0. At the moment you find a lot of 0's coming, but as we move along we will see the significant of C j minus Z j or at the movement the C j minus Z j row seems to repeat this row, because all the Z j's are 0. Later we will have situations where Z j's are non 0 and C j minus Z j will take different values. So, we complete this, we would have drawn this line and then we would evaluate the C j minus Z j, and then we complete the table by drawing this line.

Now, the value of the objective function is 0 into 21 plus 0 into 24 which is 0. So, I multiply 0 into 21 plus 0 into 24 which is 0, so we complete this table like this. Now, the value of the objective function is 0 and C j minus Z j values are these 4. Now, find the non basic variable with the largest C j minus Z j, the two basic variables are X 3 and X 4 there C j minus Z j are 0. The basic variables will always have C j minus Z j equal to 0, the non basic variables will have non 0 values.

Now, there are two non basic variables X 1 and X 2 which are not in the solution, they are at 0. So, there C j minus Z j are 10 and 9 find the non basic variable with the most positive C j minus Z j. So, if we do that now this is the variable X 1 which has the highest C j minus Z j, so variable X 1 comes into the solution. Now, when this comes into the solution it has to replace X 3 and X 4, because only two variables can be in the solution, there are two equations, so only two variables can be in the solution.

Now, which one does it replace? To find out which one it replaces, we calculate this thing called theta. So, just draw this line and write theta here and then this theta is the ratio between this 21 and this 3, 3 corresponds to the variable that is entering. So, 21 divided by 3 is your first one, so 21 divided by 3 is 7 and 24 divided by 4 which is 6 and take the smaller theta which is 6.

You also have to make sure that when you do this division, the denominator is not 0 or negative. You will realize that the numerator will be either 0 or positive and most times it will be positive, the numerator the right hand side value will not be negative. But, the denominator which is obtained from here can be 0 or negative. When we do not, when we have a 0 or negative as a denominator, we do not evaluate that value.

Now, at the moment both numerator and denominator are positive, so we got 6 and 6 is the limiting value and which is shown here. So, the variable X 4 will go out of the solution and the variable X 1 will come in to the solution. So, this is one iteration of the simplex table.

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		10	9	0	0		
C_B	X_B	X_1	X_2	X_3	X_4	RHS	θ
0	X_3	3	3	1	0	21	7
0	X_4	4	3	0	1	24	6
	$C_j - Z_j$	10	9	0	0	0	
0	X_3	0	3/4	1	-3/4	3	4
10	X_1	1	¾	0	¼	6	8
	$C_j - Z_j$	0	3/2	0	-5/2	60	
9	X_2	0	1	4/3	-1	4	
10	X_1	1	0	-1	1	3	
	$C_j - Z_j$	0	0	-2	-1	66	

Now, we proceed now and I have written the same alteration here already and I have drawn the next table using these black lines. So, now, we know that this is a variable that is coming into the solution, X 1 is the variable that is coming into the solution, X 4 is the variable that is the leaving the solution, X 1 is the variable that is coming into the solution. So, what I have do next is I write X 3 and X 1 as the basic variables, X 1 is coming into the solution and it is replacing X 4. So, I am writing X 3 and X 1 as replaced X 4 in this solution.

And as soon as write X 3 and X 1 the next thing I have to do is to write the objective function coefficients of X 3 and X 1. So, I write 0 and 10, 10 is the objective function coefficient for X 1. Now, what I do is this element which is 4 which I have shown in a different color is the intersection element between the entering variable X 1 and the leaving variable X 4 and this 4 is called a pivot element and this X 4 row is called the pivot rho, it is called the pivot element and this is called the pivot row.

So, now, take the pivot row and divide every element of the pivot row by the pivot element. So, 4 by 4 is 1, 3 divided by 4 is 3 by 4, 0 divided by 4 is 0, 1 divided by 4 is 1 by 4 and 24 divide by 4 is 6. You will also observed that you have already return this 6 here and the same 6 will come because this 6 is also 24 divided by 4. So, we have now rewritten the pivot row in terms of X 1, now we have to rewrite it in terms of X 3.

Now, to do that we do some row operations as we do in the gauss Jordan method of solving equations or inverting a matrix. Now, what are these row operations, now we realized that if X 3 and X 1 represents our solution then in the earlier one you realize that X 3 and X 4 was representing the solution and we saw the identity matrix with X 3 and X 4. Now, here X 3 and X 1 represent our solution, so we should have an identity matrix under this X 3 and under this X 1.

So, I should have a 1 here and I should have 0 here, so that I get the identity matrix under. So, in order to get a 0 in this position, now I look at the previous one which is 3 here there is 1, so 3 minus 3 times 1 will give me 0. So, what I do is, I put 0 which is 3 minus 3 times 1 is 0, now I repeat 3 minus 3 times 4, 3 minus 3 times 3 by 4. So, 3 minus 3 times 3 by 4 is 3 minus 9 by 4, 12 by 4 minus 9 by 4 which is 3 by 4. Now, here it is 1 minus 3 time 0, so I will get 1, here it is 0 minus 3 times 1 by 4 which is minus 3 by 4 I get minus 3 by 4, 21 minus 3 times 6, 21 minus 18 which is 3.

So, once again to get this row I need a 0 here, so I look at the previous element 3 minus 3 time 1 is 0. So, 3 minus 3 times 3 by 4, 3 minus 9 by 4 is 3 by 4, 1 minus 3 time 0 is 1, 0 minus 3 times 1 by 4 is minus 3 by 4, 21 minus 3 times 6 18 is 3. So, we now have a solution X 3 equal to 3, X 1 equal to 6, now we write the C j minus Z j. So, we write C j minus Z j now Z 1 is 0 into 0 plus 10 into 1 10, so 10 minus 10 is 0, Z 2 is 0 into 3 by 4 plus 10 into 3 by 4 is 30 by 4. So, 9 minus 30 by 4 is 36 by 4 minus 30 by 4 which is 6 by 4 which simplifies to 3 by 2, C 3 minus Z 3.

Z 3 is 0 into 1 plus 10 into 0 0, so 0 minus 0 is 0, Z 4 is 0 into minus 3 by 4 plus 10 into 1 by 4 which is 10 by 4 or 5 by 2. So, 0 minus 5 by 2 is minus 5 by 2 and the value of the objective function is 0 into 3 plus 10 into 6 which is 60. So, we have now completed the second iteration of the simplex algorithm, where we now have a solution with X 1 equal to 6, X 3 equal to 3 with C j minus Z j equal to 6 with the objective function value equal to 60.

Now, we wish to increase the objective function further and we realize that the basic variable have 0 C j minus Z j, the non basic variable only X 2 has a positive value. So, enter the most positive non basic variable with the most positive C j minus Z j. Now, we have to find out that X 2 has replace X 3 or X 1 to find out which one it replaces 3 divided by 3 by 4 which is 4, 6 divided by 4 by 3 which is 8 or 6 divided by 3 by 4 which is 8, 6 into 4 by 3 is 8 the limiting values is 4 therefore, the variable X 2 will now replace the variable X 3.

And then we move on to the third iteration which is shown here and in the third iteration the variable X 2 has replaced the variable X 3. So, we now write this X 2 has replaced X 3, so the basic variables are X 2 and X 1. Now, when the basic variables are X 2 and X 1 the value of the objective function coefficients are 9 and 10 respectively. Now, this is the pivot rho, now this is your pivot element and this is my pivot rho.

So, like we did last time divide everything by the pivot element, so we get 0 divided by 3 by 4 is 0, 3 by 4 divided 3 by 4 is 1, 1 divided by 3 by 4 is 4 by 3 minus 3 by 4 divided by 3 by 4 is minus 1, 3 divided by 3 by 4 is 4. Now, I have a 1 here, since I have X 2 this should be 1 0 and this should be 0 1 I require a 0 in this position, now I a require 0 in this position. Now, to get a 0 in this position only then I will have the identity matrix corresponding to X 2 and X 1, so 1 0 0 1 I need a 0 in this position.

So, I go back to the previous value which is 3 by 4, so 3 by 4 minus 3 by 4 into 1 is 0, so 1 minus 3 by into 0 will become 1. So, I repeat this and I get, so 1 minus 3 by 4 into 0 is 1, 3 by 4 minus 3 by 4 into 1 is 0, 0 minus 3 by 4 into 4 by 3 is minus 1 and 1 by 4 minus 3 by 4 into minus 1 is 1 by 4 plus 3 by 4 which is plus 1, 6 minus 3 by 4 into 4 is 6 minus 3 which is 3, so we now have a solution X 2 equal to 4, X 1 equal to 3.

Now, we compute C j minus Z j, so for the first one 9 into 0 plus 10 into 1 is 10, so 10 minus 10 is 0, for the second one 9 into 1 plus 10 into 0 is 9, 9 minus 9 0, for the third one 9 into 4 by 3 36 by 3 which is 12, 12 minus 10 is 2 so 0 minus 2 is minus 2. And for the last one 9 into minus 1 is minus 9 plus 10 into 1 10 is plus 1, 0 minus 1 is minus 1 and the value of the objective function is 9 into 4 36 plus 10 into 3 30 which is 66.

Now, we see whether we can enter another variable, the basic variables have 0 value, X 1 and X 2,the non basic variables have negative value. We are not able to enter a variable the algorithm terminates giving the solution X 1 equals to 3, X 2 equal to 4 with objective function equal to 66. So, we have now completed the problem in the form of the simplex table which gives us the same solution, other aspects of the simplex table we will see in subsequent classes.