Manufacturing Systems Management
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Lecture – 33
Theory of constraints, Product mix problem

In this lecture, we continue the discussion on synchronous manufacturing and theory of constraints.

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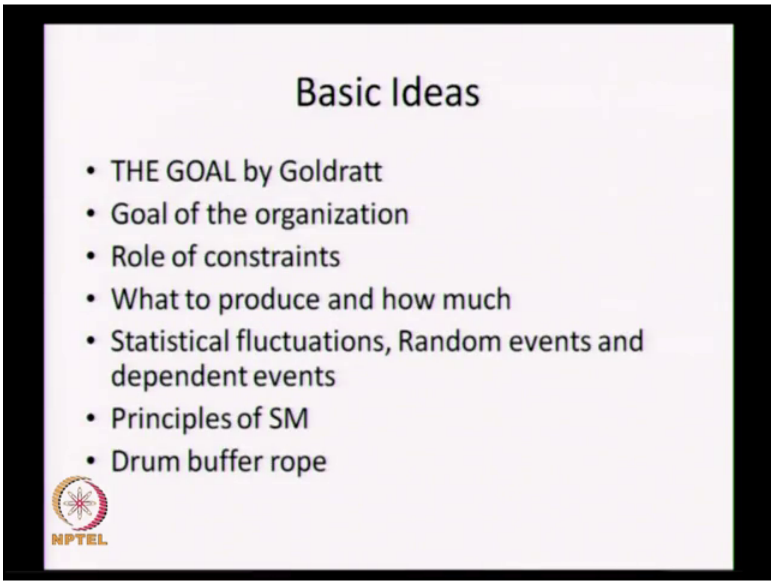

We started the discussion in the previous lecture, where we had an introduction to the book goal by Eliyhu Goldratt; we also discussed aspects such as goal of the organization role of constraints and so on.

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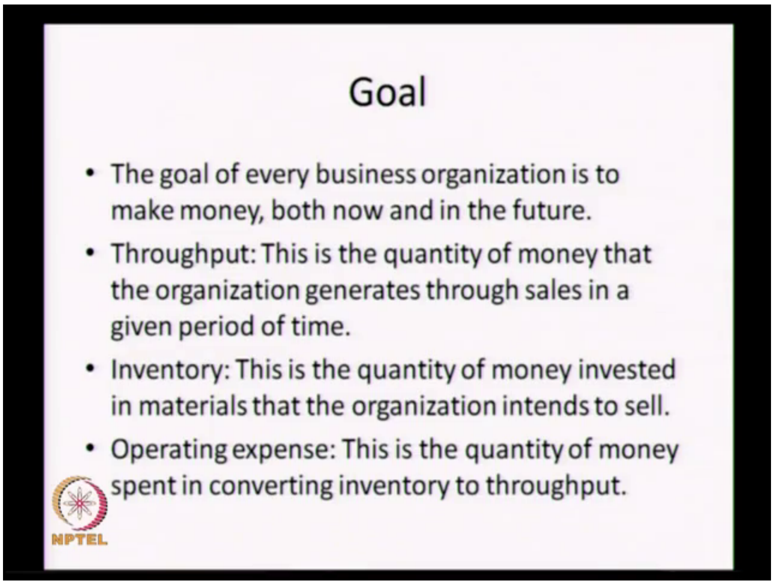

We also spoke about the goal of the organization as the effort to make money both now and in future and we also discussed 3 parameters called throughput inventory and operating expense and all the 3 of them are now defined in terms of money and also in terms of at what stage of the manufacturing, the money is either spent or received.

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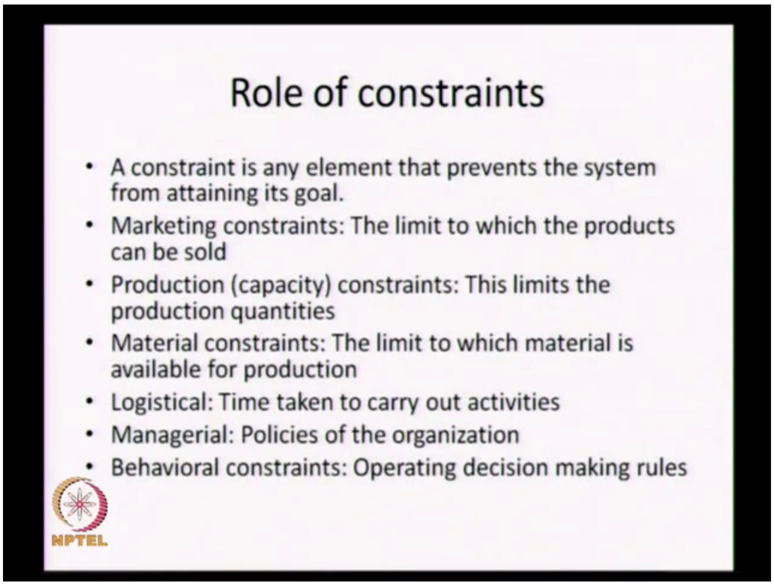

We also defined what the constraint is in the context of manufacturing and we also described several types of constraints that could happen.

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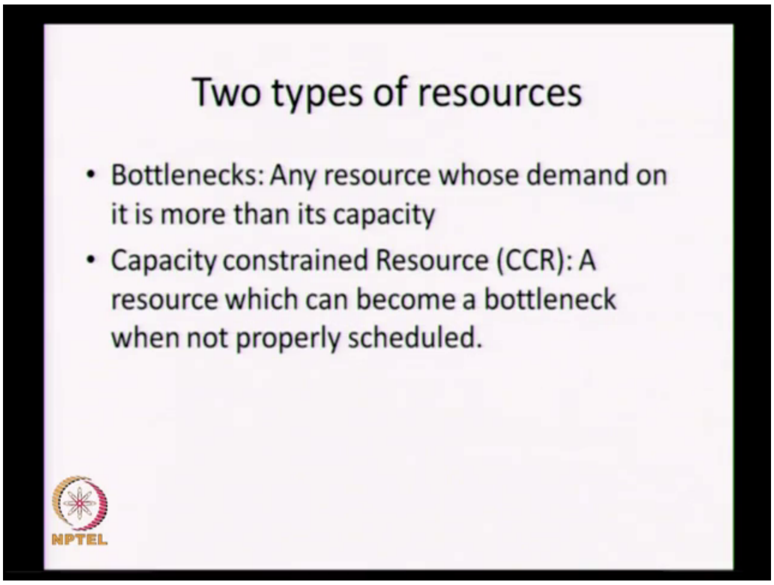

Now, we take an example, we also defined 2 types of resources which are bottlenecks and capacity constrained resource we also spoke about resources which are non bottlenecks.

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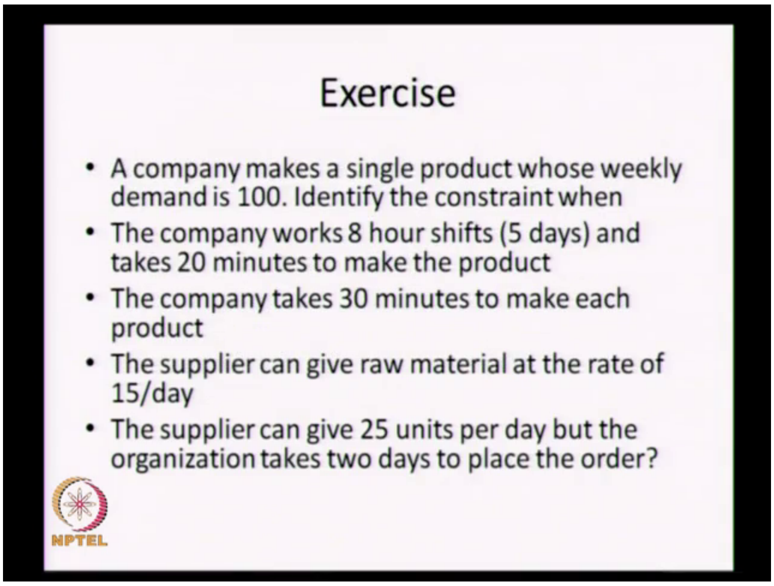

Now, we take an example to show that different constraints can affect the performance of the system. So, the example is about a company that makes a single product whose demand is 100.

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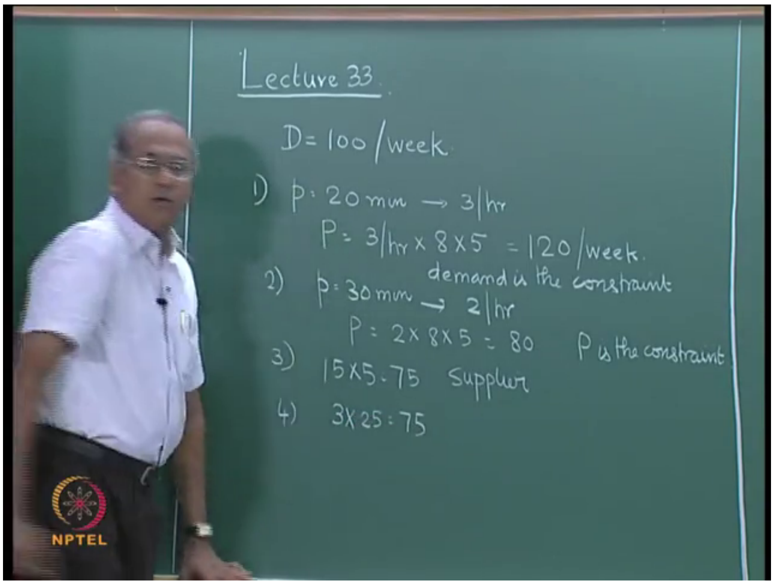

So, demand is 100 and we assume that this demand is 100 per week.

Because it is given that the weekly demand is 100. So, find out the constraint when the company works 8 hour shifts, 5 days and takes 20 minutes to make the product. So, number 1; production time is 20 minutes. In fact, the exercise talks about identify the constraint when there are 4 conditions given the company works 8 hour shifts for 5 days and takes 20 minutes to make the product 2 the company takes 30 minutes to make the product 3 supplier can give raw material at the rate of 15 per day and 4 supplier can give 25 units per day, but the organization takes 2 days to place the order. So, we look at the first one when the company works 8 hour shifts 5 days and takes 20 minutes to produce.

So, it can produce at the rate of 3 per hour and production capacity P will be 3 per hour into 8 hours per day into 5 days per week which would give us a production capacity of 120 units per week. Now the production capacity is 120 units per week, but the demand is hundred units per week. So, demand is the constraint. So, demand is the constraint which prevents us from getting or from making 120 per week and selling them in the second case the company; company takes 30 minutes to make each product.

So, P is equal to 30 which means it can make 2 per hour now capital P will be 2 per hour into 8 hours per day into 5 days a week which is 80. Now we have a situation where the demand is 100, but production capacity is only 80; therefore, production is the constraint in case 3 the supplier can give raw material at the rate of 15 per day. So, the material we can get is 15 into 5 which is 75 per week. So, supplier is the constraint and in the fourth case the supplier can give 25 units per day.

But the organization takes 2 days to place the order. So, we are going to assume that if order is placed on Monday, the supplies are going to come on Wednesday morning; therefore, to meet this hundred the orders are to be placed on Monday, Tuesday and Wednesday. So, that items are received on Wednesday, Thursday and Friday to produce. So, only 3 days we will get the items. So, 3 into 25 we will get 75. Now the supplier is not the constraint because the supplier can actually give 25 units per day, but the policies now become the constraint now where is the solution to each one of them. So, in the first case demand is the constraint demand is hundred per week production is 120 per week. So, we have to try and break the constraint which means we have to try to increase the demand and go up to 120 which is the production capacity.

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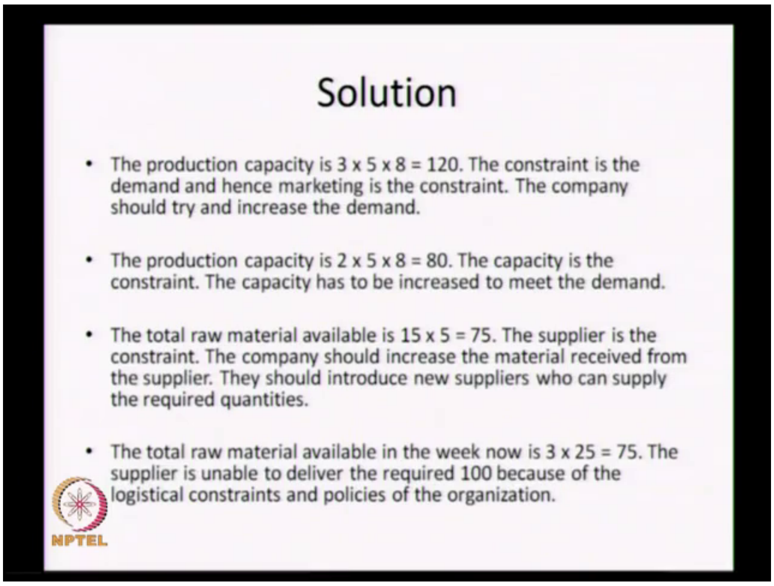

In the second case, production is the constraint it can make 80 where as demand is 100. Now we have to break this constraint increase the production and try to increase the production from 80 to 100 in the third case the supplier is the constraint because the supplier can give only 15 per day. So, we have to break this constraint by either getting these the supplier to supply more or to try and get alternate suppliers who can provide this item at the right quality in the fourth case the organizational policy is the constraint. So, if it takes 2 days then the organization should place order sufficiently early. So, that item can arrive on all the 5 days or 4 days as the case may be because the supplier can give 25 to meet the demand. So, in every case there are 2 aspects to it one is to identify the constraint and the other is to see what is to be done to break the constraint.

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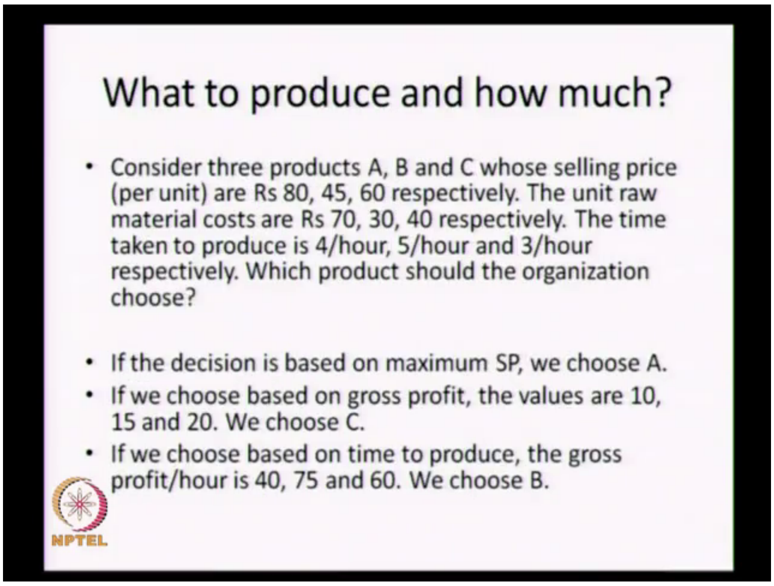

So, that the organization moves towards its goal now let us look at the next aspect which is what to produce and how much to produce.

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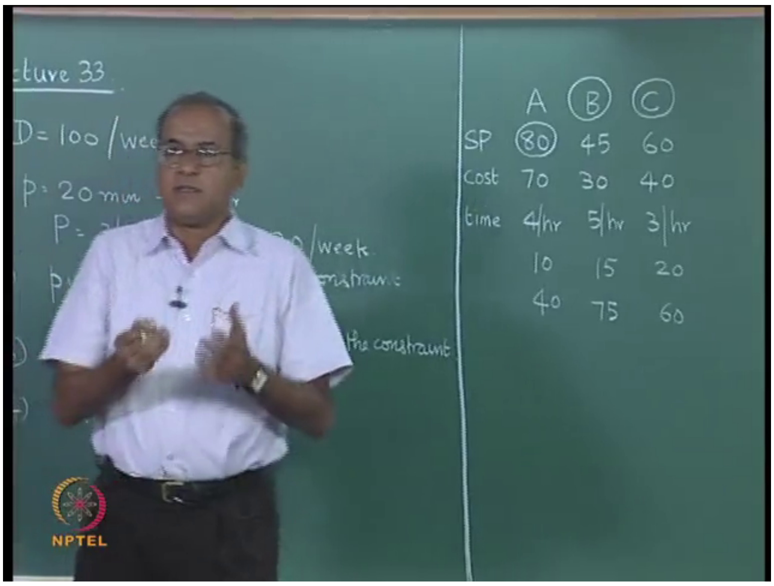

So, we do that with an example. So, consider 3 products A, B and C whose selling price are 80, 45 and 60; the raw material costs are 70, 30 and 40.

The time taken to produce 4 hours 4 per hour 5 per hour and 3 per hour which product should the organization choose and how much it should produce now there are several ways by which we can choose this actually the demands are not given in this particular example. So, if the decision is based on maximum selling price then we end up choosing item a because this gives us the maximum selling price and maximum revenue to the organization if we choose based on net profit or profit margin and where we assume that profit is equal to selling price minus cost then the profit associated with this is 10 with this its 15 and with this its 20.

So, we end up choosing C, if we try to maximize the profit if we choose based on time to produce now this can produce at 5 pieces per hour. So, gross profit per hour will be 15 into 5 because per unit we make a profit of 15 this can produce 5 per hour. So, this can give us 75 this can give us 40 per unit profit is 10, we can make 4 per hour. So, 40 and this gives us 60. So, we will end up producing B if we want to maximize the revenue that we can earn per hour.

So, what are the reasons for giving this example is to show that the item that we wish to produce will vary depending on the objective and it will vary depending on whether the organization is interested in total revenue or sale that they earn or whether it is the margin or it is the revenue that they can earn within a certain time period. So, just to bring about this idea we have given this example where depending on the criterion for measurement the product that the organization wishes to produce will change now let us look at another example to try and understand how much we have to produce and which product to produce.

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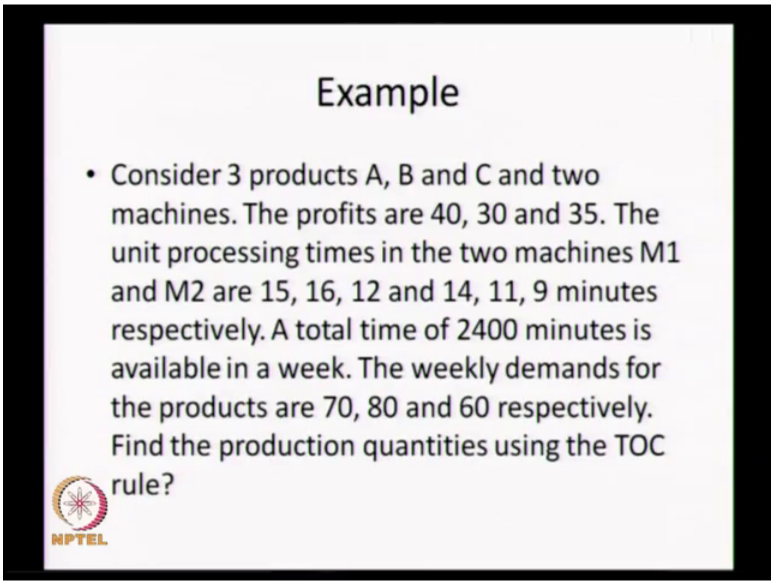

Now, in this example we consider 3 products.

That are A, B and C the profits are 40, 30 and 35; the unit processing time in 2 machines.

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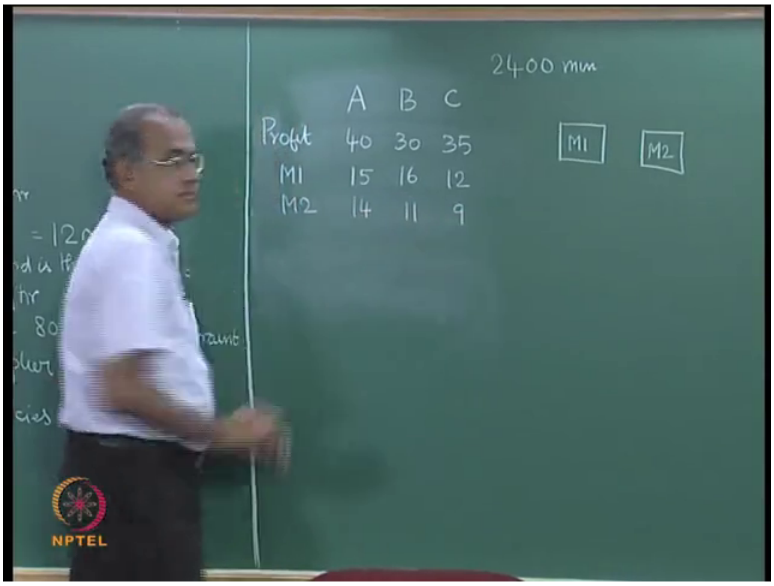

So, here we are going to assume that there is an M 1 and there is a M 2 all the 3 products go through the 2 machines. So, time required on the machines are given 15, 16 and 12 on M 1 it is 15, 16 and 12 and on M 2 the processing times are 14, 11 and 9.

Total time of 2400 minutes is available the weekly demand are 70, 80 and 60.

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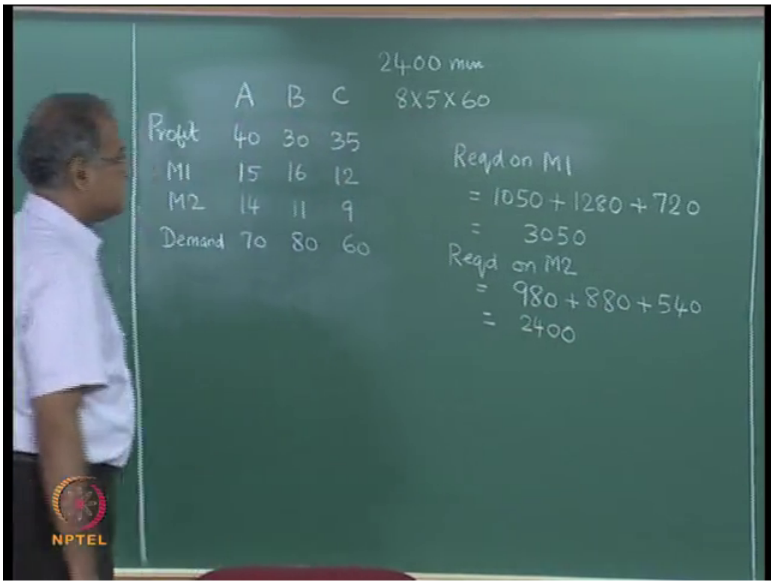

Now, this 2400 minutes comes out of 8 hours a day into 5 days of week into 60 minutes gives us2400 minutes; now ideally we would like to produce all 70 of A, all 80 of B and all 60 of C and try to make as much sales as possible or as much profit as possible. So, if we are able to produce all 70 of A, 80 of B and 60 of C, then our net profit would become 70 into 40 plus 80 into 30 plus 60 into 35.

So, at the moment the demand is actually the constraint where we can sell only 70 of A, 80 of B and 60 of C, but then we have to check whether we have enough capacity to make 70 of A, 80 of B or and 60 of C to do that we check capacity time required on M 1 is equal to demand into 15 70 into 15 is 1050 plus 16 into 80; 16 8 s are 128; 1280 plus 12 into 60 is 720.

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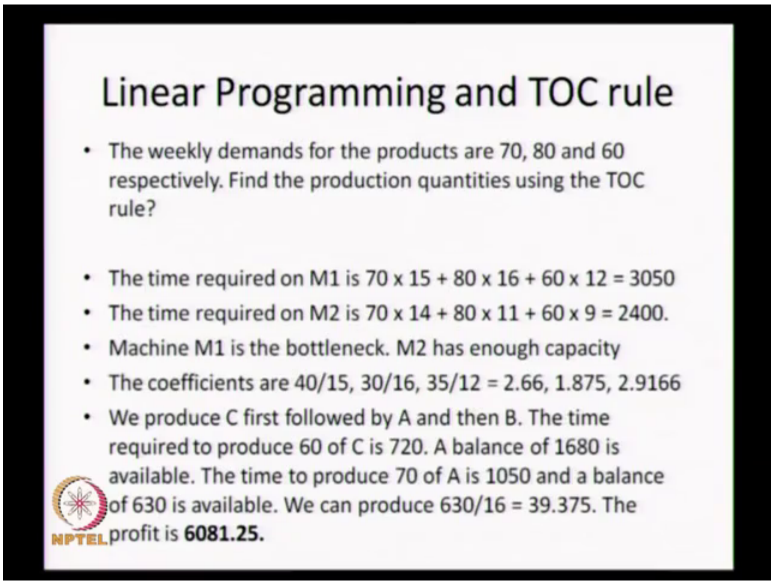

So, this would give us requirement of 3050 time units on the first machine requirement on M 2 time required on M 2 is equal to 70 into 14; 980 plus 11 into 80; 880 plus 90 into 65; 42400 is the requirement; now if you look at both the machines; this machine is available for 2400 minutes the requirement is 3050. So, the demand on the machine is more than its capacity.

So, this machine becomes a bottleneck machine now demand on this machine is 2400; available capacity is 2400. So, demand does not exceed the available capacity. So, at the moment it is not a bottleneck. So, we have enough capacity to meet the entire demand. So, when we have a situation where one of the machines now becomes a bottleneck then we will not be able to produce this 70, 80, 60 because we do not have enough capacity on M 1. So, the bottleneck machine will now be the constraint which would prevent us from making 70, 80 and 60. So, how much to produce is the next question that we have to answer and which one to produce.

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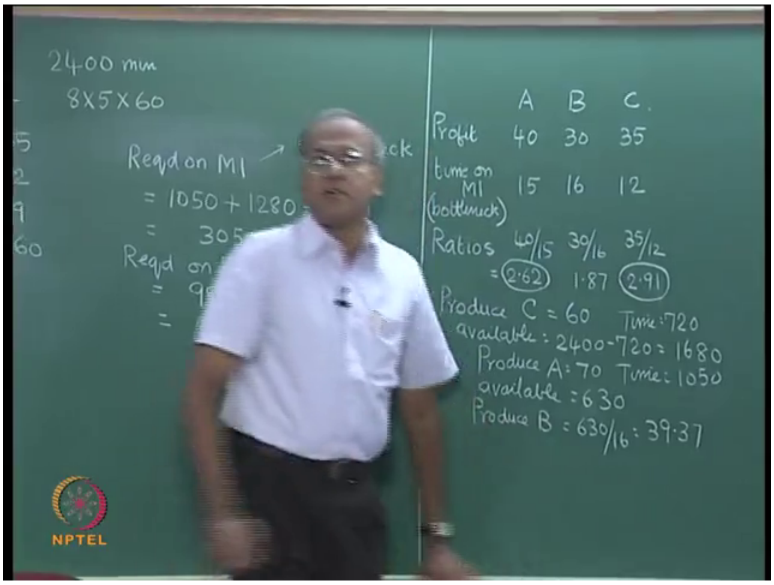

So, in order to do that we go back and write A, B and C; again the 3 products here the profits are 40, 30 and 35; the time on bottleneck is equal to 15 16 and 12 now the ratios 40 by 15; 30 by 16 and 35 by 12. So, this is 2s are 30, 100s are 96, 42s are 30 and so on; 30 by 16 ones are 38s are 128; 127s are 112 and so on and 35 by 12 is 2s are 2400 and 10 nines are 108 0 and so on 11. So, 9s are 108; 120 so 1 and so on. So, 2.91 and so on. So, these are the ratios.

Now it simply says that by consuming 15 time units and making one product the ratio of the profit by time to produce is this the one with the maximum ratio is the one that we will have to produce first. So, amongst the ratios 2.91 is the highest. So, first produce see produce C; now produce C now produce C and the 3 quantities are 70, 80 and 60. So, produce as much of C as possible 60 of C and check whether we have enough time to produce. So, 60 of C is going to take 60 into 12; 7020.

So, time available is equal to 2400 minus 720 which is 1680; now take the next one which has the highest ratio which is A and C whether we can produce demand of A is 70 time on the bottleneck is 15. So, time required is 1050 which is less than 1680. So, produce A equal to 70. So, time equal to 1050. So, time available is equal to 1680 minus 1050 is 630. Now produce B; now B is demand is 80 time required is 16. So, total time required is 16 into 80 which is 1280 which is more than this. So, we will be able to produce only 630 divided by 16; 16 3s are 48; 150, 16 9s are 144, 60, 16 3s are 48, 120, 16 7s are 112.

So, we will be able to produce 39.37 or 39.375 units of B.

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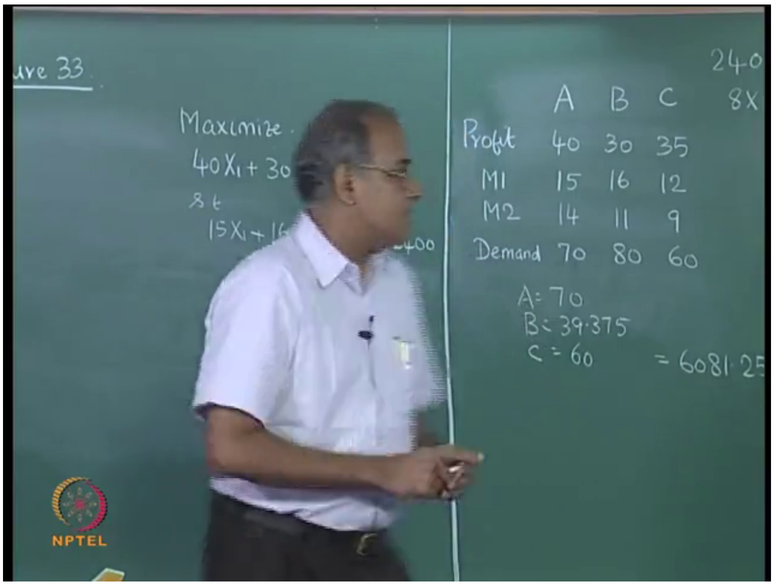

So, finally, we produce A equal to 70, B equal to 39.375 and C is equal to 60 with profit equal to 70 into 40 plus 39.375 into 30 plus 60 into 35 which would give us a total profit of 6081.25. So, this answers our question on which products to produce and in what quantities to produce particularly, when the weekly demands are given the profits are given and the processing times are also given. Now this rule by which we picked; we identified the bottleneck machine and then we computed the ratios and we sorted the ratios in decreasing order or non increasing order and then we started taking the products based on highest ratio now this rule is called the TOC rule or theory of constraints rule the literature calls this as TOC rule which is attributed to the literature on synchronous manufacturing and theory of constraints. Now the TOC rule will give the optimum solution or the best solution when there is only one bottleneck now there are 2 machines in this case one of them happens to be the bottleneck the other is not. So, in a situation where we have only one bottleneck the TOC rule will give us the optimum solution now the reason why the TOC rule will give the optimum solution is as follows if we actually formulate this problem as a linear programming problem or an integer programming problem then our problem would be to maximize.

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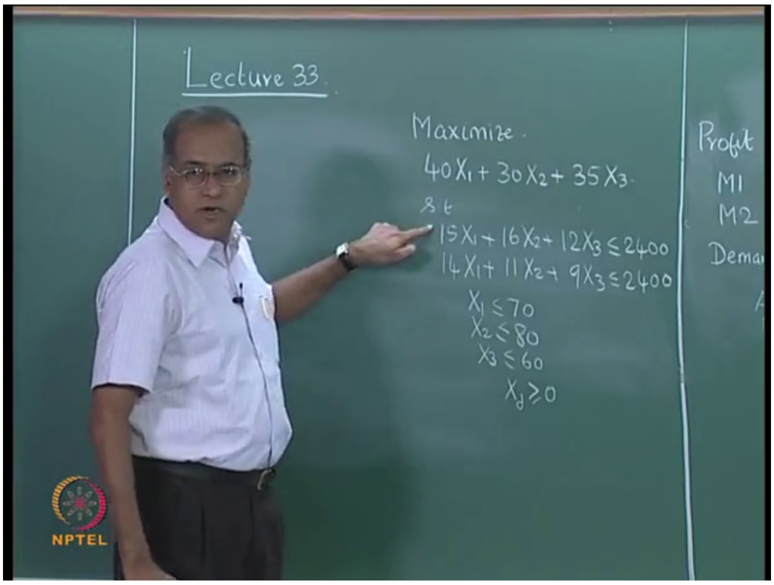

40 X 1 plus 30 X 2 plus 35 X 3 subject to 15 X 1 plus 16 X 2 plus 12 X 3 is less than or equal to 2400 14 X 1 plus 11 X 2 plus 9 X 3 is less than or equal to 2400 X 1 less than or equal to 70 X 2 less than or equal to 80 and X 3 less than or equal to 60 X j greater than or equal to 0.

Now for the moment let us assume that we are solving a linear programming problem we are not restricting the variables to be integers now what we have tried to do there is we initially computed the left hand side requirements at 70, 80 and 60 and we found out that this exceeded 2400, this is exactly equal to 2400. So, what we did was we took these 3 constraints and then we observed that this actually is redundant if we consider these 3 constraints and because it is redundant because for 70, 80 and 60 the left hand side is less than that one way to say is only based on this machine we can make 70, 80 and 60 which is what we computed.

And said that this is therefore, a non bottleneck now the other thing to do is we have found out that this constraint is redundant and can be taken away from the problem and will not affect the optimum solution. So, let us take away this constraint from this problem because we have computed that it is redundant and it will not affect the optimum solution. So, it now becomes a single constrained LP with bounds on the variables if these bounds did not exist then the optimum solution will be for one variable because there is only one constraint and that variable is the one that has the largest ratio now this is a result from linear programming which is very well known. So, if we did not have bounds on the 3 variables then we would have said make only X 3 because it has the highest ratio we have effectively computed the same 3 ratios here.

So, we said make X 3 that is only variable, but then this constraint is going to come into play and limit your X 3 to 60 which is what we did there then we look at the next one the next ratio which means it is like saying after we finish producing X 3; I am trying to solve 40 X 1 plus 30 X 2 subject to 15 X 1 plus 16 X 2 less than or equal to 1680, then the ratios are only for these 2; now this has a higher ratio. So, make X 1; X one is limited by 70 and X 2 automatically gets the other. So, essentially we are solving a single constrained LP with bounds using the TOC rule which is the well known rule from linear program therefore, TOC rule will give you the optimum solution in the case of only one bottleneck.

Or in the case of a single bottleneck now let us look at the same problem with some slight change on the parameters.

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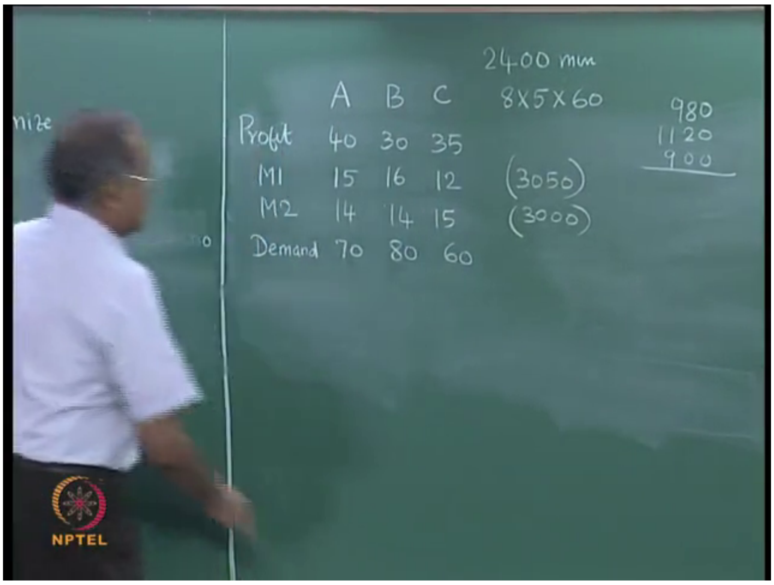

So, 3 products A, B, C; 40, 30 and 35, 15, 16 and 12 and the second one is 14; 14 and 15, 14, 14 and 15. So, now, let us look at the respective demands are 70, 80 and 60 which is also given here. So, we will now say time on M 1.

So, time on M 1 is 70 into 15 plus 16 into 80 plus 12 into 60. So, time on M 1 is 3050. So, total time on M 1 is 3050 which is the same that we calculated in the previous example, but time on M 2 is 14 into 7 is 980; 14 into 8 is 1120; 15 into 6 is 900. So, the time on M 2 is now 0003. So, the time is 3000; now both these machines are now bottleneck machines.

Actually TOC should not be applied in this case because TOC will work only when there is one bottleneck machine, but if we try to apply TOC to this situation if we apply TOC to this situation we will take M 1 because it is a higher bottleneck compared to M 2.

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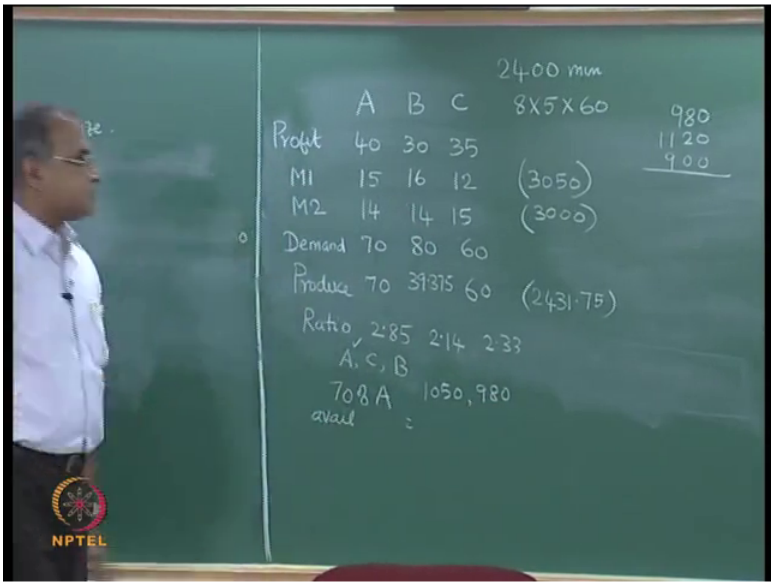

So, if we applied TOC rule based on M 1 we will get the same solution which says produce 70 of A 60 of C and 39.375 of B based only on M 1, but now for this solution we have to check whether do we have capacity to produce this now the capacity requirement is not 2400.

It is 70 into 14 plus 39.375 into 14 plus 60 in to 15 which becomes 2431.75. So, even after solving the problem with M 1 as bottleneck and applying TOC rule the resultant solution for the resultant solution M 2 continues to be a bottleneck. So, M 2 is requirement is not 2400 or less, but it is more than 2400. Now for this solution M 1s requirement is 2400, but M 2s requirement is 2431.75; now we can do TOC rule on M 2. So, if we do TOC rule on M 2 we will see what happens.

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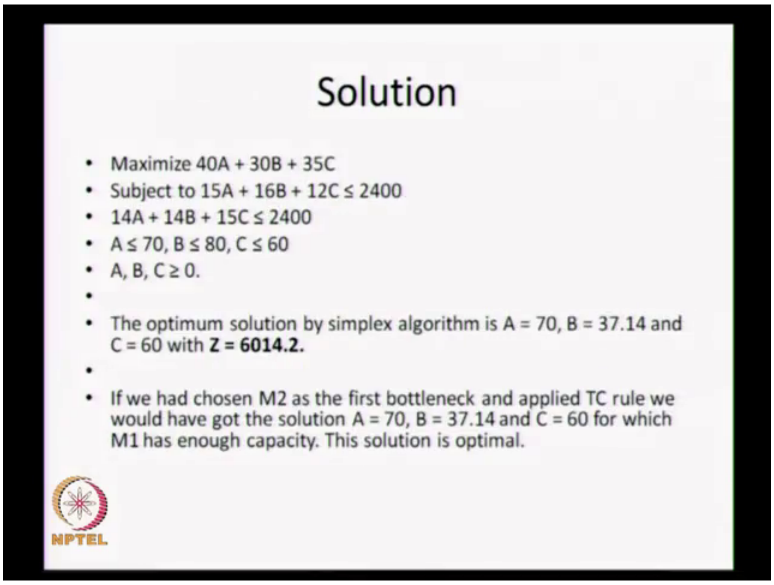

So, if we do TOC rule on M 2 we will get a solution which is like this.

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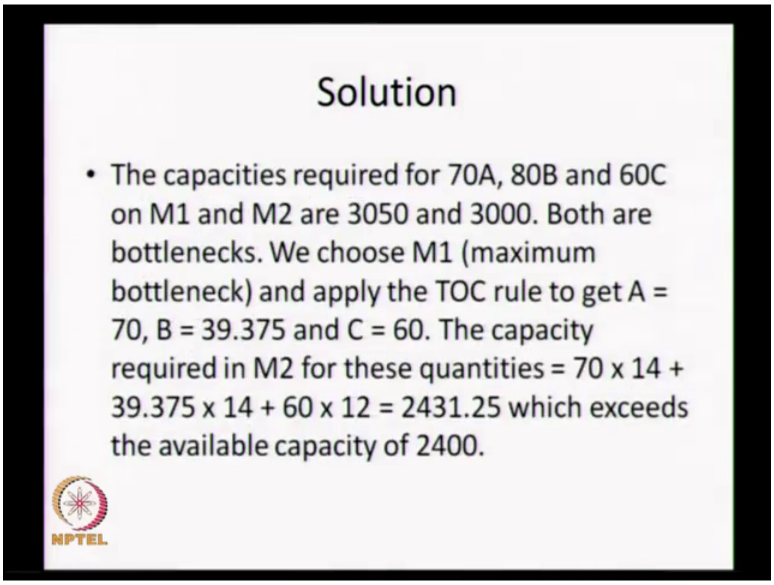

So, ratio based on M 2 is 40 by 14; 14 2s are 28; 12, 120. So, 14 8s are 112, 80, 14, 5s are 70.

Now, 16 by 14 sorry 30 by 14; 14 2s are 20, 80, 20, 14, 1s are 14, 60, 14, 4s are 56; 35 by 15; 2s are 30; 15 3s are 45 2.33.

So, we will produce a first, then C, then B, A, C and B. So, when we produce a first based on this ratio. So, we first produce A. So, A is requirement will be 70. So, produce 70 of A, now we could go back and say that the demand on both the machines. So, demand on M 1 is 15 into 70; 1050 and demand on B is 980. So, we will say availability.

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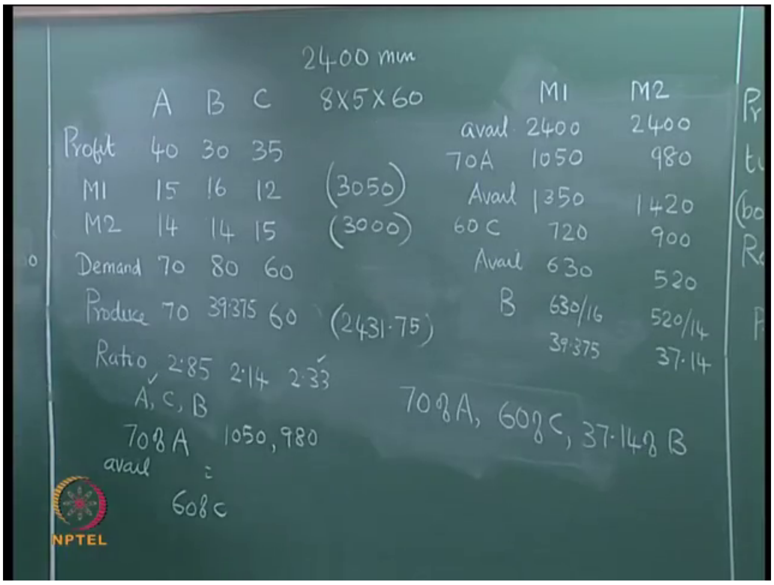

So, both machines M 1 and M 2, we are producing 70 of A. So, initial availability is 2400; 2400; 70 of 1050; 80. So, this is availability this is A; this is again availability 53; 1350; 24; 1420. So, now, we produce C which is C. So, C we make 60; C equal to 60.

So, I will say 70 A here and say 60 C here. So, 60 C on M 1 is 720; 60 C on M 2 is 900. So, again availability 36; 30, 25, 20. So, now, we look at B. So, B we start producing last. So, now, we have to go back and say this would allow 630 by 16 which is 39.375 and 520 by 12; 520 by 14 because B on this.

So, this will allow B 630 by 16 and 520 by 14; this is 39.375 14; 3s are 40; 214; 7s are 98 20. So, 14 ones are 14m 60, 37.14. So, we end up producing finally, 70 of A, 60 of C and 37.14 of B.

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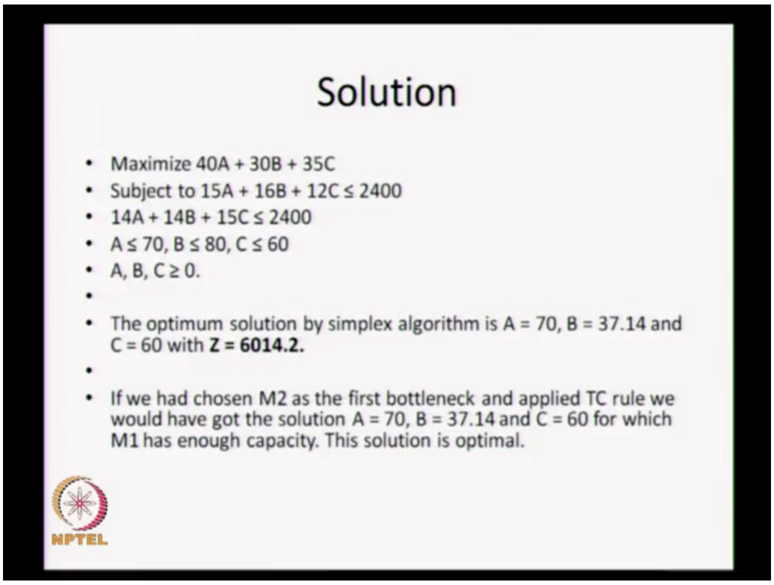

Which also happens to be the optimum solution if we solve this particular problem now the only reason we applied TOC consistently in this example is by when we were actually working with the ratios for B; we also took into consideration the effect of resource utilization of the first machine there are 2 machines when we applied TOC on the first machine, we got a solution which was which required more which required 2431; on the second machine then we looked at profits on the second machine and then we realize that when we do this and apply it simultaneously on both A and B, then we get a solution which is like this.

Now, in this particular example it turns out that this kind of an implementation actually gives the optimum solution because the corresponding linear programming problem is given in the slide. So, it will be maximized 40 A plus 30 B plus 35 C; where A, B and C here represent the number of units of A, B and C produced. So, the linear programming solution would give us 70 of A, 37.14 of B and 60 of C with 6014.02 as the optimum value interestingly; if we are chosen M 2 as the first bottleneck which we did now as a second bottleneck if we are chosen M 2 we would have got this solution which requires 2400.

Time units on the second machine M 2 and this if you see carefully 39.375 has come down to 37.14. So, it will require less than 2400 on M 1. So, if we add earlier we started by choosing M 1 as the most bottleneck and we proceeded we applied TOC rule and we got a solution, then we found out that it requires more time in M 2, then we went back and did it on M 2; we got we satisfied M 1 and therefore, we froze it to be the best solution. So, in this case if we had started with M 2 first instead of M 1 the solution obtained with M 2 as bottleneck would have satisfied M 1 still there will be instances where this approach need not give us the optimum solution.

The reason is this approach now is sequentially trying to produce A, B and C because the choice of the order of A, B and C will now depend on the bottleneck machine which is either M 1 or M 2 in this case both. So, either way we are pre determining the order A, B and C in some form based on some ratios and then taking them and then going back to the linear programming formulation and trying to reduce the right hand side values as and when we produce the simplex algorithm does not work that way to solve the linear programming problem. So, while this approach gave us the optimum solution it need not give us the give us the optimum solution in all the cases. So, we have to be careful about either of these.

One it is not that the TOC rule is incorrect the TOC rule is correct, but the scope of applicability of the TOC rule is when there is only one bottleneck we should not apply the TOC rule to a situation where there are 2 bottlenecks and try to modify it by saying in the event of multiple bottlenecks we will use it on the most bottleneck first that rule does not work the TOC rule as it is when there is only one bottleneck the TOC rule will work and give the optimum solution; now researchers understood this also understood that at some point, it can lead to the optimum solution as in this case, but then also provided ways to get the optimum solution when the TOC rule applied on multiple bottlenecks did not yield the optimum.

So, the results had to be fine tuned to come to the optimum and after a while people realized that the effort was actually equivalent to solving the dual of this linear programming problem and therefore, it is better to actually solve the primal optimally using the simplex algorithm rather than using the TOC rule. So, at the end what I am going to say is that if there is only one bottleneck then apply the TOC rule because it is the same as solving a single constraint LP with bounds if you have multiple bottlenecks do not apply the TOC rule solve the corresponding linear programming problem to get to the optimum solution. Now let us look at one more aspect of synchronous manufacturing which is extremely important to understand how manufacturing systems work.

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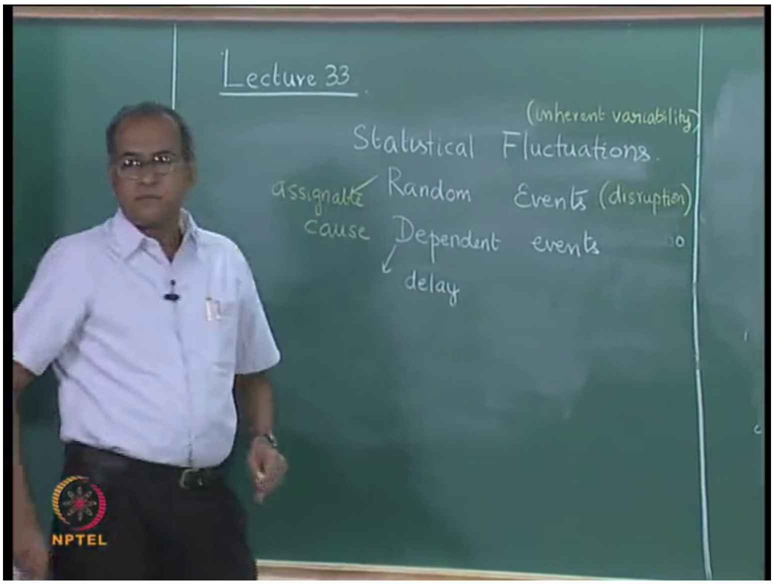

So, we are going to look at 3 things which are called statistical fluctuations random events and dependent events and we were also going to see the relationship among these 3 parameters now statistical fluctuations represent what is called inherent variability inherent variability in the system.

Now, the system has every system has a certain randomness associated with it for example, if it takes 30 minutes to come from your place of stay to the place of work and assuming that a person is cycling every day go coming by bicycle every day and let us assume that we do not have any signals or anything in between it is just about 30 minutes of cycle ride from place of stay to place of work if we actually clock every day, then there will be some days when the person would take 30 plus a few minutes, there will be some days when it will be 30 minus a few minutes and if we actually average them out it will come to 30 which is the expected value, now this fluctuation is called statistical fluctuation. So, this represents an inherent variability in the system.

There will be positive deviations there will be negative deviations, but over a period of time the positive and negative deviations will cancel out each other the variation will be very small, but the mean error will be 0 or very very close to 0. So, this is called statistical fluctuations this is this can be looked upon in the context of manufacturing where it if it takes 30 minutes to do something and an operator is doing it continuously sometimes the operator may take 32 minutes sometimes the operator can finish it in 29 minutes or 27 minutes and so on, but the average will come to 30; now random event is essentially a disruption which actually affects the performance or output of the system.

Now if we go back to the simple example of a person coming by bicycle from place of state to place of work then on a particular day while coming halfway through this person could meet a friend or somebody and maybe you would talk for 5-10 minutes which would delay the 30 to 40 or there could be a day occasionally when the tire halfway through becomes flat and it has to be fixed and there is additional time in a similar manner when we actually look at a person working on a machine and it is supposed to take 30 minutes; now halfway through there is a problem and the machine stops working and then the 30 minutes becomes 40 minutes so many times a random event is has an assignable cause it has an assignable cause which essentially delays.

What is happening and many times, it happens at there at the most inappropriate moment to delay something that is happened even though I say assignable cause these are actually chance events on which we do not have any control. So, these are causes that are uncontrollable, but effectively result in a delay in the activity a dependent event essentially depends on the previous event; now when people work in a in a factory with items then there is a dependency because there will be times the job will come to M 2 after it has completed M 1. So, if a particular thing due to either a statistical fluctuation or a random event if 30 minutes becomes 35 minutes for a particular instance.

Now, in the first one; piece would take that if I; the next piece could take 29 or 28 and it can balance out eventually, but then the one which moves to the next one will definitely cause a delay because of the dependence of the events. So, dependent events also cause delay. So, when we have dependent events the statistical fluctuations do not cancel out each other; now we will see some more aspects of statistical fluctuations and dependent events in the next lecture.