Manufacturing Systems Management
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Lecture - 30
Models in JIT

In this lecture we study some modules in Just in Time Manufacturing System. So, these models in JIT essentially try and use principles of inventory control that we have seen in earlier context, in context such as in operations management. So, what we will do in this lecture is will look at some models or some issues in JIT, and then link it up with models that are available in the operations management area and show how these models are useful, in the context of JIT or how they can be modified in the context of JIT. So, first let us look at traditional inventory control and economic order quantity that we would have seen.

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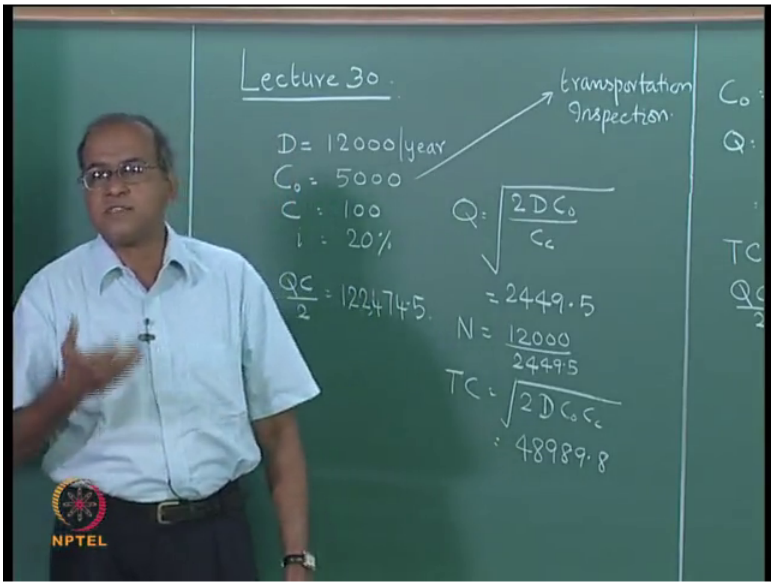

So, we take a simple example where there is a single item d. So, annual demand for the year is 12000, now order cost C naught is taken as 5000 the unit cost is 100. So, C is 100 and I is 20 percent of 100.

Now, when we apply the economic order quantity to try and find out what should be the ordering quantity from the supplier if we use this data we use the standard formula,q is equal to root of 2 D C naught by C c, where C c is the inventory holding cost, this is also equal to I into C . So, we get 2 into 12000 into 5000 divided by 20 which would give us a value of 2449.5 will be the economic order quantity. So, the number of orders per year will be 12000 divided by 249.5. So, which is roughly about 5 orders per year which also means that every order is made an order is made every 2.4 months or we have something like about 75 days of inventory which is there in every order.

So, we are not ordering daily. So, the inventory is very high in this system. So, the total cost that we are familiar with, which is sum of the order costs and the carrying costs is given by root over 2 D C naught C c and this on computation is 48000 989.8 so 49.8. So, this much of money is spent on order cost as well as carrying costs. Now the amount of money that is locked up in average inventory is QC by 2 which is 2449.5 into 100 divided by 2 which is 122474.5.

So, roughly the amount of money that spent in inventory is about 122475 rupees in terms of the average invent. So, this is how simple economic order quantity formula works, and if we are looking at an item that is being bought out from a supplier. And if we are going to make decisions based on the economic order quantity and these data, we observe that the order for about 2500 items at a time, we order roughly about 5 times a year, roughly about once in 75 days, the total money spent on order cost and carrying cost is about 48000 49000 rupees and about 122000 rupees worth of inventory is there on an average.

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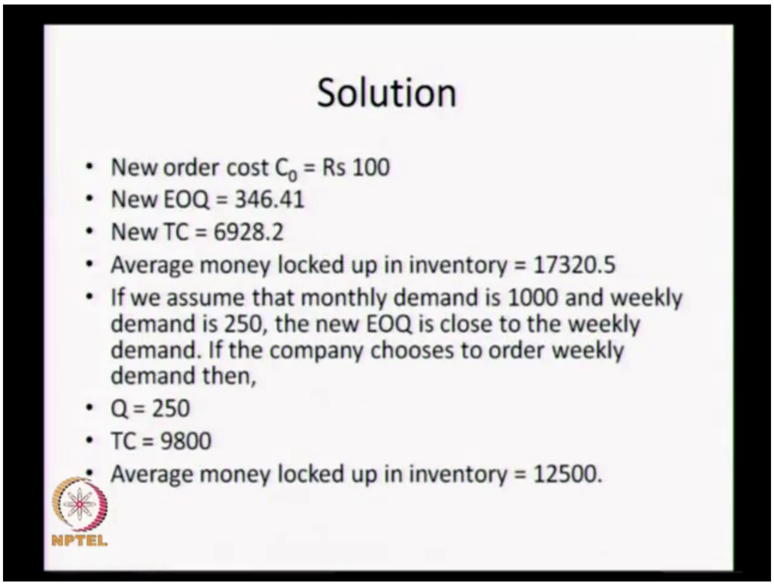

Now, let us do the same calculation by assuming that the new order cost is rupees 100. So, let us say C naught is equal to 100 in this case, instead of 5000 the order cost comes down to 100.

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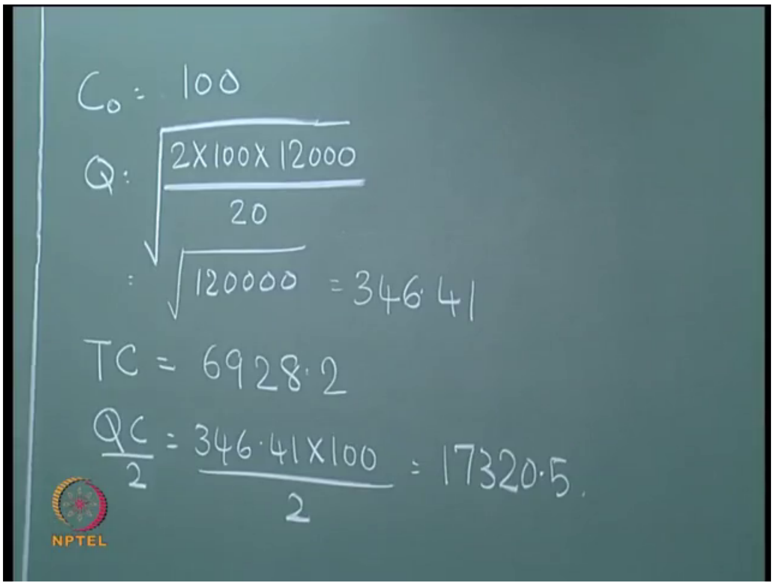

So, the economic order quantity becomes root of 2 into 100 into 12000 by C c which is 20. So, this is root of 120000, because this will go 5 times so into 10. So, this gives us 346.41. So, the order quantity comes down from 5000 from 2500 to 346.If the order cost comes down. Now the total cost that is spent in a year on inventory and order is 6928.2 and the amount locked up in inventory QC by 2 is 346.41 into 100 by 2 which becomes 17320.5. So, all that we have done is by reducing this 5000 to 100 and we see all these things are possible. Now purely from a computational point of view we have not done anything significant all that we did is just changed 1 parameter from 5000 to 100.

And then we see a very drastic change in everything. It is only common that if this comes down C naught comes down. So, root over comes down and Q will automatically have to come down. So, 2500 will become 346.Now when Q becomes 346, and if our annual demand is 12000. So, we are going to place nearly about 30 orders or so. So, we are going to place an order every 10 days or something like that because there are there are going to be 30 orders in a year and each order is roughly for about 346. So, the order cost component is going to be very high. So, roughly about 30 orders we will have, but the order cost has reduced. So, 30 into 10 is of the order of 3464. So, the inventory cost will be another 3464 giving a total cost of 6900. So, everything reduces if C naught reduces, to put it differently if we want all of these to reduce and be you know in a manner where we can control the only thing we have to do is to reduce this C naught.

Now, let us look at a third scenario now let us look at let the monthly demand be the annual demand is 12000. So, there are 12 months a year. So, the monthly demand is 1000 and the weekly demand is 250. So, let us say weekly demand is 250 per week.

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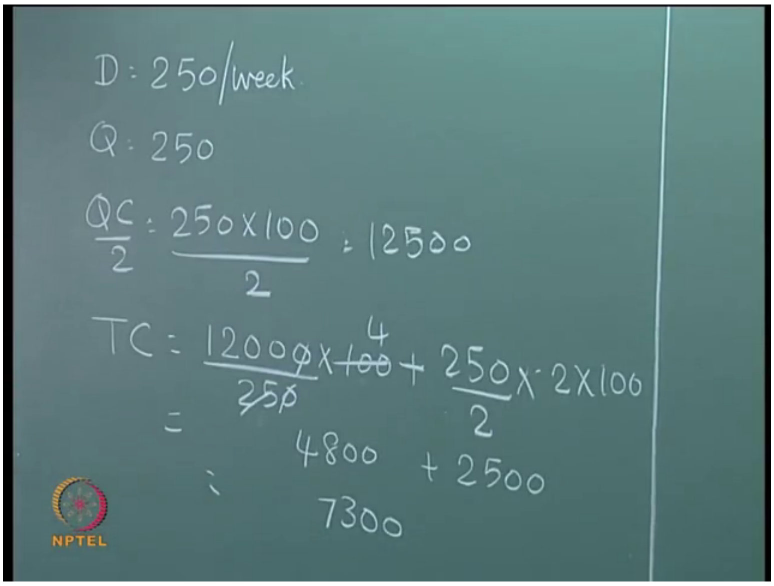

So, weekly demand is demand is 250 per week .Now if we follow a policy of ordering only once a week and ordering a weekly demand and asking the supplier to provide once a week a certain quantity, then if the order then if Q is equal to 250, every time we order we order for 250, then the average inventory that we have will be QC by 2 which is 250 into 100 by 2 which is 12500, this 17320 will become 12500. The total cost that we will have TC will be D by Q into C naught plus Q by 2 into C c. So, D by Q is 12000 by 250 into 100 plus 250 by 2 into .2 into 100. So, this will become this is 2500, because this will go 10 times 10 2500, this will become 4. So, 4800 this will become 4800 plus D by Q into C naught. So, 4800 7300 will be the value as against 6928. So, 7300 will be the value D by Q into C naught 12000 by 250 into 100 C naught is 100 plus Q by 2 into C c.

So, 4800 plus 4800 plus 2500 will be 7300. So, 6928 will go up to 7300. So, if the if we are able to bring down this 5000 to 100 then the order quantity becomes 346, but instead of using this if we follow a consistent principle of 250 then this is the expenditure. So, the only thing we have to see is can we bring down this 5000 to 100 or how much can we bring down this 5000. So, that we are able to implement that now if we see very carefully if this 5000 has several components order cost has several components 2 important components are transportation and inspection. If we are able to bring the supplier physically close to the factory then the cost of transportation is going to reduce significantly

For example if the manufacturing company is located in Chennai and if the supplier is located up north in Jalandhar, then the cost of transportation will be very high, but if the supplier is located nearby and the supplier is also located in Chennai say then this cost can be reduced considerably. If the inspection inwards goods inspection if we follow a quality at source and if inwards goods inspection is left to the supplier and not to the organization this cost also comes to. So, 2 important aspects of JIT which is about few and nearby suppliers, network of suppliers quality at source can help us reduce this 5000. So, this can help us reduce this 5000 and if this is brought to a number which is reasonably close to this not saying 100, but reasonably close to this then something like this is possible.

So, even though we spend a little bit of extra amount of money here if you are able to order 250 per week which is the weekly requirement, then automatically the amount of money that is locked up in inventory comes down 12500 versus 122475, and the amount spent on order cost and carrying cost. So, the economic order quantity formula actually can also has relevance when we try and bring in these ideas into it. So, when we look at it purely from a JIT point of view we say that bring the supplier close. So, their transportation cost can be reduced give the quality of the inward goods inspection to the supplier .Now if we are able to capture these 2 things into the order cost and we are able to reduce the order cost, we will see that the economic order quantity solution is invariably as good as the solution that we will have when we implement these principles of JIT.

So, we will be able to have time voice production slots given. So, we will have a weekly delivery with the customer. So, all these things are enabled and now we see how traditional e o Q model can be made suitable in the context of just in time manufacturing system.

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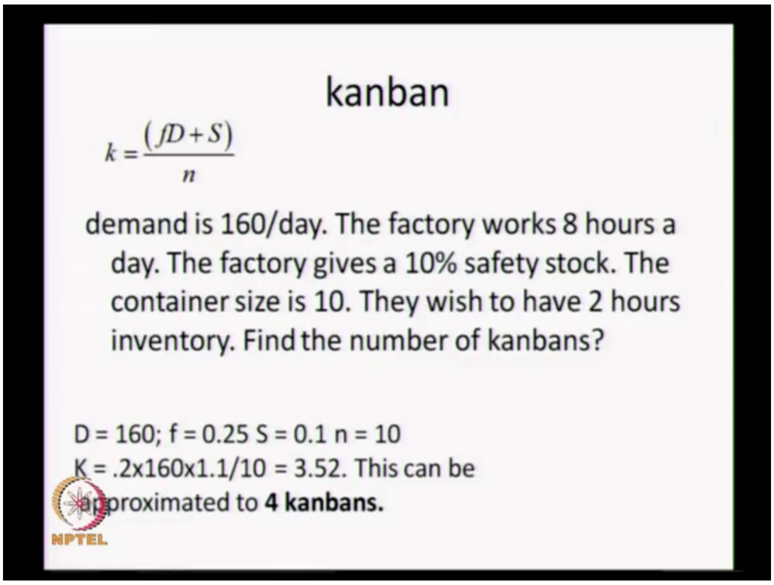

Now, let us look at the next thing which is the formula for kanban and see how we do that.

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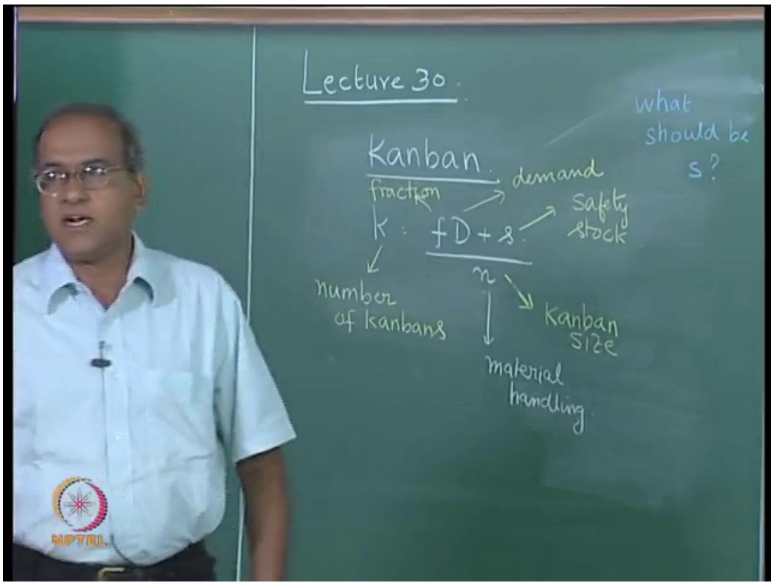

So, let us look at the kanban system. So, the formula that we have in for kanban is k is equal to f D plus s divided by n. So, let us explain the notation for each one of these. So, that we understand this now k is the number of kanbans, n is called the kanban size, s is the safety stock, D is the demand and f is some fraction of demand.

So, let me explain this. So, let us look at a situation where demand is 160 per day.

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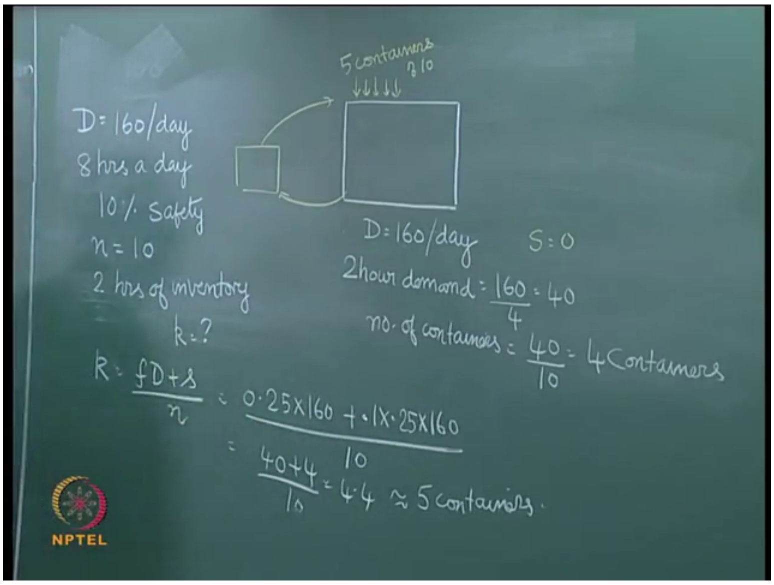

Demand is 160 per day, the factory works 8 hours a day, 10 percent safety stock container size is 10, I wish to have 2 hours of inventory find the number of kanbans. So, let us go back to what we discussed in a previous lecture. So, daily demand is D is equal to 160 per day. So, simple calculation is my demand is 160 per day I wish to have I work for 8 days a week and I wish to have 2 hour 8 hours a day and I wish to have 2 hours of inventory.

So, 8 hour demand is 160. So, 2 hour demand is 160 by 4 which is 40. So, 2 hour demand is 40 I have container size of 10. So, I will have 4 containers in the system. So, 8 hour demand is 160, 2 hour demand is 160 by 4 which is 40. So, I will have. So, 8 hour demand is 160, 2 hour demand is 160 by 2 hour demand is 160 by 4 which is 40. So, number of containers is equal to 40 by 10 which is equal to 4. So, if we actually substitute it in this formula k is equal to f D plus s by n. So, f becomes 2 by 8, which is 1 by 4, which is .25 into 160, f D is .25 into 160 plus s is 10 percent of this.

So, plus .1 into .25 into 160 divided by 10. So, this is 40 plus 4 by 10 is equal to 4.4 containers. So, we go back and correct this to 25 which will give us 4.4 and this will become 5 containers. Now 4.4 is now approximated to 5 containers, now this formula was a direct computation which involved s is equal to 0 there was no safety is stock given here. So, the division was directly 160 by 4 which is 40, but as mentioned earlier in an earlier lecture if I wish to have 2 hour inventory here then I wish to have 25 percent of the demand which is 40 in this system and then it is enough for me to release 4 kanbans.

But I realized that 4 kanbans will be very tight. So, I have to give a safety stock. So, that safety stock is now assumed as 10 percent. So, that we have 4.4 point 4 kanbans in the system and 4.4 becomes 5. So, there will be 5 containers that will be present in this system at any point in time there will be 5 containers of size 10. So, the inventory there will be 50 which is a little more than the 4.4 or 44 that we looked at. So, at any point in time there will be 50 units here. So, we could have a system where as soon as one container is over they will come and place it in the container area. So, there it can goes either to assembly or to some other place and then the card is replaced and some other container that comes here to work. So, there will be 5 containers of 10 each. So, if we do not have safety or if s is equal to 0, then we would have 4 containers we would have 4 containers

But then if we wish to have a safety of 10 percent then we get into 5 containers .Now if we have a safety of 20 percent we would still get 4.8 which is rounded off to 5 containers if we have safety of 25 percent it would still come to 5 containers, but if we have a safety of 30 percent then this will become 40 plus 30 percent of 40 which is 12, 52 by 10 which will go to 6 containers. So, the larger the safety that we have in this system the more containers we have. So, logically it leads us to several questions now what is S. So, how should we even though this formula is a very simple one and this is in some way the only formula to determine what should be S is the leading question what should be the safety stock, given S we can easily do this computation, but in a normal system how do we compute this S how do we say that a system requires 10 percent safety or 25 percent safety or whatever other things are all as I said other things are all known in the sense the container size is known. So, one could think in terms of

How we define the container size. So, that could be another problem how do we find the container size daily demands are given this change with every day the amount of inventory that I wish to have is also a parameter that we have. So, we could think in terms of 2 hour inventory or 1 hour inventory and. So, on whereas, if this said instead of 2 hour inventory this is 1 hour inventory then 1 hour inventory would mean 20. So, 20 plus 10 percent of 22 by 10 will give us 3 containers. So, there are 2 important questions, but the more important one is what should be the safety because that decides the number of kanbans in the system. So, more the safety more the kanbans little more the inventory in the system, tighter it gets less inventory in the system. So, this is usually calculated the kanban size is determined largely by the material handling systems

And the transportation systems suppose the individual himself or herself can carry these items are small and they can carry. So, it will depend on the weight and size of the item. So, that an individual can carry it if an individual is going to simply lift it and take it. So, we could have the size n depending on the size of the item the volume as well as the weight.

If we are going to have forklift trucks that are going to move them which means that the items are little heavy then the n will become slightly larger because it will be not so economical to transport very small quantities using forklift trucks and so on. So, n gets a little larger. So, depending on the weight and the size and the type of material handling equipment and transportation equipment that we use the n gets fixed now these 2 are daily decisions how do we fix S is the key issue that we address next .

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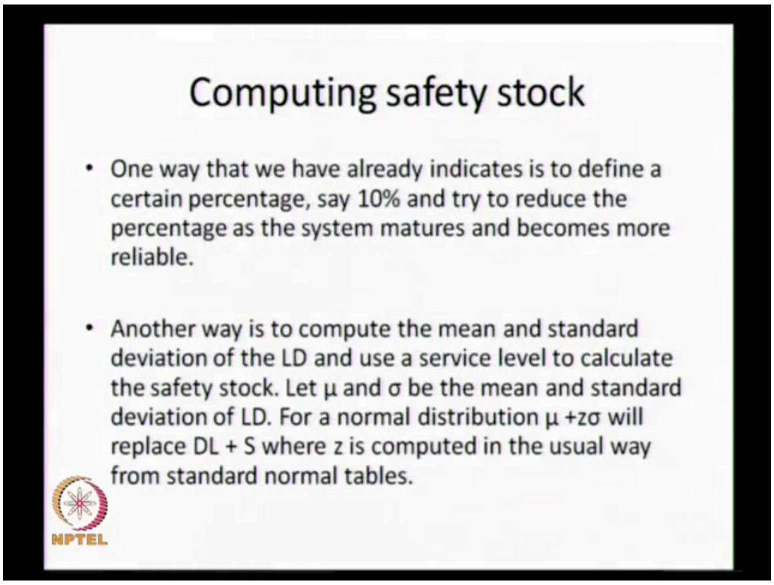

So, how do we compute the safety stock or how do we compute the S. So, computing the safety stock is the next issue that we will look at.

Computing the safety stock one way that we can do is to define a certain percentage which is right to begin with we would say that we could keep 25 percent safety stock one way that we have already indicated is to define a certain specific percentage that we see for example, 10 percent or for example, 25 percent. So, if we had put 25 percent as a safety stock f D by n in this case f D is 40 25 percent of 40 is another 10. So, safety is 10 this is 40, if safety is 25 percent then it becomes 50 divided by 10 is 5 containers. So, we will initially begin with a certain percentage say 10 percent or 25 percent, let the system mature in the sense that then we do this suppose we have given a 25 percent safety and we are letting 5 containers in the system.

Now, what are we trying to do here what we are trying to do is we are capable of doing 160 per day, a day is 8 hours. So, 160 units in 8 hours implies 20 units in 1 hour. So, each kanban of 10 is going to take half an hour.

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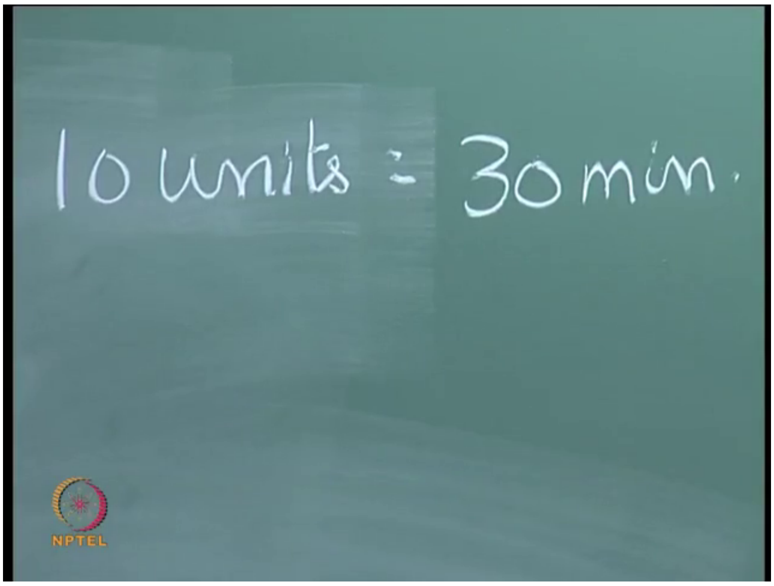

So, each kanban time taken to produce 10 units is equal to 30 minutes, time taken to produce 10 units is 30 minutes, 20 units is 60 minutes which is 1 hour and per day it is 160. So, what we do now is we released 5 containers to begin with at some point, these 5 can come from here or can come from here and. So, on now the system starts now each there are there are a set of machines here in this cell there is unidirectional flow let us say it ends here. So, what we do is we expect every container to come here at 30 minutes.

If they come in 30 minutes then they will go here and they are replenished and they will go back, now 2 or 3 things can happen it will not a need not be exactly 30 minutes you could have a situation where there is a small pause in one of these machines or one of the operators is working a little slowly or there is some correction that is needed one item gets into a reject and therefore, another item has to be brought in. So, all these little complications can happen and these things can come out at times 31 or 30 or 35 or 40 they can just come out at that kind of times now if we follow what is called a dual kanban system and then we say that this is transported here, but when this is transported something has to be replenished.

Now for something to be replenished that should be available it may have to come from some other cell, it may have to come from the supplier. So, there could be issues where this is not available when it is required to be replenished. So, even the replenishment may happen at 38 minutes or 40 minutes I may be able to give put this at 31 minutes, but then I do not have it.

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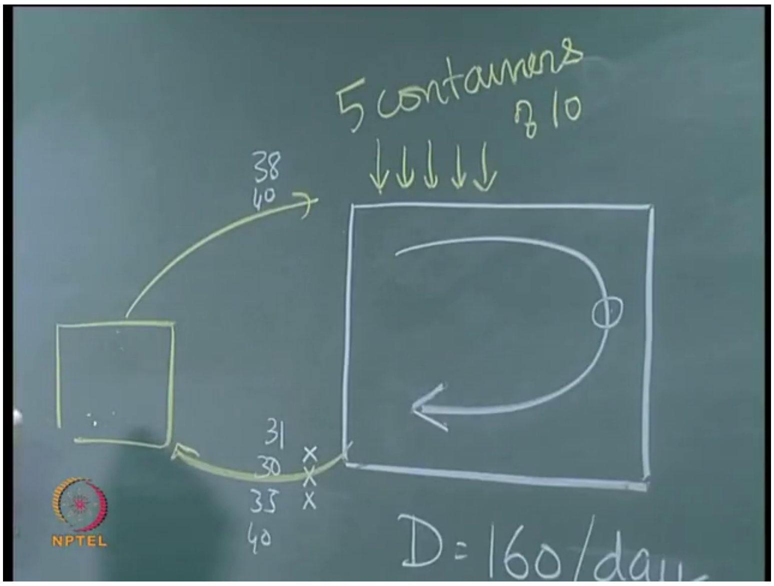

So, the time between 2 replenishments will now change to 38 minutes. So, what happens is the system is now enabled to do it exactly at 30. So, if the system is doing exactly at 30 minutes, then it is enough for me to do 4 containers if I wanted 2 hour inventory and not 5 containers, but because I am not able to do it every 30 minutes I need an extra buffer. So, initially I expect the system to behave in a in a not so tight manner in the sense all these variations can happen and if all these variations can are happening then I need an extra cushion in the system for me to run a 2 hour inventory. So, 4 will become 5, but as things get better and better as I implement preventive maintenance systems as I implement employee empowerment as I implement quality at source as I implement smaller changeovers.

So, I start implementing all of these together, but the effect is seen and now I am able to maintain or bring down this variation and bring it closer to this thirty then I realize that I do not have to work with 25 percent safety it is enough to work with 10 percent safety or it is enough to work with 5 percent safety. Therefore, automatically the safety will come down and the number of kanbans will also come down. So, this is the first way to compute the safety stock the second way to compute the safety stock is to compute the mean and standard deviation of the lead times and use a service level to calculate the safety stock now let us go back temporarily to some inventory models.

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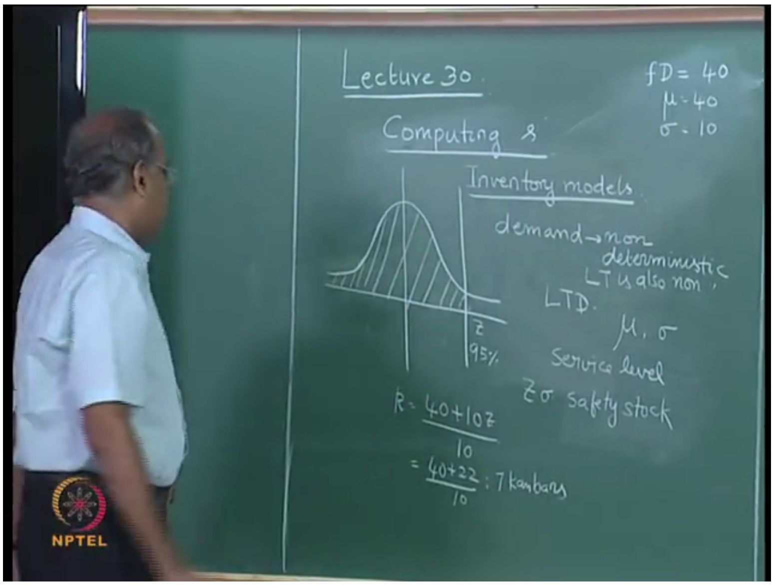

Now, when we looked at inventory models we always looked at safety stock, we looked at safety stock when the demand is not deterministic and the lead time is also non deterministic or we said the lead time demand shows variation and follows a certain distribution.

So, we said let mu and sigma be the mean and standard deviation of the lead time or lead time demand as the case may be depending on what we take and if mu and sigma represent the mean and standard deviation of the lead time demand, then for a normal distribution what we do is we assume a service level, we assume a certain service level and then based on the service level we compute the area under the normal distribution and then find out the z value. So, we will do this we will take the normal distribution and if we define a service level of say 90 percent or 95 percent we now go back to this normal distribution and find out a z such that area under the normal curve a standard normal curve is .95 standard normal curve area is given between 0 and 1. So, the full area is seen as one minus infinity to z the area is .95 find that z now z sigma is the safety stock. So, z sigma is the safety stock and D l plus s lead time demand plus safety stock is the reorder level

Now, z is computed by looking at the standard normal tables now we can do this. So, if we look at the second way that we just described in this particular thing now we will go back and say that now f D is the lead time demand that we talked about which is what we want to have that is the amount of inventory that we want to have now suppose now in this D is 160, f D becomes 25 percent of 160 which is 40.

So, this is 25 percent of 160 which is 40 which is the lead time demand that we are talking about now we say that mu is 40 and sigma the variation that we can think of let us say is some 10. Now if we want a 95 percent service level in the system then we go back and find out a z for which area under the normal curve is 95 percent and then z sigma is the safety. So, we go back to the standard normal curve and then we find out a particular z and whatever z we calculate 10 z will be 10 is the sigma. So, 10 z will be the safety stock. So, the actual kanban will be 40 plus 10 times z divided by 10, now z is to be calculated using the standard normal tables. So, that way we can find out. So, if we calculate z for a 95 percent then the k the number of kanbans in this case will be 40 plus 10 z divided by 10, this 10 is the sigma and this 10 is the container size.

So, if we assume that in this case for a 95 percent z will be approximately around let us say 1 22, roughly. So, this will become 40 plus 22 by 10 which will become 6.2 and it will give us 7 kanbans. Higher the service level higher will be the value of z. So, we say 90 percent service level it will roughly come to 1.645. So, 40 plus 16 would give us 6 kanbans. So, this is another way to find out just as we use ideas from probabilistic inventory we go we fall back on the z sigma idea go to the normal distribution and then z into sigma will give us the safety stock use the same safety stock here not defining it as a percentage of the demand see in this formula we said 10 percent of the demand is taken as safety stock.

Therefore here we wrote 10 percent of this is the lead time demand. So, 10 percent of this was written now instead of defining it as 10 percent of the lead time demand we now say the lead time demand follows a normal distribution with mean 40 and sigma 10 and what do we do for a 90 percent confidence level or for a 95 percent confidence level. So, we can use this idea from the normal distribution and then we can try and compute the safety stock which is used in the formula to find out the number of kanbans.

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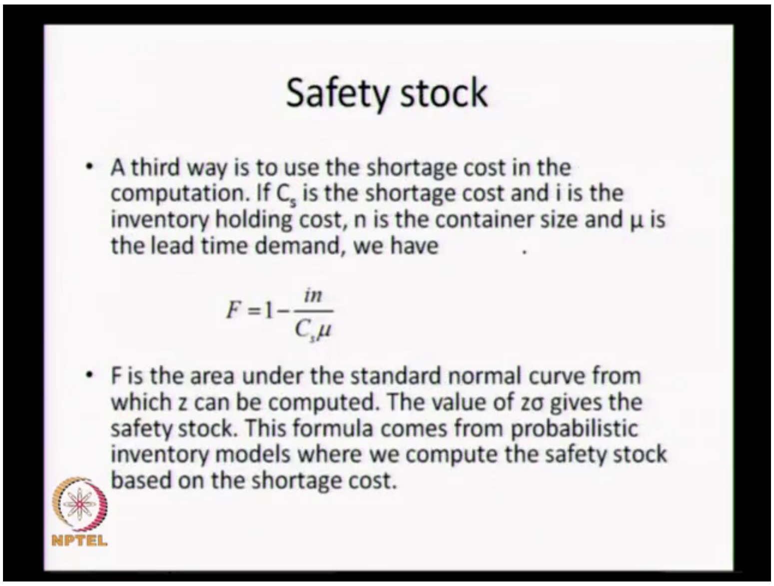

Third way is to use the shortage cost in the computation. So, if C s is the shortage cost and I is the inventory holding cost, then f is equal to 1 minus I n by C mu and the area under the standard normal curve from which z can be calculated.

So, let me explain this. So, in inventory control we have to find out this f D stock safety stock comes in because lead time demand follows a distribution with mu and sigma with mean mu and standard deviation sigma. So, a lead time demand is not known only expected values of lead time demands are known. So, some safety is built into the system what is the extent of safety is what we are looking at now one way to define is let us give a 10 percent safety which is what we did here the other way to define as I have is I want to maintain a service level of some 95 percent or whatever. So, I want to maintain a service level. So, when I maintain a service level automatically from assuming it is normal with the given mu and sigma one could go to the standard normal tables and for a 95 percent service level which is area under this curve being 95 percent which is a service level that we have define then we can go back and find out the z for 95 percent and use z sigma.

The third is to try and do some optimization now when we try and do some optimization in all standard inventory problems.

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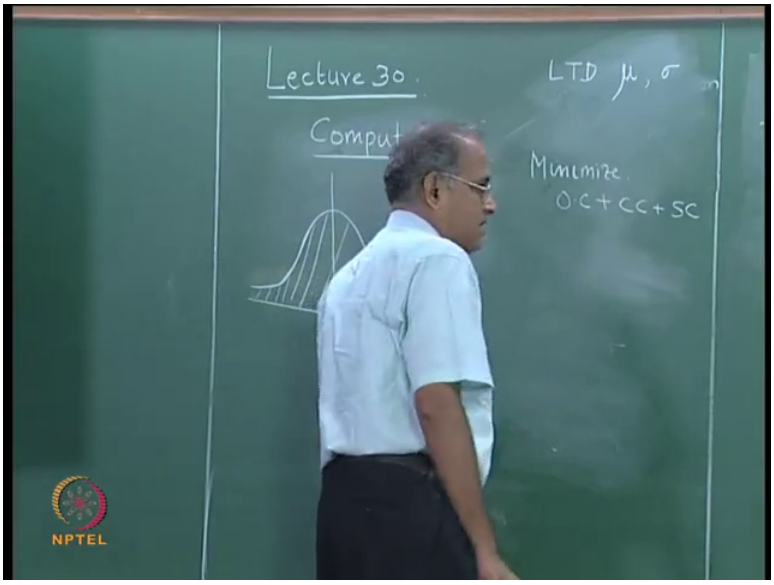

We will try and minimize some of order cost plus inventory carrying cost plus shortage cost. So, there are 3 things that can happen. So, there is going to be an order cost an inventory carrying cost and there is going to be some shortage cost. So, in the usual notation that we talk about the inventory problem will be like this.

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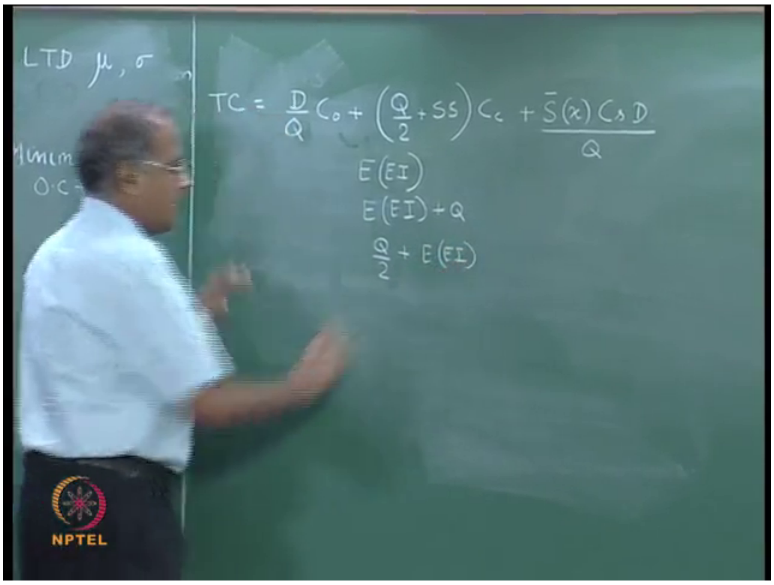

The total cost of in the inventory system the total cost will be the sum of order cost plus inventory holding cost plus shortage cost. So, in a normal inventory problem it will be D by Q into C naught which is the order cost because every time the order we place an order for Q items, plus Q by 2 plus safety stock into C c.

Now, when we have a safety stock built into the system. So, we will have Q by 2 plus safety stock into C c Q by Q is the quantity ordered Q by 2 is the average inventory in the system now this will be the safety stock that is brought into the system. So, safety stock is the excess stock over and above the average inventory that we have that we will bring into the system, plus the shortage cost for the formula for the shortage cost let us write that s bar of x into C s into D by Q this is the number of orders per year this multiplied by the order cost per order will give total order cost, here what we do is we can write this either as Q by 2 plus safety stock or we could go back and write this as there is an expected value of ending inventory in the system.

So, we have a we have something called expected value of ending inventory in every cycle. So, there is a every cycle ends with the inventory which is the ending inventory in every cycle. So, beginning inventory of every cycle will be ending inventory of every cycle plus Q because as at the end of the cycle we are going to get a q. So, average inventory in this will be Q by 2 plus expected value of ending inventory.

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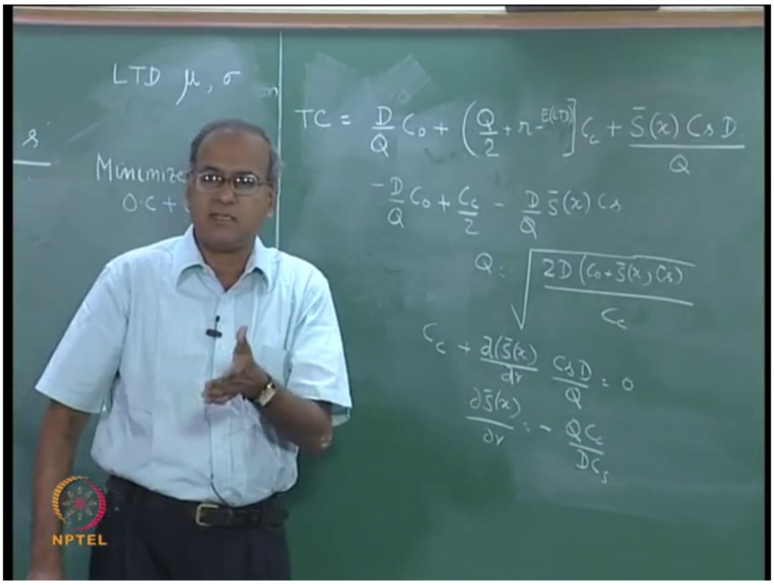

So Q by 2 plus expected value of ending inventory is the average inventory in this cycle. Ideally we would expect this expected value of ending inventory to be 0, this is the ideal situation; so Q plus expected value of this by 2.

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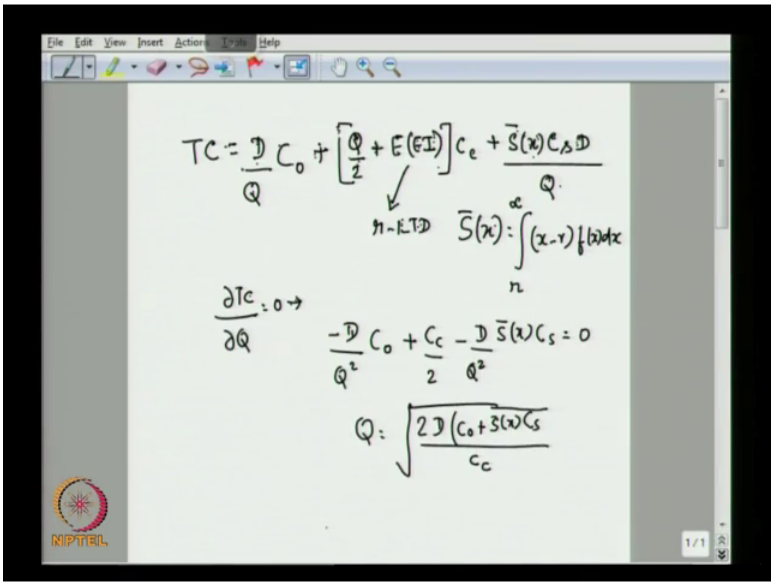

Now, total cost is given by TC is equal to D by Q into C naught plus Q by 2 plus expected value of ending inventory into C c plus s bar of x C s into D by Q .

Now, there are 3 terms the first term D by QC naught is for the order cost Q by 2 plus expected value of ending inventory into C c is for the inventory holding cost and s bar of x into C s D by Q is the expected shortage cost, C s is the expected shot is the shortage cost per unit short, s bar of x is the expected value of the shortage and D by Q is the number of cycles in a year. So, s bar of x which is the expected value of the shortage in a cycle is given by integral r to infinity x minus r f x D x now shortage occurs when the demand exceeds the reorder level. So, the limits of integration are from the reorder level r to infinity and x minus r f x is the expected shortage or the expected quantity by which the demand exceeds the reorder level.

Now, in order to we have to find 2 parameters Q and r. So, in order to find Q we differentiate this with respect to Q to get the expression for q. We can also write this expected ending inventory as r minus lead time demand or r minus expected value of lead time demand now in order to get the value of Q we partially differentiate this to get dou TC by dou Q equal to 0 gives minus D by Q square C naught plus C c by 2 minus D by Q square into s bar of x C s equal to 0 now this term Q by 2 into C c will give us C c by 2 on differentiation the rest of these terms are differentiated this way and from this we get the expression for Q is equal to root of 2 D into C naught plus s bar of x C s divided by C c .

Now, differentiating with respect to r we will get this is C c because differentiating with respect to r would give us a C c here plus will give this, D of s bar of x or dou s bar of x by dou r into C s D by Q is equal to 0. So, this would give us something like dou s bar of x by dou r is equal to minus Q C c by D C s. So, we get this term called minus Q C c by D C s that keeps coming here and then from this we could say that the expected shortage that will happen will be 1 minus Q C c by D C s. So, differentiating this is r minus f of x D x. So, this would give when we differentiate this we will give this is r to infinity. So, we measure this Q C c by D C s from here and treat this Q C c by D C s as a way to estimate the expected shortage. So, Q C c by D C s though we have a minus sign here through a computation.

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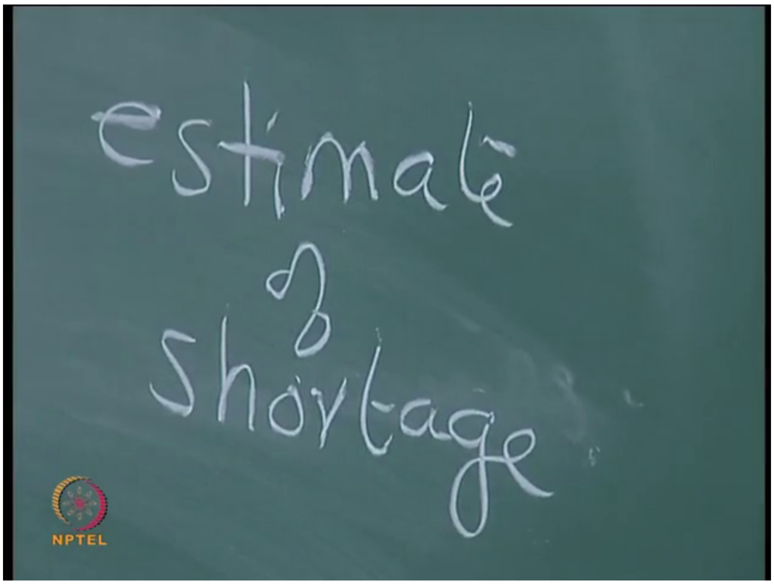

We will later show how this minus sign is taken care of, but Q C c by D C s would give us an estimate of shortage of shortage.

So, once we know the estimate of shortage or the probability of shortage 1 minus probability of shortage is the service level. So, we could compute the probability of shortage using this Q C c by D C s. We will explain this through a numerical illustration in the next lecture, and then show how we compute Q C c by D C s and then use it. Now the formula that is given in the slide is 1 minus I n by C s mu which is actually 1 minus Q C c by D C s. So, there is a relationship between Q C c by D C s and the shortage and service level which we will see in the next lecture.