Introduction to Operations Research

Prof G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Module – 01
Linear Programming – Introduction and formulations
Lecture - 01
Product Mix problem and Notations

Welcome to the course introduction to operations research. Operations research is a field of study where, we optimize performance under given constraints. Operations research also helps us develop models for decision making. Among the several topics in operations research, linear programming is the most important and the most popular one. Linear programming problems have applications in the various fields some of which include manufacturing systems, publics systems, business, supply chain management and analytics. In linear programming, we try to optimize a linear objective function subject to linear constraints and with non-negativity restrictions on the variables. Every activity is carried out with an objective in mind and it is important to optimize the objective.

These activities are also carried out under some constraints or conditions and linear programming helps us understand these and helps us to formulate and solve problems which, come out of these practical situations where several activities are modeled. We start linear programming with the simple example to understand the notation and also to understand the aspects of formulating a problem into a linear programming problem.

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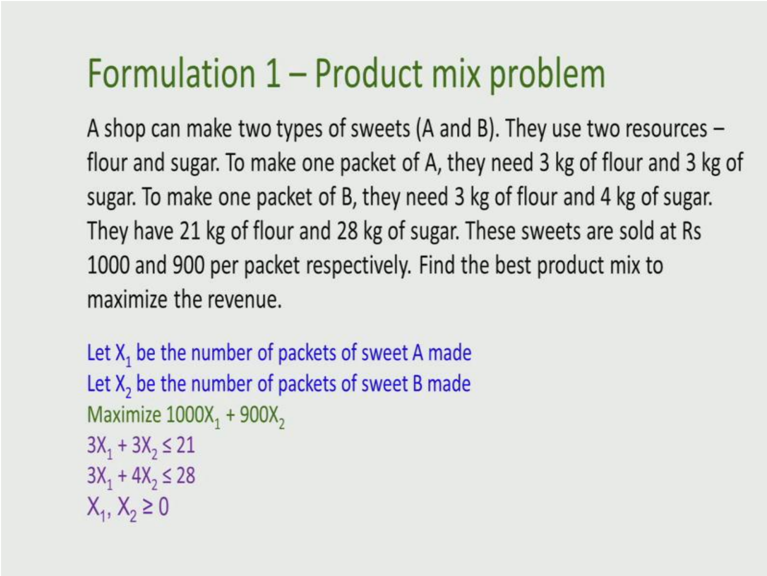

The first formulation that we will be looking at is called a product mix problem where, we try to find out what is the mix of products that an organization should produce. We start with a very simple example, you can see the text of the example and then we convert this into a formulation and then we define the terms and notation related to linear programming.

Now, let us consider a shop that makes two types of sweets or two types of products these are called a and b. The shop uses two resources which are indicated here as flour and sugar they make this sweets in packets. So, to make one packet of a they need 3 kg of flour and 3 kg of sugar. To make one packet of b they need 3 kg of flour and 4 kg of sugar, they have with them 21 kg of flour and 28 kg of sugar. Now, these sweets when made are sold at rupees 1000 and rupees 900 per packet respectively. Find the best product mix to maximize the revenue generated by the shop.

Now, we will look at this example and we will try to formulate this as a linear programming problem. The first thing that the shop has to decide is how many packets of the two sweets that are going to be made. So, we start defining these variables to this problem as x 1 and x 2 which are two variables where x 1 represents the number of packets of sweet a made and x 2 represents the number of packets sweet b made. And in general, we could say x 1 x 2 would represent the number or the quantity of the products that are being made. Now, by making sweets and by selling them the revenue generated is rupees 1000 per packet for a and 900 per packet of b. So, if we make x 1 number of packets of a then the revenue generated is 1000 into x 1 and if we make x 2 packets of sweet b the revenue generated is 900 into x 2.

Therefore, the total revenue generated is 1000 x 1 plus 900 x 2 which is shown here 1000 x 1 plus 900 x 2 is the revenue that is generated by the production and sale of these products. Now we write this revenue function as 1000 x 1 plus 900 x 2. Now obviously, we want to maximize the revenue that is generated and therefore, we wish to maximize the function 1000 x 1 plus 900 x 2. Now, the availability of these resources which are flour and sugar they limit or restrict the values that x 1 x 2 can take or in general restrict the values that the quantity of production can take. Now, these restrictions are written as constrains and they are shown here. Now, we know that 21 kg of flour is available to make one packet of a, we need 3 kg of flour to make one packet of b we also need 3 kg of flour.

So, to make x 1 packets of a, we need 3 x 1 kg of flour to make x 2 packets of b we need 3 x 2 kg of flour. Therefore, the total quantity of flour required is 3 x 1 plus 3 x 2 which is shown here. The quantity of flour available is 21 which is shown in right hand side and now this 21 is going to limit the values that x 1 and x 2 can take. Therefore, 3 x 1 plus 3 x 2 which is the quantity of flour required cannot exceed 21 which is the quantity of flour available. This is modeled as 3 x 1 plus 3 x 2 is less than or equal to 21 which means 3 x 1 plus 3 x 2 cannot exceed 21.

Similarly, we have 28 kg of sugar which is going to restrict the values that x 1 and x 2 can take in a certain way. To make one packet of a, we need 3 kg of sugar to make x 1 packets of a v require 3 into x 1 kg of sugar. To make one packet of b we need 4 kg of sugar to make x 2 packets of b we require 4 into x 2 kg of sugar and 3 x 1 plus 4 x 2 which is the requirement, cannot exceed 28 which is the availability. So, the two resources availability of the two resources which is 21 kg of flour and 28 kg of sugar are going to restrict or limit the values that x 1 and x 2 can take and that is represented by 3 x 1 plus 3 x 2 less than or equal to 21 and 3 x 1 plus 4 x 2 less than or equal to 28.

We also have this condition x 1 and x 2 greater than or equal to 0 which means we would like to make non-negative quantities of both the products which also implies that we cannot make negative quantities of any of the products. So, the formulation of the product mix problem for the given situation is, let x 1 be the number of packets of sweet a made let x to be the number of packets of sweet b made maximize 1000 x 1 plus 900 x 2 subject to 3 x 1 plus 3 x 2 less than or equal to 21 and 3 x 1 plus 4 x 2 less than or equal to 28 x 1 and x 2 greater than or equal to 0. So, this completes the linear programming formulation of the given product mix problem. We will now go on to define what is a linear programming problem? And y this problem that we have formulated comes under the category of linear programming.

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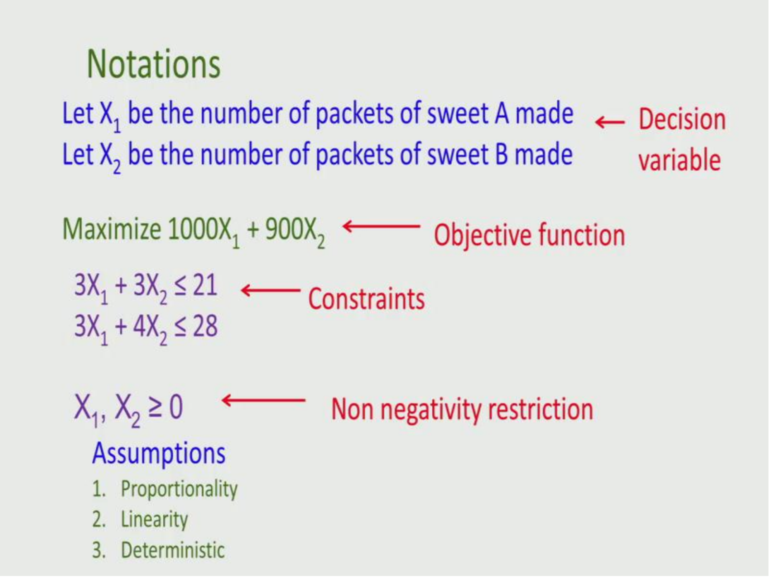

So, we start by saying let x 1 be the number of packets of sweet a made, let x 2 be the number of packets of sweet b made. So, we first wrote these variables and these variables that we wrote are called decision variables. These variables represent the decisions that have to be made in the product mix problem. So, these variables that we have defined are called decision variables. We then wrote the revenue function in terms of the decision variables as 1000 x 1 b plus 900 x 2 if x 1 packets of sweet a are made and x 2 packets of sweet b are made. Now, this function that we wrote is called the objective function and represents the objective of the problem which is to generate the revenue and to maximize or try to have as high a value of revenue as possible.

So, the revenue function is the objective function which is to be maximized and therefore, maximize 1000 x 1 plus 900 x 2 is the objective function associated with the problem. We then wrote 3 x 1 plus 3 x 2 less than or equal to 21 and 3 x 1 plus x 4 x 2 less than or equal to 28. Now, these two conditions or constrains as they are called limit the value that the decision variables can take. And these constraints are shown here there are two constraints in this problem. Last we wrote x 1 x 2 greater than or equal to 0 and said that we are not going to produce negative quantities of the products therefore, we have explicitly stated a non-negativity restriction on the variables.

So, the same formulation that we made has now been described in terms of decision variables, in terms of the objective function, in terms of constraints and non-negative restriction on the decision variables. So, first part of a linear programming formulation is to define the decision variable, then we define the objective function as a function of the decision variable and in this case we have a linear function of the decision variable. The objective function is 1000 into x 1 plus 900 into x 2 which is a linear function of x 1 and x 2 the function is not non-linear.

For example, this function is not 1000 x 1 square plus 900 x 2 square and so on. It is 1000 into x 1 plus 900 into x 2 which is a linear function if we draw the curve associated with this we would get a line so, its linear function of the decision variables. The constraints, have a left hand side a right hand side and a relationship. In this case the relationships are inequalities and both the inequalities are of the less than or equal to ten, The constraints left hand side we have linear functions of the decision variables 3 x 1 plus 3 x 2 is a linear function of x 2 3 x 1 plus x 1 and x 2 3 x 1 plus 4 x 2 is also a linear function of x 1 and x 2. So, the constraints or linear functions of the decision variables and we have an explicit non negativity restriction on the decision variables.

So, the linear programming problem is one where the objective function is a linear function of the decision variables. The constraints are all linear inequalities or equations as the case may be. In this example, we have only inequalities we do not have equations and there is an explicit non-negativity restriction on the decision variable. So, if we are able to model or write formulation which has an objective function which is a linear function of the decision variables, constraints which are linear inequalities or equation and explicit non-negativity restriction on the decision variables, we have formulated in linear programming.

Therefore, the problem that we just now formulated, the product mix problem is a linear programming problem that has an objective function which is to maximize the revenue which is a linear function. The constraints are linear inequalities and there is an explicit non-negativity restriction. We also take a look at some of the assumptions in these problems. Three important assumptions are proportionality, linearity and deterministic nature of the problem let me explain all of this. Proportionality comes from saying that if I make one packet of sweet a I get 1000 rupees by selling it, if I make x 1 packets I would get 1000 into x 1 if I make half a packet I would get 500, if I make two packets I would get 2000 and so on.

Here, for every packet I need 3 kg of the resource. So, if I make two packets I would require 6 kg. If I make half a packet I would require one and half kg this is the personality assumption. The additivity or linearity assumption in this case is the revenue made by selling x 1 kg is 1000 x 1 and the revenue to made by selling x 2 kg of sweet b is 900 x 2 the total revenue is 1000 x 1 plus 900 x 2. So, there is the additivity or linearity assumption. In a similar manner, if 3 x 1 kg of the resource is required to make sweet a and 3 x 2 is required to make sweet be then the total requirement is sum of these two requirements and we have the linearity or the additivity of assumption that way. Deterministic in the sense the values are known and values are known with certainty when we actually formulate the problem.

The availability is a constant which is 21 kg of this resource and is known well in advance and deterministic. So, the three important assumptions are proportionality, linearity and deterministic nature of the numbers or the resources that are available for the product mix problem. In next unit of this module on linear programming we will continue to look at some more formulations where we try to understand minimization problems that require greater then or equal to constrains.