Manufacturing Systems Management
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Lecture - 37
Flexible Manufacturing System, Part selection problem

In this lecture we study Flexible Manufacturing Systems. In this lecture series we have seen some aspects of cellular manufacturing just-in-time manufacturing and synchronous manufacturing, now we move to the last part of this course which is on flexible manufacturing systems.

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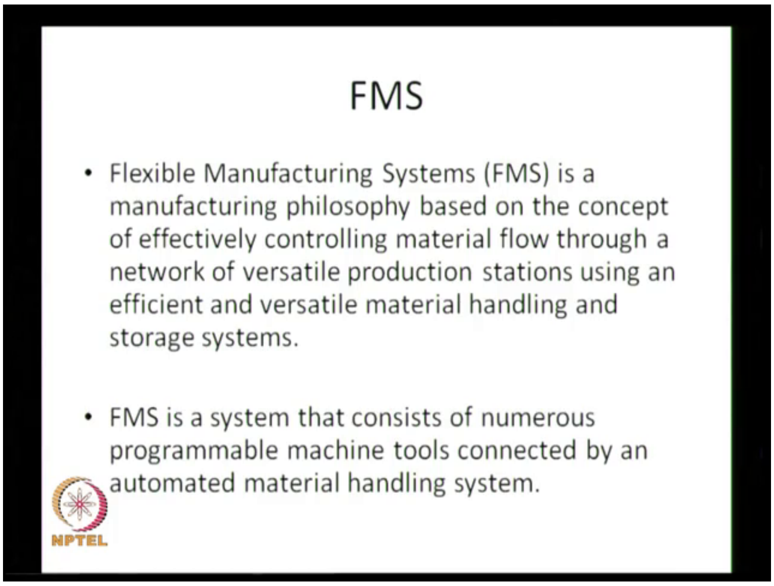

So, flexible manufacturing systems is a manufacturing philosophy based on the concept of effectively controlling material flow through a network of versatile production stations using an efficient and versatile material handling and storage system.

So, this is one of the definitions of flexible manufacturing systems. Flexible manufacturing systems are called FMS in short. The important key words here are control effectively controlling material flow, network of versatile protection stations which is important, efficient and versatile material handling and storage systems. So, the essential components of an FMS are effective material control, network of versatile production stations, efficient material handling systems and efficient material storage systems. So, we integrate manufacturing material handling and storage into a single system and that is called a flexible manufacturing system.

Another definition is FMS is a system that consists of numerous programmable machine tools connected by an automated material handling system. So, in this definition the important keywords are numerous programmable tools, tools meaning machine tools connected by which talks of integrating a machines with material handling system and an automated material handling system. So, 3 important things programmable machine tools connected are interlinked with a material handling system and an automated material handling systems all these 3 things come together to form a flexible manufacturing system.

So, we combine both these definitions we understand that an FMS consists of many programmable machine tools. So, there is a lot of automation and computer usage, it is connected by an automated material handling system, so material handling is integrated and is part of the manufacturing system. FMS have been there for about 25 to 30 years now since the early 80s they have been there and they provide a very efficient way to produce a very large volume and they also have the ability to handle variety.

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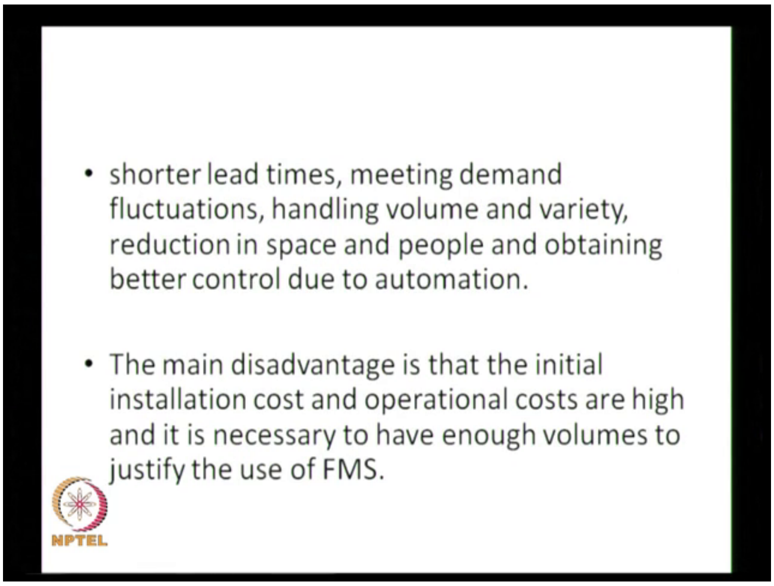

Advantages of FMS our shorter lead times, shorter production lead times, ability to meet demand fluctuations, ability to handle both volume and variety, reduction in space and people and obtaining better control due to automation. So, 5 advantages are given here and all these advantages are evident when we look at the definition which talks of programmable machines connected by an automated material handling system.

Again as I mentioned 3 things programmable machines automated material handling system and they are connected. So, when we do that we will be able to have shorter lead times. Production lead times are smaller because most of the production is carried out by automatic machines; material handling is an integral part of the manufacturing system so automatically lead times are short times. Lead times, production lead times depend on setup or change over time, manufacturing time, transportation time and good scheduling. So, these 4 things are important to reduce lead times.

In an FMS setup times are smaller largely because of what is called changeover flexibility or set up flexibility about which we will see a little later. The ability to change over while production is happening change over on some other for some other product or ability to set up another product while machining is happening on some other piece is called changeover flexibility so that reduces the changeover times.

Machines are automated and have higher capabilities and therefore, they reduce the manufacturing time material handling is integrated with the manufacturing and therefore, time to retrieve and handle material transport and retrieve material is reduced and since the entire FMS is computer controlled scheduling also becomes a part of the computerized control system and therefore, good scheduling algorithms are used or good scheduling practices are followed. And therefore, all the 4 components of lead time are reduced in an FMS. So, lead times become shorter.

Ability to meet demand fluctuations comes largely because of its ability to handle large volume as well as certain amount of variety within the aggregated volume. So, FMS have ability to meet demand fluctuations largely because of the capability of the machine tools which are automated. Production in space is expected space and people because in these type of manufacturing systems a single machine is capable of doing what 2 or 3 other semi automatic or manual machines are capable of doing and since there is a large extent of automation people requirement is reduced. And be able to get better control due to automation happens simply because automation is able to provide a lot more production control the manual intervention is very less and there is better production control due to automation. So, these are the advantages of flexible manufacturing systems.

Flexible manufacturing systems are used largely in manufacturing electronic goods. One of the disadvantages and in fact, the main disadvantage of FMS is that the initial installation cost and operation costs are very high and it is also necessary to have enough volume to justify the use of FMS. Initial cost is very high largely because of the components of the FMS, a large a network of machines, a large set of programmable machine tools each of them is extremely costly, automated material handling and retrieval systems are costly they can be rail guided or AGV automated guided vehicles computerized control systems are costly to develop and implement and therefore, FMS is a costly proposition size is small, but the cost is very high.

And secondly, in order to have an FMS in place the organization should justify it through large volumes. So, if the volumes are large then organizations go for FMSs, but then the disadvantage is that the organization should also be should have enough volume to justify the use of FMS, sometimes FMS are installed and then there is an effort to increase the product volumes.

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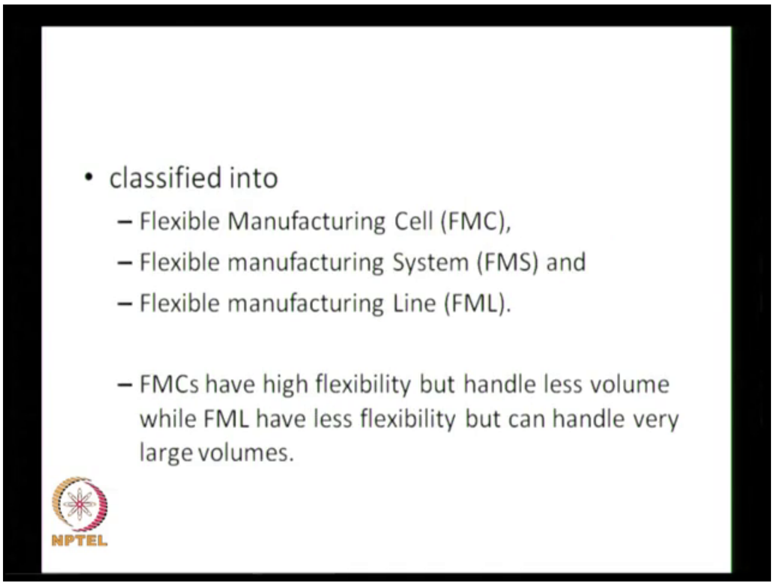

FMSs are classified into 3 types flexible manufacturing cell, flexible manufacturing system and flexible manufacturing line, so FMC, FMS and FML. All these are essentially terminologies. Flexible manufacturing cell by definition is like a single cell as in cellular manufacturing having 3 or 4 different types of automated machines coming together and all the parts are produced within the cell and the parts are made and sent to customers or to assembly.

Flexible manufacturing system by definition is a collection of flexible manufacturing cells at least 2 or more flexible manufacturing cells each cell having a certain number of automated machines as well as computer controlled material storage system and material handling system more than one cell parts are made in different cells can be assembled or sent to the customer. Except for the size and aggregate volume FMC and FMS are the same. Flexible manufacturing line is like a production line, it is like a flexible manufacturing cell, but it handles more volume and very few products compared to a flexible manufacturing cell.

So, if the volumes are very very high or if a cell is required to make only one product then it becomes a flexible manufacturing line, if it makes more than one then it becomes a flexible manufacturing cell is a little more variety and a collection of flexible manufacturing cells make a flexible manufacturing system. So, let us written here FMC s have high flexibility handle less volume while FML have less flexibility, but can handle very large volumes. So, depending on the volume we call it an FML or FMC depending on both volume and variety.

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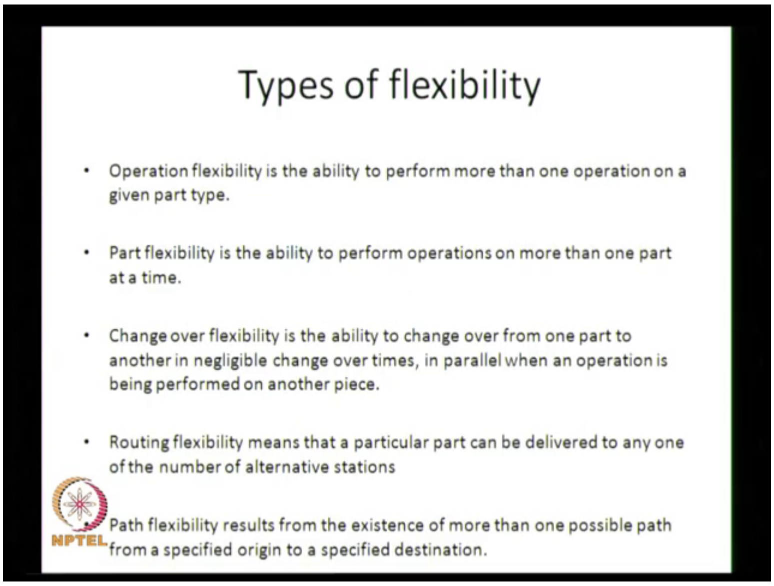

Now, serious a system is called as a flexible manufacturing system it has to be flexible. So, we try and define what are the types of flexibilities, that are part of a flexible manufacturing system. Several flexibilities exist, now we concentrate on 5 important aspect of flexibilities and these 5 flexibilities are operation flexibility, part flexibility, changeover flexibility, routing flexibility, path flexibility.

So, there are 5 flexibilities that are defined and each of these are explained as follows. Operation flexibility is the ability to perform more than one operation on a given part type. So, the given part type is fixed it set up on the machine, but then we have multiple tools working on the part and that is called operation flexibility. Due to operational flexibility the time taken to produce reduces significantly many things change, normally the time taken as far as the computations are concerned is a sum of all the operation times, but when the FMS has operation flexibility and can do 2 operations at the same time on the piece then the time is the maximum of the 2 and not the sum of the 2. So, the very approach or the way to calculating the time taken changes, so operation flexibility brings in a completely new perspective with respect to manufacturing.

Part flexibility is the ability to perform operations on more than one part at a time. So, there are there are two parts and then there are tools that come and work on both the parts. Now, this again changes the way one does computations and scheduling time taken for a batch is always the batch size into unit processing time plus changeover time. Now, if it can do 2 parts at a time or 3 parts at a time then we have to do batch size divided by 2 into unit processing time.

So, as a result of both we find that the time taken to process reduces significantly. Then we have changeover flexibility changeover flexibility is the ability to change over from one part to another in negligible changeover times in parallel when operation is performed on another piece. So, change of our flexibility is like this there could be 2 tables, on one table and operation or some manufacturing is happening on a piece on the other table we are able to change over from one piece to another and once this is over they can index, and the one that is already fixed or its already set up on the machine the manufacturing happens on that the one on which the manufacturing is over is removed and another piece is brought in.

Now, changeover time happens in parallel when manufacturing or production is happening on another piece so as the ability to change over from one part to another in very small or negligible changeover times. Many a times these changeovers also happen with the help of robos or with the help of computer controlled systems and they happen very very fast. So, once again the very definition of time to produce will change because usually the time on a batch is the changeover time plus batch size into unit processing time if we bring all the 3 together the changeover time becomes negligible because it can be done in parallel while something else is happening.

Due to operation flexibility maximum time of the operations is now taken, due to part flexibility if two parts can be done at the same time the batch size is divided by 2 and all the 3 components of the processing time of the batch reduces therefore, the time taken to produce reduces considerably in the case of a flexible manufacturing system.

Routing flexibility means that a particular part can be delivered any one of the number of alternate stations. So, if we have a flexible manufacturing cell with let us say 4 workstations and there is a particular part which requires a certain operation and more than one can handle it, in a normal conventional manufacturing system the routing is done before the manufacturing begins many times the routing is predetermined and fixed and then we say it will follow a particular route till all the operations are over. Now, because we have routing flexibility dynamically as and when the part is coming a machine amongst the alternate machines can be chosen and since the entire thing is computer controlled it is linked to the material handling system and the part is quickly taken by the material handling system to the machine to which it is assigned.

So, routing flexibility gives the advantage of higher utilization of the machines as well as reduced waiting times because that particular machine could be working could be busy while there could be another machine that is idle and one could go to that machine. And since all these are controlled by computer there is no disruption, there is no change in change that affects the production and things are done much faster. So, routing flexibility gives the advantage of higher utilization and less waiting time for all the parts.

Another advantage is that if we compare this with a cellular manufacturing system or a cell within a cellular manufacturing system it is quite unlikely that for a particular operation we could have more than one machine in the cell because the cell invariably contains a set of functionally dissimilar machines. Whereas, in the flexible manufacturing system if there are 3 or 4 workstations its quite likely that more than one workstation can handle every operation because most of these workstations are so versatile that they are capable of handling a large number of operations unlike machines in a manufacturing cell as in cellular manufacturing system. So, routing flexibility is something you need to FMS and helps in reducing the time to produce.

Path flexibility results from the existence of more than one possible path from a specified origin to a specified destination. We mentioned that within a cell or within a flexible manufacturing cell FMC if it has multiple workstations parts move from one workstation to another. We also mentioned that material handling or transportation within the FMC is automated using rail guided vehicles or using AGVs. So, if a part has to go from one particular machine to another machine in a normal traditional system only one route is specified because the entire system is computer controlled we could have alternate routes and the system will have the ability to choose the alternate routes for example and we could think of these 4 machines within an FMC and we could have.

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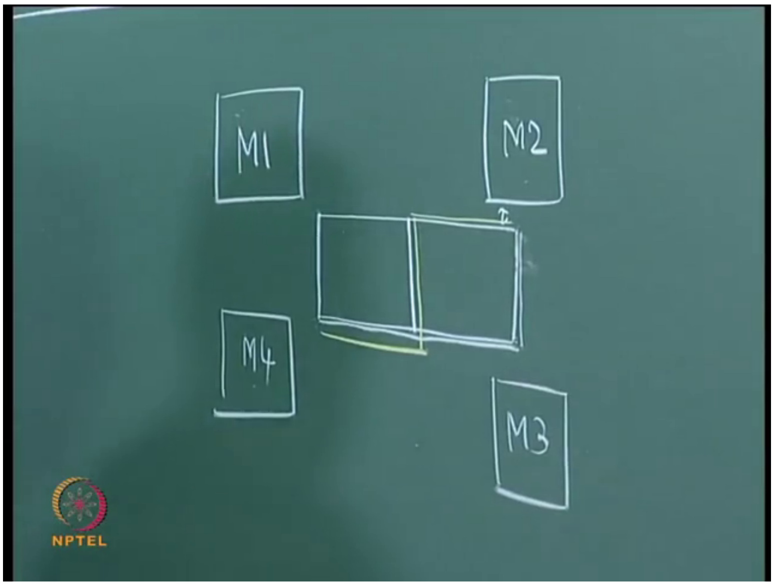

Now, let us assume these are the 4 machines, now let us say one particular part is over here and it has to go to M 4. Now it could take either from M 2 this way to M 4 or it could go this way to M 4 or it could come this way and go to M 4. So, this is an example of having alternate path between M 2 and M 4. So, if the original path is this and for some reason there is something happening here then it may not be able to take this part now this path will now be changed to either this part or the other part. So, this is an example of path flexibility.

So, we have seen 5 types of flexibilities and these flexibilities all of them essentially help in minimizing the delay as I mentioned earlier the time taken to produce is the time to set up or change over from one part to another, time taken to actually produce an manufacturer for the operations, time taken for material handling transportation and retrieval and 4 time taken or time taken if machines are idle due to scheduling issues.

Now, once again if we look at all these 4 times from the point of view of the flexibilities that we talked about changeover flexibility reduces the changeover time change overs are also quick because they sometimes are computer controlled and robot controlled. So, changeover times are smaller. Changeover flexibility which is the ability to do change over on one side when operations are carried out on the other reduces the changeover time from the calculations because it is done in parallel. So, changeover times are reduces. The actual time to machine or do the carry out the operations reduces due to operation flexibility and part flexibility operation; flexibility as the ability to perform more than one operation, so it could be haft.

Part flexibility is more than one part. So, the batch size effectively reduces for the purpose of calculation while routing flexibility and path flexibility reduces the delay in transportation, material, handling etcetera. By changing the route and utilizing a machine that is free the scheduling delays are minimized. So, routing flexibility helps in reducing the delay and idleness due to scheduling and path flexibility reduces the time taken to transport by considering alternate paths from one machine to another. So, all these types of flexibilities also help a flexible manufacturing system to cut down the shot the first important gain which I mentioned is shorter lead times. So, lead times are shorter due to these flexibilities which are part of the flexible manufacturing system.

Now, we get into some aspects of modelling in flexible manufacturing systems.

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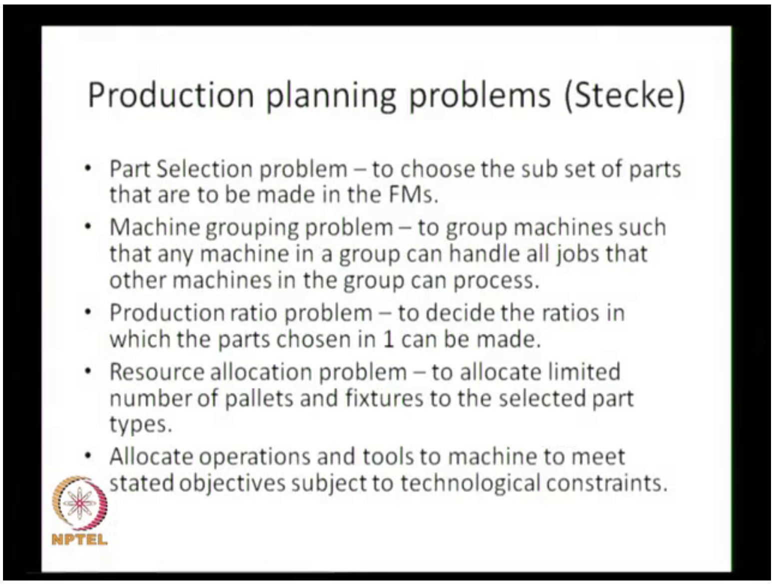

So, several researchers worked on different types of problems in flexible manufacturing, some of these problems are unique to flexible manufacturing while the problems are unique to flexible manufacturing the methodologies used to solve them are new in some cases and traditional in some other cases. So, we will look at some interesting production planning problems in the context of FMS and we will look at them in detail through numerical examples and we will also try and solve them using newer methodologies as well as existing methodologies.

So, one of the important work in the area of flexible manufacturing is by stick who gave 5 production planning problems in flexible manufacturing she defined 5 important problems which a production planner should consider while implementing a flexible manufacturing system. So, the first one is called the part selection problem, second one is called the machine grouping problem, third is a production ratio problem, fourth is a resource allocation problem and the fifth is tool and machine tool allocation problem are the 5 important production planning problems in FMS.

Now, what is the part selection problem? Part selection problem talks about choosing that subset of parts which are going to be made in the FMS. Now we are going to make an assumption here that the entire factory is not going to operate on the FMS mode, we are going to assume that is a certain portion of the factory is going to operate on the FMS mode. If the entire factory is going to operate on the FMS mode then the part selection problem does not exist because all parts have to be made in the FMS.

So, we are going to assume that some parts are going to be made in the FMS while some parts are not going to be made in the FMS. So, among these parts which are going to be made in the factory which are the ones that are going to be made in the FMS is the part selection problem. And we look at the part selection problem in detail through a numerical example which we will try to solve.

Second one is called the machine grouping problem which is to group machines such that any machine in a group can handle all the jobs that other machines in the group can process. So, in this case the author is talking about grouping similar machines into one group like a functional or a departmental kind of a system except that the number of such machines will be very small and these machine groups will also be located physically close to each other and not very far away from each other. Flexible manufacturing system reduces the size of the plant considerably, there are few are machines each machine is extremely versatile and fast and can handle a lot of variety few are machines and they are all connected by a material handling system.

So, in principle they cannot be very far away from each other because it will be extremely expensive to transport from and to these machines which are very far away from each other. So, they are very close to each other physically located and space required is very less. So, a machine grouping problem is where the machines are grouped such that all the machines in the group can handle all the jobs which other machines can process. So, that if there are a set of jobs that are allocated to these groups of machines now which job will go to which machine is called a loading problem which we will see much later loading problem with additional constraints we will see later, but it helps in the machine loading problem where groups of similar machines are made.

Third problem is called the production ratio problem to decide the ratios in which parts chosen in one can be made. Now, there is a certain time available in the FMS and that time has to be used to produce the parts that are chosen in problem number 1, so what percentage of the demand of these parts are going to be met by the FMS. So, that is a production ratio problem resource allocation problem to allocate limited number of pilots and fixtures to the selected part types. FMS contains pallet us and fixtures tools, these are all sometimes scarce resources we do not have too many tools that are available, but the tool the same we made have just one tool of a particular type and that tool now has to be scheduled it can be fixed on different machines at different points in time to handle different parts. So, resource allocation problem essentially allocates all the scarce resources to the machines over time.

So, there is an allocation and there could also be a scheduling which actually comes in the fifth problem to allocate operations and tools to machines to meet stated objectives. Making a part would involve a set of operations and each operation would require a set of tools. Now it is not necessary that all the operations go to the same machine. So, if there are more than one machine in a group then some operations can go to machine 1 some operations can go to machine 2. Now, this helps in balancing the loads which essentially brings out when we are producing a large batch it brings out items quickly. So, there is a load balancing objective where when operations are allocated to when jobs are allocated to different machines and operations and allocated to different machines.

So, sometimes the entire job is allocated to one machine there we do allocation of jobs to machines, sometimes the jobs contain several operations and some operations are carried out in one machine while some other operations are carried out in some other machine. Now, when certain operations are allocated to certain machines then the corresponding tools have to be allocated, so there would also be constraints about the availability of these tools. So, these tools are to be allocated to various machines and scheduled over a period of time so that the tools can be used from one machine to another. So, that all these operations can be carried out. So, all these are production planning problems in the context of flexible manufacturing systems.

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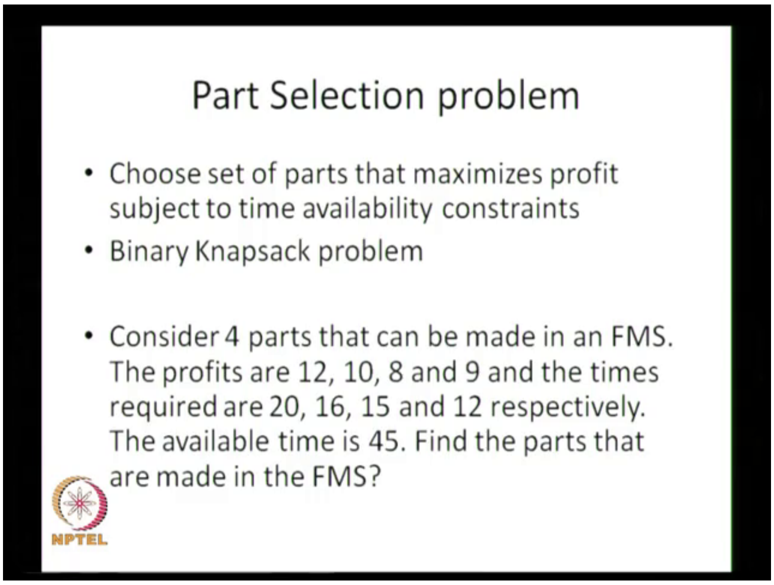

So, we will now look at some of these problems in a little more detail. So, the first problem that we will look at from a modelling perspective is called the part selection problem. So, choose the set of parts that maximizes profits subject to time availability constraints. So, consider 4 parts that are made in an FMS the profits are 12, 10, 8 and 9 and the time required are 20, 16, 15 and 12, available time is 45 in the parts that have to be made or that can be made in the FMS is the first problem that we will take from a modelling perspective.

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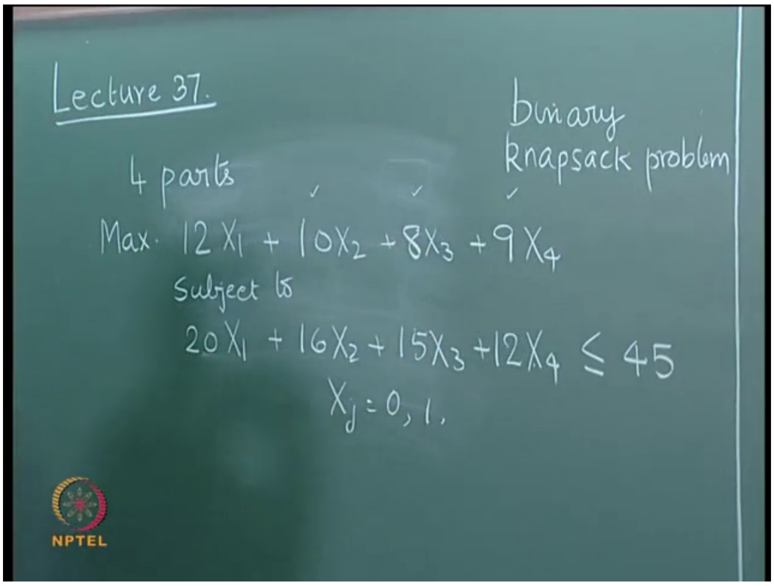

We look at 4 parts the profits are 12, 10, 8 and 9 and the times are 20, 16, 15 and 12, 15 and 12. Now, we have to find out whether we which are the, which of the 4 parts are to be made on the FMS. The reason is the time available on the FMS is 45 whereas the time required on all the 4 put together exceeds 45. So, all of them cannot be made, so which are the ones that are going to be made. So, we define X 1. So, X 1 plus 16 X 2 plus 15 X 3 plus 12 X 4, so X j equal to 1 if part j is going to be made in the FMS X j equal to 0 otherwise. So, the profit will now become 20 X 1 plus 10 X 2 plus 8 X 3 plus 9 X 4 which is to be maximized subject to a single constraint and X j is binary it is either 0 or 1.

Now, this problem is called a binary knapsack problem is called a binary knapsack problem it is like having a sack a knapsack with volume equal to 45 and there are 4 items each item volume is 20, 16, 15 and 12, now which are the items I can put into the bag such that the bag is able to hold the items. So, X equal to 1 if I choose that item X equal to 0 if I do not choose that item it is binary because there is only one item available of each type. So, either take it and put it into the bag or I leave it.

Now, from a operational research perspective it is a simple integer programming problem binary IP weather vary with 1 constraint and a maximization objective. So, this problem can be solved comfortably up to a reasonable size even though very theoretically this problem is a difficult problem up to a reasonable size it can be solved extremely comfortably. So, we could simply solve this problem using a simple solver to get a solution. Now, the optimum solution is Y 1 equal to Y 4 Y 1 equal to Y 2 equal to Y 4 equal to 1.

Now essentially, so to begin with we can actually directly solve this problem as an IP there is no difficulty at all. So, the solution is 9 10 and 8, 10 8 and 9 these are the solutions that make up. So, we can actually solve this with the X itself I later explain why we define the y's, but let us we can solve it with the X itself. So, this would give us 16 plus 15 is 31 plus 12, 43. So, 43 units is used and it and the profit will be 10 plus 8 18 plus 9, 27 with X 2 equal to X 3 equal to X 4 equal to 1 is the optimum solution if it were solved using a particular solver, but if we do not have a solver.

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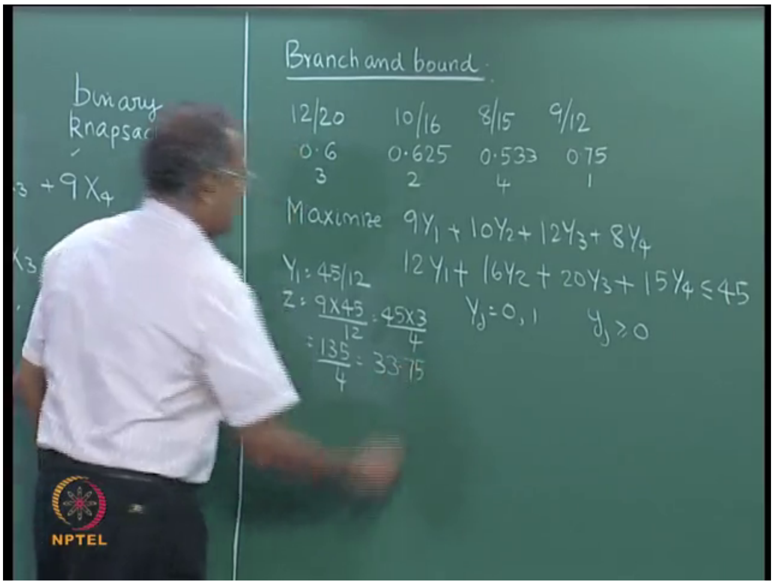

And if we wish to solve it optimally then we could do what is called a branch and bound algorithm which I will describe now. Now, this is something which we have done earlier also what we do in this cases find out the ratio of the objective function to the constraint whenever you have a single constrained integer programming either binary or integer you find out the ratios. So, the 4 ratios are 12 by 20, 10 by 16, 8 by 15 and 9 by 12. So, these ratios are 12 by 20 is 0.6, 6 by 10 which is 0.6, 10 by 16 is 0 in 6 Za 96 40, 16 2 Za are 32, 0.625; 8 by 15 is 0, point 80 5 Za 75, 0.533 and 9 by 12 is 0.75.

Now arrange these coefficients or arrange the variables in the decreasing order of coefficients, so this will become 1 the highest coefficient this will become 2 this will become 3 and this will become 4. Now, rewrite the problem by renaming the variables. So, this will become Y 1, this will become Y 2, this will become Y 3 and this will become Y 4. So, X 4 will become Y 1 X 2 will become Y 2 X 1 will become Y 3 and X 3 will become Y 4 therefore, the problem will take the form that is given in the slide which is to maximize 9 Y 1 plus 10 Y 2 plus 12 Y 3 plus 8 Y 4 subject to 12 Y 1 plus 16 Y 2 plus 20 Y 3 plus 15 Y 4 less than or equal to 45.

Now, Y j is by 1 now the reason for doing this is for renaming the variables and writing the redefining the variables in the decreasing order of c j by a j as it is called or c j by t j as it is called in this case is. If we now solve an l p by leaving out this a single constraint the l p will have only one variable in the solution and because the variables are already sorted in the decreasing order of c j by t j the one with the highest c j by t j or the leftmost variable is the one that will be in the solution. For that reason the variables are rearranged, so that when we solve an l p of it now, this is a binary I p because of the 0 1 it is not a linear programming problem if we solve a linear programming problem out of it then such a linear programming problem the where is the variable in the solution will be the leftmost variable that we have. For that reason this is being done. This also helps us to develop a branch and bound algorithm which we will see now.

We start the branch and bound algorithm by solving the linear programming problem first. So, when we solve an l p relaxation of it now this is solving Y j greater than or equal to 0 if we solve an l p relaxation Now, this is a single constrained l p therefore, only one variable will be in the solution and we have already sorted the variables in the decreasing order of c j by t j. So, leftmost variable will be in the solution. So, this would give us a solution Y 1 is equal to 45 by 12 and Z is equal to 9 into 45 by 12 which is 45 into 3 by 4 which is 135 by 4, 12 15 4 3 za 12, 33.75.

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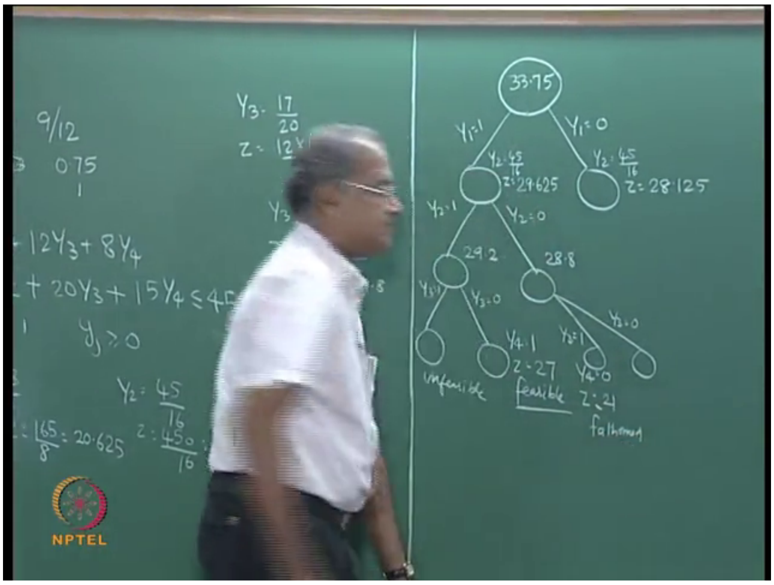

So, this would give us a solution with 33.75, it is a maximization problem therefore, linear programming will be an upper bound l p relaxation will be an upper bound to the binary or IP optimum. So, this would tell us that the optimum is less than or equal to 33 it cannot be more than 33 or it cannot be more than 34.

So, now we proceed and create 2 nodes where Y 1 is equal to 1 and Y 1 is equal to 0. So, this will be Y 1 is equal to 1 this will be Y 1 is equal to 0. So, when we force Y 1 equal to 1 then we are making this. So, this variable comes out of the solution 12 units are already used so now, what is available is 33 because this 12 is already used in Y 1. So, when Y 1 equal to 1 the corresponding l p solution will now become Y 2 equal to 45 by 16. So, this solution will be Y 2 equal to 45 by 16 and Z is equal 10 into 45 by 16, 5 into 45 by 8, 225 by 8, 8 2 za 16, 65 8 8 za 64 sorry, Y 1 equal to 1 means 12 units are already used up so, only 33 units are available. So, this will give a solution Y 2 equal to 33 by 16 and Z is equal to 330 by 16 which is 165 by 8 which is 20.5 by 8 50, 8 6 za 48, 20.625 plus another 9 because Y 1 is equal to 1. So, upper bound is 29.625. Now you see that it has come down from 33.75 to 29.625.

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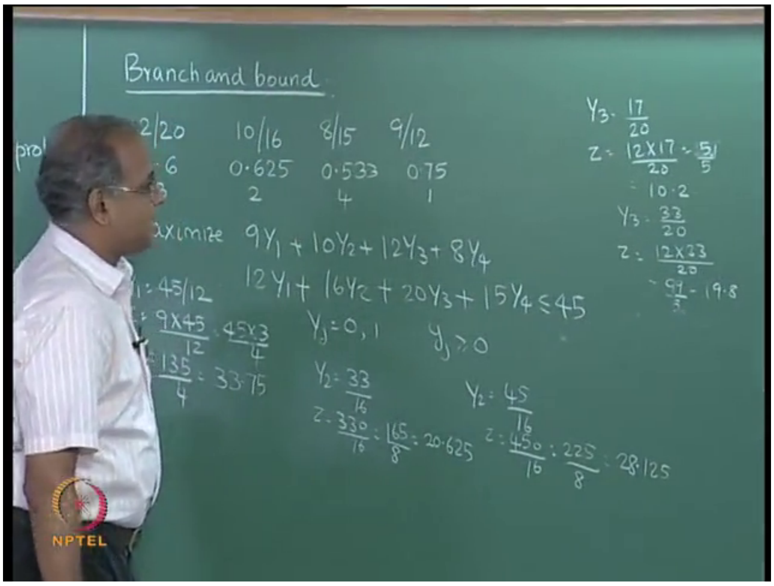

Now Y 1 equal to 0 would give us a bound now Y 1 is equal to 0, now Y 1 variable is out, but all the 45 resources are available. So, the solution will be Y 2 equal to 45 by 16 and Z is equal to 450 by 16 which is 225 by 8, 8 2 za 16 65 za 64 10 1, 28.125. So, this would give us Y 2 is equal to 45 by 16 and Z is equal to 28.125.

Now we branch from the node that has the higher value of this. So, now, Y 2 is equal to 1 and Y 2 is equal to 0 Y 2 equal to 1 Y 2 equal to 0. So, Y 1 equal to 1 Y 2 equal to 1 takes away 12 plus 16 28 units. So, 17 units are available here. So, Y 3 will be 17 by 20 Z is equal to 12 into 17 by 20 which is 68 by 5, 51 by 5 which is 10.2 plus 19, so you get

19.2, 29.2. The other one will now become Y 1 equal to 1 Y 2 equal to 0, so we are operating here Y 1 equal to 1 takes away 12. So, 33 units are remaining. So, Y 3 will become 33 by 20 and Z is equal to 12 into 2 33 by 20 which is 99 by 5 which is 19.8 plus Y 1 is 128.8 so this will become 28.8.

So, now, again Y 3 can take 0 or 1. So, Y 3 is equal to 1 Y 3 equal to 0. So, Y 1 equal to 1 Y 2 equal to 1 Y 3 equal to 1 is 12 plus 16 20 8 plus 20 48 is infeasible, is infeasible. Now 16 plus 12 20 8 plus 15 43 is feasible. So, this when Y 3 equal to 0 we come to the last variable which is Y 4, so Y 4 can take 0 or 1. So, Y 1 equal to 1 Y 2 equal to 1 takes away 20 8 units 17 units are available. So, that can give us Y 4 equal to 1 and Z is equal to 9 plus 10 19 plus 8 27 which is feasible, so 27 which is feasible to this problem,

Now, we have to go back to the other things and branch. So, when we branch from here we have we create 2 nodes which is Y 3 equal to 1 and Y 3 equal to 0. So, now, when Y 3 is equal to 1 Y 1 is 1, Y 3 is 1, Y 2 is 0, Y 1 is 1, by 3 is 1, Y 2 is 0. So, we have 12 plus 20 32 that is consumed, Y 3 is 1 30 2 is consumed sp automatically Y 4 will have to be 0. So, Y 4 will be 0. So, this will give a feasible solution Y 1 equal to 1, Y 3 equal to 1 with 12 plus 9 Z equal to 21 and since its less than 27 this is fathomed and this is not going to (Refer Time: 48:47).

The forth 1 is Y 1 equal to 1, Y 2 equal to 0, Y 3 equal to 0 will give us enough resource to do Y 4. So, Y 4 equal to 1 and this will give us Z equal to 17 which is again fathomed. So, right now we have a solution with Z equal to 27 which is our best solution, but the algorithm does not stop because we also have to branch from this side because from here the upper bound is 28.125 so we could get a solution with 28 because these are all integers and these are also integer values binary. So, the objective function value at the optimum will be an integer. So, we have to proceed from this because there is still the chance of getting a solution with 28 which we have to explore; right now the best value is 27.

Now, we will continue the branch and bound algorithm and try to get to the optimum solution in the next lecture.